With these illustrations we rest our case concerning the nonexistence of a single universally applicable time constant for characterizing transient light fields in extensive optical media. Perhaps, if a single time constant were demanded which could be pressed into use more often than all the other time constants discussed in the present chapter, then we might tentatively suggest T_α for consideration. For T_α appears quite often in the energy context and most critically in the radiance context of (10) of Sec. 5.7. Furthermore, T_α is based on the one inherent optical property (namely α) of optical media which is the most thoroughly documented and which is the most readily measured member of the basic trio α, σ, a.

Finally, we observe that all our preceding deliberations concerned unbounded media--or very extensive media in which their boundaries played a negligible role. For a discussion of the theory of time constants in bounded media in which the sensitivity of radiometer instruments also plays a role the reader may consult the papers in parts IV, V of [236]. These references are part of a set of five reports in which the main discussion centers on the study of the general *metric properties* of time dependent light fields. The theory of the time constant found in [236] is one of the several applications of the general metric theory developed in the series.

5.12 Global Approximations of General Radiance Fields

In this and the following section some of the theory of time-dependent n-ary radiant energy fields will be applied to two general problems of radiative transfer theory. In the present section attention will be directed to the problem of finding relatively simple approximations of time dependent and steady state radiance fields in optical media. In particular it will be shown how the n-ary radiant energy fields may be used to obtain approximations of the observable radiance field such that the approximations are *exact on a global level* over the given medium.

The precise meaning of this phrase will become clear during the course of the constructions of the approximations, to which we now turn. Unless specifically stated otherwise, all constructions will take place on a general optical medium X with arbitrary source conditions.

We begin with the observation that the operator formula

$$N^n = N^1 S^{n-1} \quad ,$$

based on the theory of Sec. 5.1, suggests the following simple approximation, where we write:

$$"N_g^n" \quad \text{for} \quad \frac{U^n}{U^1} N^1 \tag{1}$$

Here U^n, $N \geq 1$, is the n-ary radiant energy in X, and N^1 is the primary radiance function in X. N_g^n is called the *global approximation* of N^n for $n \geq 1$.

The reason for such a name and structure of N_g^n lies in the following observations. Note first that N_g^n has scattering order "dimensions" of n-ary radiance. Next, observe that the global approximation for N^n yields the estimate:

$$\frac{1}{v(x)}\frac{U^n(t)}{U^1(t)}\int_{\Xi} N^1(x,\xi,t)\ d\Omega(\xi)$$

for the radiant density function u in X. If we write "u_g^n" for this function, then we see that:

$$u_g^n = \frac{U^n}{U^1}u^1 \tag{2}$$

for $n \geq 1$. Finally:

$$\int_X u_g^n(x,t)\,dV(x) = \frac{U^n(t)}{U^1(t)}\int_X u^1(x,t)\,dV(x)$$

$$= \frac{U^n(t)}{U^1(t)} \cdot U^1(t)$$

$$= U^n(t)$$

This shows that the approximation N_g^n to N^n has the property:

$$U^n(t) = \int_X \frac{1}{v(x)}\left[\int_{\Xi} N_g^n(x,\xi,t)\ d\Omega(\xi)\right] dV(x) \tag{3}$$

In other words, N_g^n yields the same radiant energy content of X at each time t as does N^n, the actual n-ary radiance function on X. Thus N_g^n yields an exact prediction of approximation of N^n on an overall (or global) basis. The directional or local structure of N^n is approximated by that of N^1, a relatively easily computed function.

The global approximation of N^n may be used to obtain a global approximation of the directly observable radiance N by means of the natural solution representation of N_g^*, where we have written:

$$\text{"}N_g^*\text{"} \quad \text{for} \quad \sum_{j=1}^{\infty} N_g^j \tag{4}$$

For, by the definition of the N_g^j we have:

$$N_g^* = \sum_{j=1}^{\infty} N_g^j = \sum_{j=1}^{\infty} \frac{U^j}{U^1} N^1 = \frac{U^*}{U^1} N^1 \quad . \tag{5}$$

The requisite global approximation of N is obtained by writing

$$\text{"}N_g\text{" for } N^o + N_g^* \tag{6}$$

It follows that:

$$U(t) = \int_X \frac{1}{v(x)} \left[\int_\Xi N_g(x,\xi,t)\, d\Omega(\xi) \right] dV(x) \tag{7}$$

so that N_g indeed endows X with the same radiant energy content as N, the actual observable radiance function on X. The function N_g may then be used to assign to each x in X, and ξ in Ξ at time t the radiance:

$$\boxed{N_g(x,\xi,t) = N^o(x,\xi,t) + \frac{U^*(t)}{U^1(t)} N^1(x,\xi,t)} \tag{8}$$

where, in case standard growth conditions are in force in X, $U^*(t)$ (alias $U(t;s)$) and $U^1(t)$ are given by (14) of Sec. 5.9 and (12) of Sec. 5.10. In the steady state attained under standard growth conditions, (8) yields:

$$\boxed{N_g(x,\xi) = N^o(x,\xi) + \frac{1}{1-\rho} N^1(x,\xi)} \tag{9}$$

which is defined for $0 < \rho < 1$.

