$$\lim_{s\to\infty} \Psi_{-+}(s,s+d:0,\infty) = R_\infty e^{-kd}\left[1 + \frac{\rho R_\infty}{2k}\right] \tag{134}$$

$$\lim_{s\to\infty} \Psi_{--}(s,s+d:0,\infty) = e^{-kd}\left[1 + \frac{\rho R_\infty}{2k}\right] \tag{135}$$

An unexpected dividend accrues from the preceding array of Ψ-factor relations. Observe first that (128)-(131) agree with our intuitive ideas that the relation between the source irradiance $H^o(s,+)$ and the response field $H(s,+)$, should be the same as that between $H^o(s,-)$ and $H(s,-)$ when boundaries are far away from level s (because the medium is optically symmetric about very deep levels). By the same intuitive expectations, Ψ_{+-} and Ψ_{-+} of (126) and (130) should be numerically equal. Apparently, this does not seem to be the case. However, if we rely on the correctness of our principles and algebraic manipulations, to yield up (129) and (130) then on the basis of our intuition we are led to conclude that:

$$\frac{\rho}{2k} = R_\infty\left[1 + \frac{\rho R_\infty}{2k}\right] ,$$

or equivalently that:

$$\boxed{\frac{\rho}{2k} = \frac{R_\infty}{1 - R_\infty^2}} \tag{136}$$

where ρ is the local reflectance (i.e., backscattering) factor for the one-D theory and k and R_∞ are respectively the decay rate and reflectance factor associated with irradiance fields in $X(0,\infty)$. It follows that all the preceding results (128)-(131) may be characterized in terms of R_∞ only. The reader may establish (136) independently of the preceding argument by using the connections (105) above, with (32) of Sec. 7.3, which holds in the irradiance context also. Still further connections between k, ρ, R_∞ and related concepts are available in Chapters 9 and 10.

8.8 A Model for Vector Irradiance Fields

The purpose of this section is to apply the vector theory of the irradiance field to an important class of scattering-absorbing optical media, namely the class of natural hydrosols consisting, e.g., of oceans, harbors, and lakes. The application is of practical value in that it yields explicit expressions for the depth-dependence of the irradiance vector in terms of its components at the surface and certain of the optical properties of these media. Furthermore, the discussion presents particularly simple interpretations of the quasi-potential and related functions, arising in vector analysis and which are pertinent to the description of natural light fields. These vector interpretations emerge naturally from the geometry and physics of the present application, and will be given as the discussion proceeds. In this way we add to the evidence that the formalism of the "photic field" as developed by Moon, Spencer, and others (Refs. [187], [188]) is of more than academic interest, and in fact provides an elegant tool for the study of the light vector **H**(s) in the practical settings encountered in the study of hydrologic optics.

While the practical context of the present discussion is limited specifically to that of natural hydrosols, the mathematical arguments apply equally well to any arbitrary plane-parallel scattering-absorbing medium in which the light vector possesses a quasi-potential. The radiometric prerequisites for the present discussion are given in (2)-(16) of Sec. 2.8. A useful text on vector analysis is Ref. [30].

The Quasi-Irrotational Light Field in Natural Waters

We fix the stage of the present discussion by adopting a stratified plane-parallel medium X(0,b) with stratified light field. The present discussion makes use of the concept of a quasi-irrotational light field, i.e., a light field in which at each depth z of a natural hydrosol, the irradiance vector satisifes the condition:

$$\mathbf{H}(z) \cdot \left[\nabla \times \mathbf{H}(z) \right] = 0 \tag{1}$$

In general, **H**, the *vector-irradiance function*, (or *light field*), is defined at each point x (which stands for the ordered triple (x,y,z)) of an optical medium by writing:

$$\text{"}\mathbf{H}(x)\text{"} \quad \text{for} \quad \int_{\Xi} \xi N(x,\xi)\, d\Omega(\xi) \quad , \tag{2}$$

where Ξ is the unit sphere (the collection of all unit vectors) in euclidean three-space E_3, and $N(x,\cdot)$ is the radiance distribution at point x (see Sec. 2.8).

As will be shown in (12) below, the justification of the use of the relation (1) rests on the following two vectorial versions of well-known experimental facts about the spatial and directional distribution of light in natural hydrosols:

(i) For every fixed $z \geq 0$ in X(0,b), **H**(x,y,z) is independent of x and y .

