CHAPTER 7

Invariant Imbedding Techniques for Light Fields

7.0 Introduction

We return in this chapter to the general circle of ideas introduced in Chapter 3, our purpose being to give detailed derivations of functional relations holding among the various interaction operators introduced there and discussions of how those operators can be evaluated in practice. For expository reasons we shall at first limit the discussions for the most part to the case of light fields in isotropic plane-parallel media. However, the techniques displayed are all extendable in principle to light fields in arbitrarily shaped anisotropic media. By carrying out the present program we not only add to the store of solution techniques for light fields discussed in Chapters 4-6, but illustrate within the domain of radiative transfer an important procedure of modern theoretical physics, the invariant imbedding solution technique. This technique gives rise to functional equations governing various physical processes by means of certain general group-theoretic and limit-theoretic arguments. These functional equation representatives of the physical processes give insight into the processes and occasionally result in useful numerical methods of determination of the processes. Some of these methods will be illustrated in this chapter.

From the great number of results on functional relations for various radiative transfer operators obtained in recent years by means of invariant imbedding techniques, we select the following for exposition in the present chapter: first, the derivations of the differential equations governing the reflectance and transmittance operators R and T for plane-parallel media. The steady state version of the derivation is given in Section 7.1, the time-dependent version is given in Section 7.2. A particularly interesting feature of these derivations is the statement of the local forms of the principles of invariance and their conceptual relation to the usual (global) forms of the principles of invariance. In Sections 7.3-7.5 it is shown how new and possibly useful functional relations can be discovered for the various interaction operators by treating the operators as algebraic entities and the equations in which they appear as algebraic statements which are occasionally subject to simple limit arguments. As a result of these heuristic manipulations three novel means of determining light fields in natural optical media, which occur in Sections 7.4-7.5, are selected for further study in Sections

7.6-7.8. An example of an actual numerical computation of the R and T operators based on the functional relations of Section 7.1 is given in Section 7.9 for the case of homogeneous source-free plane-parallel media with isotropic scattering. This numerical method is generalized in Section 7.10. In Section 7.11 the preceding results are consolidated and generalized. Section 7.12 is concerned with the conditions of homogeneity and isotropy and related ideas, which will help simplify theoretical and numerical work and help classify optical media in general. Section 7.13 develops some deep connections among the various standard and invariant imbedding operators within media with internal sources. Finally, in Section 7.14, it is observed how the Laplace and Fourier transform techniques, which have proved so useful in the classical formulation of the transport phenomena, can be combined with the invariant imbedding approach to simplify the functional relations of the latter approach and to encourage their applications to time-dependent problems, point source problems, and other transport problems which ordinarily involve higher numbers of variables.

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7.1 Differential Equations Governing the Steady State, R and T Operators

In Sections 3.6 and 3.7 we saw how the R and T operators of plane-parallel (and other) media were used in both theory and practice to determine light fields in natural optical media. In this section we show how the four R and T operators generally associated with stratified plane-parallel media may be determined from knowledge of the volume scattering and volume attenuation functions within the medium. This will be done by deriving the differential equations governing the operators as a function of the thickness of the medium. Thus, if we know the R and T operators for a given layer of material the differential equation will show how the operators change by addition of a very thin layer of the material to the given layer. By letting the given layer grow continuously from some given thickness, we will therefore know how its R and T operators evolve from their given values, and how they may be computed in both theory and practice. We turn now to the details of the derivations.

Local Forms of the Principles of Invariance

We begin the derivations by casting the equation of transfer for a stratified plane-parallel medium into a pair of equations which are strongly reminiscent of the two main principles of invariance for such media (Ex. 3, Section 3.7); the main difference being the presence of derivatives of N in the new equations. Thus under the assumption that all functions (radiance distributions and optical properties) depend only on depth y in the medium (cf. Fig. 7.1) Equation (3) of Sec. 3.15 becomes: