unified set of analytical techniques for solving internal-source problems in general optical media.

6.8 Bibliographic Notes for Chapter 6

The discussions of Sec. 6.1 leading to (36) of that section are based on some elementary properties of complete orthonormal families of functions, which in turn find their rightful place in Hilbert space theory, or general vector space theory. For an exposition of these ideas, see, e.g., [104]. The isolation of the two properties, namely: the *finite recurrence property* of the orthonormal family and the *isotropy property* of the medium led to the finite forms (26) of the abstract harmonic equations in Sec. 6.2. This explicit delineation of the necessary properties to be held jointly by orthonormal families and optical media, which lead to the abstract harmonic equations (26) of Sec. 6.2, appears to be new.

The exposition of the classical spherical harmonic method in Sec. 6.3 is based on that of Refs. [175] and [314]. The solution procedures of the classical spherical harmonic equations for plane-parallel media in Sec. 6.4 are based on modern algebraic methods in differential equation theory, such as those in [47]. Some innovations in numerical procedures in the spherical harmonic method may be found in [323] and [325]. The manner of approach to diffusion theory in Sec. 6.5 is dictated by the specific needs and outlook of geophysical radiative transfer theory. The classification of diffusion processes in Sec. 6.5 is of course only partially complete; a systematic investigation of such classified processes appears to be of some interest to radiative transfer theory, and offers interesting physically based problems in partial differential equation theory.

The general solutions of the classical diffusion equations in the opening paragraphs of Sec. 6.6 are widely known, useful formulas for scalar irradiance. The various primary scattered flux source methods and those based on higher ordered scattered flux sources in the latter part of Sec. 6.6 offer some novelty in the otherwise quite thoroughly formed classical method of treatment of the diffusion of light through scattering media. Furthermore, the particular needs of hydrologic optics and meteorologic optics has caused some emphasis to be placed on the representation of the radiance distribution $N(x,\cdot)$ throughout diffusing media. This resulted in derivations of formulas for $N(x,\xi)$ in general diffusion contexts, such as (29) of Sec. 6.5; and (14) and (40) of Sec. 6.6, which do not appear to be too widely known.

The solutions of the exact diffusion equations in Sec. 6.7 for the case of infinite media are based on the work in [40]. This work also contains many useful tables and graphs of associated solutions. The theory of semi-infinite media with point sources is relatively unexplored. However, reference [88] forms a definitive beginning of such a theory, and the latter half of the discussions in Sec. 6.7 are based on the results of [88].

Further References

Further references beyond those mentioned above and which contain contributions to the classical theory of transport phenomena may be briefly mentioned here. First of all there is the early definitive work by Hopf [111] on mathematical problems of radiative transfer in media which are in thermodynamic equilibrium. This work contains the germ of the modern operator theoretical approach to transfer problems which is continued in [37] and [143], and more recently in [251]. Another early definitive work on classical radiative transfer theory is that of Chandrasekhar [43] which develops a minor variant of the spherical harmonic method of the kind formulated by Wick in [319]. Applications of the Chandrasekhar theory are made by Lenoble in [108], [155], [156]. By far the most significant contribution in [43] is that of the principles of invariance, which were discussed in general in Chapter 3 above and which will be considered further in Chapter 7 below. The reference [62] also contains much useful mathematical information which is applicable to practical radiative transfer contexts. A relatively recent survey of radiative transfer theory and classical and exact diffusion theory may be found in [288].

Some tabulated solutions of the equation of transfer are given in [53], [91], and [11]. Diffusion theory from the point of view of Monte Carlo techniques is explored in [41] and [176]. Some recent numerical solutions for light fields in homogeneous slabs (with isotropic scattering) which blend the spherical harmonic method and the technique of invariant imbedding are given in [15] and [16].

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