(ii) For every fixed pair (x,y), **H**(x,y,z) lies in a fixed vertical plane for all $z \geq 0$.

Of course, some variations of **H** on horizontal planes, and some oscillations of the vertical plane containing **H** do occur in all natural hydrosols. However, properties (i) and (ii) summarize the two most readily apparent permanent gross features of the light field in natural waters, on which it is possible to develop a mathematical theory of the light vector **H**(x) .

Interpretations of the Integrating Factor

Since our interests lie principally in the physical and geometrical aspects of natural light fields, it would be instructive to develop some physical interpretations of (1) and concepts immediately related to it. This we now do.

The general theory of vector fields asserts that to each quasi-irrotational light field one may associate two real-valued functions Φ and ζ, defined on the appropriate subset of $X(0,z_1)$ representing the optical medium. These functions have the property that:

$$\mathbf{H}(x) = \frac{1}{\zeta(x)} \nabla \Phi(x) \quad . \tag{3}$$

Φ is the *quasi-potential function*, and ζ is the *integrating factor*, unique to within a multiplicative constant, associated with Φ. Equation (3) is the necessary and sufficient condition that:

$$\mathbf{H}(x) \cdot [\nabla \times \mathbf{H}(x)] = 0$$

at each x of the medium. (See, e.g., Sec. 105, Ref. [30].)

In the present context the function ζ has particularly simple and interesting geometrical and physical interpretations. We begin with the geometric interpretation.

Figure 8.10 defines a terrestrially based coordinate system usually adopted for the discussion of the light fields in natural hydrosols (re: Sec. 2.4). The fixed plane referred to in property (ii) is taken as the xz-plane, and thus lies in the plane of the figure. The standard unit vectors **i** and **k** are positioned as shown. The unit vector **j** along the posituve y-axis is normal to the plane of the figure and directed away from the reader.

Consider an arbitrary rectangular path ABCD in the xz-plane such that its sides are parallel to the coordinate axes. According to (3) and properties (i) and (ii) of the light field in natural hydrosols, it follows that:

$$\int_{ABCD} \zeta(x)\mathbf{H}(x) \cdot d\mathbf{s} = 0 \quad ,$$

so that:

$$\int_{AB} \zeta(x)\mathbf{H}(z_1) \cdot ds = \int_{DC} \zeta(x)\mathbf{H}(z_2) \cdot d\mathbf{s}$$

The condition (i) suggests that ζ can be independent of x and y. By (13) of Sec. 2.8:

