

## PREFACE

It is a relatively rare occurrence in applied mathematics that one encounters a method of solution of a given type of equation that is both effective numerically and rich in physical imagery. With the advent of the principles of invariance into radiative transfer theory, the equation of transfer underlying the theory received its natural solution companion. Together the principles of invariance and the equation of transfer form a combination which illustrates that rare occurrence alluded to above.

In the present work we use this combination to explore the transfer of radiant energy through general optical media (exemplified by the atmosphere and the sea) and develop numerically effective procedures of strong intuitive content. The Method of Groups is a case in point. It is summarized in Equations (6)-(9) of Sec. 7.11 and shows in outline how the complex problem of radiant energy scattered in a general three-dimensional medium (such as a cloud) may be reduced to an ostensible one-dimensional sweep method--the hallmark of the invariant imbedding idea.

Over the years this useful combination of an equation of transfer and the principles of invariance has been extended to other fields of physics. In linear hydrodynamics, e.g., the counterpart to the equation of transfer is the set of dynamic and continuity equations. Instead of radiance (upward and downward into the sea) we have water wave elevation and fluid volume flux over the surface of a fluid basin, such as the sea. The principles of invariance go over essentially unchanged into the hydrodynamic setting. Consequently, all of the visualizable physical notions of invariant imbedding are transferable intact to hydrodynamics, such as the transmittances and reflectances of bodies of water--canals, bays, oceans. Moreover, the numerical efficiency of the imbedding technique is once again realized in this new setting. Work in this direction has proceeded far enough to show the thoroughgoing analogy between radiative transfer of light in optical media and the linear transport of water waves in natural bodies of water.<sup>1-5</sup>

The development of the invariant imbedding idea continues in still other fields and may be pursued in a recently compiled bibliography.<sup>6</sup>

The work in this volume was essentially done in the period 1964-1965 while I was with the Visibility Laboratory at Scripps Institution of Oceanography, and has been essentially unchanged in its conversion to manuscript form. My recent application of invariant imbedding to linear hydrodynamics has served to check the correctness of the theory below, and to reinforce my confidence in its universal applicability to all linear transport phenomena including light, ocean wave, electromagnetic or acoustic fields.

The final manuscript was typed by Ms. Judith Marshall.

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