Global Approximations of Higher Order

The global approximation N_g^o in (1) above is but the lowest rung on an infinitely high ladder of global approximations of the radiance function in the medium X. We now formulate the global approximation to N of arbitrarily high order. Thus let us for every $n \geq 1$, write:

$$\text{"}N_g^n\text{" for } \frac{U^n}{U^{(k)}} N^{(k)}$$

Here we choose to use the same name "N_g^n" for the approximating function, and we have now written, *ad hoc:*

$$\text{"}N^{(k)}\text{"} \quad \text{for} \quad \sum_{j=1}^{k} N^j$$

and

$$\text{"}U^{(k)}\text{"} \quad \text{for} \quad \sum_{j=1}^{k} U^j$$

N_g^n is the *global approximation of the kth order of N^n*. It is easy to verify that N_g^n again is globally exact in the general sense of (3). Defining N_g as in (6) and N_g^* as in (4), now for the kth order context, by stopping the sums in (4) and (6) at j = k, it follows that:

$$\boxed{N_g^{(k)}(x,\xi,t) = N^o(x,\xi,t) + \frac{U^*(t)}{U^{(k)}(t)} N^{(k)}(x,\xi,t)} \qquad (10)$$

we call $N_g^{(k)}$ in (10) the *global approximation of the kth order of N.* $N_g^{(k)}$ is globally exact in the sense of (7), i.e., using $N_g^{(k)}$ in (7) will yield $U^{(k)}(t)$. Observe that this approximation also has the virtue of converging to N as $k \to \infty$. That is:

$$\lim_{k\to\infty} N_g^{(k)} = N \qquad (11)$$

This follows from (10) and the facts that:

$$\lim_{k\to\infty} U^{(k)}(t) = U^*(t) \qquad (12)$$

and that:

$$\lim_{k\to\infty} N^{(k)} = N^* \quad .$$

In this way we see that the global approximations to N have one additional property over the truncated solutions of Sec. 5.5, namely the global exactness property. The steady state limit version of (10) attained under standard growth conditions is:

$$\boxed{N_g^{(k)}(x,\xi) = N^o(x,\xi) + \frac{1}{1-\rho^k} N^{(k)}(x,\xi)} \qquad (13)$$

and which is defined for $k > 1$, and $0 < \rho < 1$. Under standard growth or decay conditions, one may use in (10) the expressions for $U^*(t)$ and $U^n(t)$, developed in Sec. 5.11, to generate useful approximations to time-dependent radiance fields. First or second order global approximations should suffice for many practical settings.

We note in passing that preliminary and informal numerical studies seem to indicate that the shapes (the directional structure) of N^n appear to be spherical (or very nearly so) when n is larger than some integer p which depends on the medium X and ρ. If this conjecture can be proved in general, (probably by means of the set up in 10.5) then an enormous advance in the practical utility of (13) can be made. This conjecture of the limiting shape of N^n as $n \to \infty$, bears a striking analog to the asymptotic radiance theorem studied elsewhere in this work (cf., e.g., Chapter 10). An important application would be to diffusion theory (see (78) of Sec. 6.6).

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5.13 Light Storage Phenomena in Natural Optical Media

The applications of the natural mode of solution of radiative transfer problems in optical media discussed in this chapter will now be concluded with a definition and discussion of the light-storage phenomena in such media.

Everyday Examples of Light Storage

Those who have looked out of a window of an airplane as it descended into a sunbathed cloud layer may recall the sudden transition to a brilliant ambient field of light, and how the sensation of brightness in every direction increased to dazzling proportions as the airplane descended further into the upper regions of the cloud. This phenomenon is but one of many common examples of the storage of light by the mechanism of scattering. One can also see evidence of light storage on overcast nights on the outskirts of large cities: the cloud layer hovering low over the city is deeply and extensively illuminated from the street and building lights below. Flashes of lightning in storm clouds can light up an extensive cloud layer from horizon to horizon even though the actual volume taken up by the network of electrical discharges is a minute fraction of the illuminated volume. Lighthouses on densely fogged nights pour a well-defined beam of light into a surrounding fog with the result that the beam and the lighthouse are imbedded in a field of scattered light which, under suitable conditions, may be observed by approaching mariners far sooner than the light of the revolving beam. As one descends into a lake or the ocean on a sunny day, there is a shallow region near the surface in which the radiance measurably increases with increasing depth for various horizontal and upward-looking lines of sight.

These examples illustrate the phenomenon of the storage of light in scattering media. The sense of the work "storage" is used in its everyday sense: the accumulation or building up of radiant energy in the scattering material that surrounds the source of the energy. If one were to quickly extinguish the light source, the stored light would not immediately disappear with the extinction of the source; rather the scattered