$$\bar{H}(z,\mathbf{i}) = \mathbf{H}(z) \cdot \mathbf{i}$$

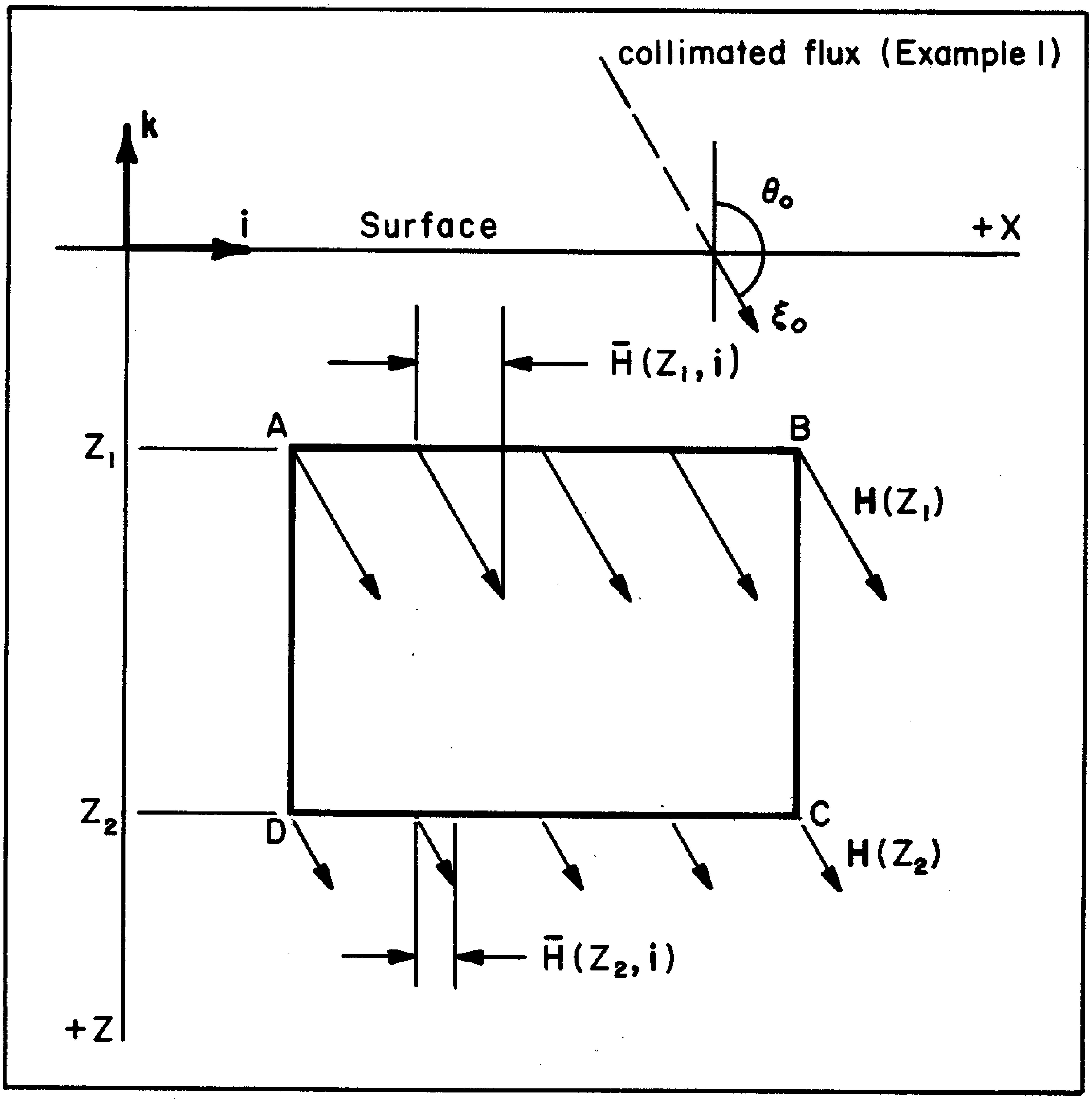

FIG. 8.10 How to visualize a quasi-irrotational irradiance vector field.

so that the integral equality above implies:

$$\zeta(z_1)\bar{H}(z_1,\mathbf{i}) = \zeta(z_2)\bar{H}(z_2,\mathbf{i}) \quad . \tag{4}$$

From this and the fact that ζ is determined only to within a multiplicative constant, we can set $\zeta(0) = 1$ and so:

$$\zeta(z)\bar{H}(z,\mathbf{i}) = \bar{H}(0,\mathbf{i}) \quad , \tag{5}$$

for all z in $[0,b]$. Thus ζ may be selected as a dimensionless quantity which stretches the horizontal component $\bar{H}(z,\mathbf{i})$ to $\bar{H}(0,\mathbf{i})$ at every depth z in $X(0,b)$.

Next we consider the physical interpretation of $\zeta(z)$. The invariance with depth of the product:

$$\zeta(z)\bar{H}(z,\mathbf{i})$$

shows that the depth dependence of $\zeta(z)$ is such that its logarithmic derivative is equal, to within an algebraic sign, to the logarithmic derivative of the net horizontal irradiance $\overline{H}(z,\mathbf{i})$. That is:

$$\frac{1}{\zeta(z)} \cdot \frac{d\zeta(z)}{dz} = \frac{-1}{\overline{H}(z,\mathbf{i})} \cdot \frac{d\overline{H}(z,\mathbf{i})}{dz} \quad . \tag{6}$$

This logarithmic derivative of $\overline{H}(z,\mathbf{i})$ will be denoted by "$\overline{K}(z,\mathbf{i})$". Now the logarithmic derivative $\overline{K}(z,\mathbf{i})$ is but one member of an important family of apparent optical properties used in modern hydrologic optics, as we shall see in Chapter 9. This family of optical properties includes such well-known quantities as $k(z)$, where:

$$k(z) = -\frac{1}{h(z)} \frac{dh(z)}{dz} \tag{7}$$

and where $h(z)$ is scalar irradiance of depth z :

$$h(z) = \int_{\Xi} N(z,\xi)\,d\Omega(\xi) \tag{8}$$

Thus, in the terminology of Chapter 9, the logarithmic derivative of ζ is none other than the *k-function* for the net horizontal irradiance in the $\mathbf{i}$-direction. According to (6) we may represent $\zeta(z)$ as:

$$\zeta(z) = e^{C(z)} \tag{9}$$

where we have written:

$$\text{"}C(z)\text{"} \quad \text{for} \quad \int_0^z \overline{K}(z',\mathbf{i})\,dz' \tag{10}$$

