The classical radiative transfer setting entities are paired off with the abstract setting entities of the preceding theorem as follows:

	In Radiative Transfer Theory	In the Theorem
a)	Set $\mathcal{D}$ of all radiance functions on an optical medium X	$\mathcal{D}$
b)	The radiometric d, as in (2) or (2a)	d
c)	The operator L, as in (1)	L

We will make one final remark on the existence of the solution f of the general operator equation f = Lf. This is the observation that the solution f defined in (5) is independent of the initial function $f^{(o)}$ starting the chain of iterations $L^n f^{(o)}$. This fact becomes clear, at least logically, by noting the uniqueness property (ii) above. For if $f^{(o)}$ and $g^{(o)}$ are two distinct initial functions, then construction of their iteration sequences yields f and g such that property (i) holds for each.

5.15 Bibliographic Notes for Chapter 5

The natural mode of solution of the equation of transfer studied in this chapter, as noted in the introduction, plays a unique, fundamental role in radiative transfer theory. The formal power of the method and its intuitive simplicity cannot be overemphasized. For some historical notes on the natural mode of solution, see Secs. 26 and 42 of Ref. [251]. For recent modifications of the iterative concept of solutions of functional equations, especially for numerical purposes, see [171].

The development of the natural solution, as presented in Secs. 5.1 and 5.4, follows in the main that given in Ref. [251]. The canonical representation of primary radiance in (8) or (9) of Sec. 5.3 is occasionally referred to as "Seeliger's formula," and is to be conceptually distinguished from the more useful and accurate representation of $N_{\uparrow}^{*}$ given in (5) of Sec. 4.4. The only common feature of the two radiance representations is that they both fall within the purview of the basic canonical formula (4) of Sec. 4.7.

The discussion of the "optical ringing problem" in Secs. 5.7 and 5.8 is based on the natural-solution approach to the time-dependent radiative transfer problem, and is designed to be more precise than simple time-dependent *classical* diffusion theory (Sec. 6.6). The approach outlined in these sections is drawn from the results in Ref. [211]. A related approach to the optical ringing problem from the point of view of temporal metric spaces was tentatively explored in the series of reports [236]. Further approaches to time-dependent radiative transfer problem are possible via the higher-order diffusion equations. See Table 1 of Sec. 6.5. The truncated natural-solution inequalities in Sec. 5.7 are based on [239]. Further inequalities in this circle of ideas may be found in Ref. [67].

The material of Secs. 5.8 to 5.12 is drawn, with minor revisions, from Ref. [211]. The light storage discussions in Sec. 5.13 are based on Ref. [237]. The abstract overview of the natural mode of solution in Sec. 5.14 uses advanced concepts of functional analysis (in particular, the principle of contraction mappings) which may, e.g., be studied in Ref. [140].

In the opening remarks of Sec. 5.11, it was emphasized that the dimensionless forms of the equations describing n-ary radiant energy fields are shared by many natural processes, some quite distinct conceptually from the time-dependent evolution of radiant energy in optical media. For a brief exploration of such alternate processes governed by the same equations, see Chapter 14 of Ref. [39] and the footnotes in that chapter.

The analogies between radiative transfer phenomena and other transport phenomena discussed in Sec. 5.11 also can be pursued further, e.g., in [259] and [312].

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