The Curl and Divergence of the Submarine Light Field

The vectorial concepts of curl and divergence have been applied very often in hydrodynamic theory, and other field theories. However, there is also a useful role for these concepts in the description of light fields in natural hydrosols. Under the present assumptions (i) and (ii) about the light field, and with the adopted coordinate system, the curl of $\mathbf{H}$ may be verified to be of the form:

$$\nabla\times\mathbf{H}(z) = -\,\mathbf{j}\,\frac{d\overline{H}(z,\mathbf{i})}{dz}$$

so that:

$$\boxed{\nabla\times\mathbf{H}(z) = \mathbf{j}\overline{K}(z,\mathbf{i})\overline{H}(z,\mathbf{i})} \tag{11}$$

It follows from this and property (ii) that:

$$\boxed{\mathbf{H}(z) \cdot [\nabla\times\mathbf{H}(z)] = 0} \tag{12}$$

The derivation of the divergence relation for the light field in (source-free) scattering-absorbing media will require the use of the equation of transfer for radiance:

$$\xi \cdot \nabla N(z,\xi) = -\,\alpha(x)N(x,\xi) + N_*(z,\xi) \quad , \tag{13}$$

where the path function N_* is given by:

$$N_*(x,\xi) = \int_{\Xi} N(x,\xi')\sigma(x;\xi';\xi)\,d\Omega(\xi') \tag{14}$$

Here σ and α are, respectively, the volume scattering function and the volume attentuation function.

Returning to (13) and integrating each side over Ξ , we have:

$$\nabla \cdot \mathbf{H}(x) = -\,\alpha(x)h(x) + s(x)h(x) \quad ,$$

which reduces to the required divergence relation:

$$\boxed{\nabla \cdot \mathbf{H}(x) = -\,a(x)h(x)} \tag{15}$$

by using the facts that:

$$s(x) = \int_{\Xi} \sigma(x;\xi;\xi')\,d\Omega(\xi') \tag{16}$$

and that:

$$\alpha(x) = a(x) + s(x) \quad , \tag{17}$$

and which were formally introduced in (3) and (4) of Sec. 4.2.

For the present geometry (15) reduces to:

$$\boxed{\frac{d\bar{H}(z,\mathbf{k})}{dz} = a(z)h(z)} \tag{18}$$

where

$$\bar{H}(z,\mathbf{k}) = \mathbf{H}(z) \cdot \mathbf{k} \tag{19}$$

is the vertical component (the net upward irradiance) of the light vector at depth x . (Observe that the derivatives of $\nabla \cdot \mathbf{H}$ are along the directions of **i**, **j**, **k** , and recall that z is measured positive in the direction $-\mathbf{k}$.) If the medium

X(0,b) has sources, then the term $h_\eta(z)$ is added to the right side of (18). The reader will find it instructive to return to (62) of Sec. 8.4 and view it in the light of (15). Furthermore, it should now be clear that by adding together the equations of (19) of Sec. 8.3, we obtain (18).

General Representation of the Submarine Light Field

The starting point of the present derivation is taken as theorem 9 of Ref. [187] which, applied to the present concepts of radiative transfer in scattering-absorbing media, states that if:

$$\nabla \cdot \mathbf{H}(z) = - a(z)h(z)$$

and:

$$\mathbf{H}(z) \cdot [\nabla \times \mathbf{H}(z)] = 0$$

then a quasi-potential Φ and integrating factor ζ exist such that:

$$\nabla^2 \Phi(x,y,z) = \frac{1}{\zeta(z)} \left[- a(z)h(z) + \nabla \left(\frac{1}{\zeta(z)} \right) \cdot \nabla \Phi(x,y,z) \right] \qquad (20)$$

Furthermore, it follows from properties (i) and (ii) and the preceding equation that Φ can be at most linear in x and y. We can use this fact along with (3):

$$\mathbf{H}(z) = \frac{1}{\zeta(z)} \left[\mathbf{i} \frac{\partial \Phi}{\partial x} + \mathbf{j} \frac{\partial \Phi}{\partial y} + \mathbf{k} \frac{\partial \Phi}{\partial z} \right] \qquad (21)$$

to deduce the requisite representation of $\mathbf{H}(z)$. Now, since Φ is at most linear in x:

$$\frac{\partial \Phi}{\partial x} = A \quad , \qquad (22)$$

where A is a constant. Furthermore, since Φ is at most linear in y and by virtue of the present coordinate system we can set:

$$\frac{\partial \Phi}{\partial y} = 0 \qquad (23)$$

Hence Φ may be represented in the present context as follows:

$$\Phi(x,y,z) = Ax + f(z)$$

According to (21) it remains to determine A and $f'(z)$ ($= df(z)/dz$). Toward this end, equation (20) may be written:

$$f''(z) - \overline{K}(z,\mathbf{i}) - \zeta(z)a(z)h(z) = 0$$

The integrating factor for this differential equation is clearly

$$\frac{1}{\zeta(z)} = e^{-C(z)} \quad ,$$

so that:

$$(f'(z)/\zeta(z))' = a(z)h(z) \quad ,$$

and:

$$f'(z)/\zeta(z) = f'(0) + \int_0^z a(z')h(z')dz' \quad .$$

Finally, from (21) it follows that:

$$\boxed{\mathbf{H}(z) = \mathbf{i}\overline{H}(0,\mathbf{i})e^{-C(z)} + \mathbf{k}\left[\overline{H}(0,\mathbf{k}) + \int_0^z a(z')h(z')dz'\right]} \quad (24)$$

which is the desired general representation of the vector irradiance function **H** .

Example 1: The case of Isotropic Scattering

For the remaining portion of this section we will assume that the medium X(0,b) is homogeneous, i.e., that α (and hence σ , s, and a) is independent of depth z . The present example will be concerned with an illustration of the particular form that **H**(z) takes in a medium that scatters isotropically and which is irradiated at the upper boundary by collimated flux incident at an angle θ_o = - arc cos μ_o , $\phi_o = 0$, and where ϕ_o is an azimuth angle measured in a horizontal plane from the positive x-axis. The number μ_o is defined as in (70) of Sec. 8.5.

Now it is easy to verify that the diffuse component of the light field (i.e., that part consisting of all radiant flux scattered one or more times) is symmetrical about the z-axis. Hence the net horizontal irradiance receives no contribution from the diffuse light field. Therefore:

$$\overline{H}(z,\mathbf{i}) = \overline{H}(0,\mathbf{i})e^{-\alpha z/\mu_o} \quad (25)$$

so that $\overline{K}(z,\mathbf{i})$ in this case is represented by:

$$\overline{K}(z,\mathbf{i}) = \alpha/\mu_o \quad (26)$$

and C(z) in the general theory above reduces to:

$$C(z) = \alpha z/\mu_o \quad (27)$$

Example 2: Asymptotic Form of the Light Field

In optically infinitely deep media (i.e., $b = \infty$ in $X(0,b)$) the values $k(z)$ of the function k defined in (7) rapidly approach, with increasing z, a fixed magnitude k which is independent of the external lighting conditions and which depends only on the inherent optical properties of the medium. This and related facts we shall explore in some detail in Chapter 10. In view of this fact it is permissible, for most engineering calculations, to assume that there is a depth $z_o \geq 0$ below which $k(z) = k$. From the divergence relation (18) we see that in general:

$$\bar{H}(z_2,\mathbf{k}) - \bar{H}(z_1,\mathbf{k}) = \int_{z_1}^{z_2} a(z)h(z)\,dz \tag{28}$$

so that in particular:

$$\bar{H}(z,\mathbf{k}) - \bar{H}(0,\mathbf{k}) = \int_0^z a(z')h(z')dz'$$

Furthermore, in the present case:

$$\begin{aligned}\bar{H}(z,\mathbf{k}) - \bar{H}(z_o,\mathbf{k}) &= a\int_{z_o}^{z} h(z')dz' = ah(z_o)\int_{z_o}^{z} e^{-k(z'-z_o)}dz' \\ &= \frac{-ah(z_o)e^{kz_o}}{k}\left[e^{-kz} - e^{-kz_o}\right] \\ &= \frac{ah(z_o)}{k}\left[1 - e^{-k(z-z_o)}\right]\end{aligned}$$

It follows that the $\mathbf{k}$-component of $\mathbf{H}(z)$ in (24) may be written:

$$\begin{aligned}\bar{H}(z,\mathbf{k}) &= \bar{H}(0,\mathbf{k}) + \int_0^z a(z')h(z')dz' \\ &= \bar{H}(z_o,\mathbf{k}) + \int_{z_o}^{z} a(z')h(z')dz' \\ &= \bar{H}(z_o,\mathbf{k}) + \frac{ah(z_o)}{k}\left[1 - e^{-k(z-z_o)}\right]\end{aligned}$$

Now since $\bar{H}(z,\mathbf{k}) \to 0$ as $z \to \infty$ (see, e.g., the two-D models in Secs. 8.5 and 8.6), it follows that we may set:

$$\bar{H}(z_o,\mathbf{k}) = -\left(\frac{a}{k}\right) h(z_o) \quad ,$$

so that (24) reduces to:

$$\mathbf{H}(z) = \mathbf{i}\bar{H}(z_o,\mathbf{i})e^{-C(z-z_o)} + \mathbf{k}\bar{H}(z_o,\mathbf{k})e^{-k(z-z_o)} , \quad (29)$$

for $z \geq z_o$. A further simplification is effected if we can find a suitable approximation for the function C . Thus observe that for $z \geq z_o$, the diffuse component of the light field is essentially symmetrical about the z-axis (Chapter 10) so that, as in the isotropic case (27):

$$\bar{K}(z,\mathbf{i}) = D^o\alpha$$

where we have

$$D^o = \frac{1}{\mu_o}$$

for clear sunny skies with sun at $\theta_o = -\arccos \mu_o$ from the zenith, or for some fixed D^o , with

$$1 \leq D^o \leq 2$$

for overcast days (see (2) of Sec. 8.5). With these assumptions, (24) takes the particularly simple approximate form:

$$\boxed{\mathbf{H}(z) = \mathbf{i}\bar{H}(z_o,\mathbf{i})e^{-D^o\alpha(z-z_o)} + \mathbf{k}\bar{H}(z_o,\mathbf{k})e^{-k(z-z_o)}} , \quad (30)$$

where D^o may take any of the above special values. For most engineering applications it is permissible to take $z_o = 0$ in (29) or (30).

We conclude this example by making a few observations on the limiting directions of **H** as $z \to \infty$. First, if $s \neq 0$, then $k < \alpha$, (see, e.g., (9) of Sec. 6.6). If, in addition, we also have $a \neq 0$ then from (30) and the fact that $\bar{H}(a,\mathbf{k})<0$, it is clear that:

$$\lim_{z\to\infty} \frac{\mathbf{H}(z)}{|\mathbf{H}(z)|} = -\mathbf{k} \quad .$$

If, on the other hand, we have $a = 0$, then $\bar{H}(z,\mathbf{k}) = 0$ and:

$$\frac{\mathbf{H}(z)}{|\mathbf{H}(z)|} = \pm\,\mathbf{i}$$

for all z ; the direction being that of $\bar{H}(0,\mathbf{i})$. Finally, if $s = 0$, then the problem of the explicit determination of **H**(z) for all z reduces to a relatively trivial (although sometimes tedious) calculation. In this case the limiting direction of **H** depends in a simple way only on the external lighting conditions. If $N^o(0,\xi)$ represents the incident radiance distribution at $z = 0$, suppose $|\xi_o \cdot \mathbf{k}|$ is the largest value for which $N(0,\xi_o) \neq 0$. Then the limiting direction of **H**(z), as $z \to \infty$, is along the line defined by ξ_o .

Global Properties of the Irradiance Field

The curl and divergence of the irradiance field $\mathbf{H}(x)$ show how the field behaves in the neighborhood of a point. In other words, the curl and divergence of $\mathbf{H}(x)$ are *local properties* of $\mathbf{H}(x)$. An interesting and important global property associated with the irradiance field comes from an application of the divergence theorem to (15) and we shall now derive that property.

Let X be an arbitrary connected, bounded homogeneous subset of X(0,b), with boundary surface S and with steady light field. Then on the one hand from (15):

$$\begin{aligned}\int_X \nabla \cdot \mathbf{H}(x)\,dV(x) &= -\int_X ah(x)\,dV(x)\\ &= -av\int_X u(x)\,dV(x)\\ &= -avU(X) \qquad . \end{aligned} \tag{31}$$

The latter two equalities rest on (4) and (12) of Sec. 2.7. On the other hand if $\mathbf{n}(x)$ is the unit inward normal to S at x, then by the divergence theorem, we have:

$$\int_X \nabla \cdot \mathbf{H}(x)\,dV(x) = -\int_S \mathbf{H}(x) \cdot \mathbf{n}(x)\,dA(x) \tag{32}$$

and we shall denote the integral on the right side by "$\overline{P}(S,-)$" which thereby represents the *net inward radiant flux across S* (cf. (8) and (9) of Sec. 2.8). Combining (31) and (32) we have:

$$\boxed{\overline{P}(S,-) = avU(X)} \tag{33}$$

This equation shows how the net inward flux $\overline{P}(S,-)$ across the boundary of S is related to the energy content U(X) of X and the volume absorption coefficient a of X. Equation (33) may have some practical interest in laboratory procedures of determining the volume absorption coefficient a. The radiant energy content U(X) of X may be obtained by systematically probing X with an instrument which measures scalar irradiance h(x). By numerically integrating the measured values h(x) throughout X, the term vU(X) may be obtained. Further, the term $\overline{P}(S,-)$ may be obtained by traversing the boundary S of X with a subtracting janus plate (re: Sec. 2.8) to find $\overline{H}(x,\mathbf{n}(x))$, and then integrating the measured values. It is clear that this method would be independent of the directional structure of the light field within X. This fact may be used in laboratory setups to prearrange the light field so as to require a minimal amount of measuring throughout X. If this can be achieved, novel and simple means of determining the volume absorption coefficient will thereby be attained.

<< Table of Contents << Title Page >> Next Page >>

8.9 Canonical Representation of Irradiance Fields

We close this chapter on models for irradiance fields with a derivation which parallels the canonical representation of the radiance field given in (5) of Sec. 4.5. It is possible to derive the requisite relation so as to be a proper generalization of (5) of Sec. 4.5, and we shall now follow such a course.

Let X be an arbitrary optical medium. Let x be an arbitrary point of X and to x associate a direction $\mathbf{n}(x)$ and a set $\Xi_o(x)$ of directions. Let us write:

$$\text{"}\mathbf{H}(x,\Xi_o(x))\text{"} \quad \text{for} \quad \int_{\Xi_o(x)} N(x,\xi)\,\xi\,d\Omega(\xi) \quad . \tag{34}$$

This is a generalization of the irradiance vector $\mathbf{H}(x)$. The latter is obtained by requiring $\Xi_o(x) = \Xi$ (cf. (2) of Sec. 2.8, and also (4) of Sec. 2.4 for the numerical instance of (34); and (41) of Sec. 8.6 for an alternate version of (34)). Further, let us write:

$$\text{"}H(x,\mathbf{n}(x),\Xi_o(x))\text{"} \quad \text{or} \quad \text{"}H(x,\mathbf{n},\Xi_o)\text{"} \quad \text{for} \quad \mathbf{n}(x)\cdot\mathbf{H}(x,\Xi_o(x)) \tag{35}$$

It is clear that $H(x,\mathbf{n},\Xi_o)$ is the quantity measured by a subtracting janus plate (Sec. 2.8) whose collecting surfaces are exposed to the set $\Xi_o(x)$ of directions and whose pointer is directed along $\mathbf{n}$ (cf. Figs. 8.11 and 2.21). Associated with $H(x,\mathbf{n},\Xi_o)$ is the scalar irradiance $h(x,\Xi_o)$, where we have written:

$$\text{"}h(x,\Xi_o)\text{"} \quad \text{for} \quad \int_{\Xi_o} N(x,\xi)\,d\Omega(\xi) \tag{36}$$

$h(x,\Xi_o)$ is measured in practice by a spherical irradiance collector exposed to the direction set Ξ_o.

We pause to observe that by suitable choice of Ξ_o, $H(x,\mathbf{n},\Xi_o)$ can generate the usual irradiances $H(x,\xi)$ and the radiances $N(x,\xi)$ (see Fig. 8.11). In the former case we need only set $\mathbf{n}(x) = \xi$ and $\Xi_o(x) = \Xi(\xi)$. In the latter case, we let $\Xi_o(x)$ be a variable circular conical set with central direction ξ. Then:

$$N(x,\xi) = \lim_{\Xi_o \to \{\xi\}} \frac{H(x,\xi,\Xi_o(x))}{\Omega(\Xi_o)} \tag{37}$$