fact may be used in laboratory setups to prearrange the light field so as to require a minimal amount of measuring throughout X. If this can be achieved, novel and simple means of determining the volume absorption coefficient will thereby be attained.

8.9 Canonical Representation of Irradiance Fields

We close this chapter on models for irradiance fields with a derivation which parallels the canonical representation of the radiance field given in (5) of Sec. 4.5. It is possible to derive the requisite relation so as to be a proper generalization of (5) of Sec. 4.5, and we shall now follow such a course.

Let X be an arbitrary optical medium. Let x be an arbitrary point of X and to x associate a direction $\mathbf{n}(x)$ and a set $\Xi_o(x)$ of directions. Let us write:

$$"\mathbf{H}(x,\Xi_o(x))" \quad \text{for} \quad \int_{\Xi_o(x)} N(x,\xi)\,\xi\,d\Omega(\xi) \quad . \tag{34}$$

This is a generalization of the irradiance vector $\mathbf{H}(x)$. The latter is obtained by requiring $\Xi_o(x) = \Xi$ (cf. (2) of Sec. 2.8, and also (4) of Sec. 2.4 for the numerical instance of (34); and (41) of Sec. 8.6 for an alternate version of (34)). Further, let us write:

$$"H(x,\mathbf{n}(x),\Xi_o(x))" \quad \text{or} \quad "H(x,\mathbf{n},\Xi_o)" \quad \text{for} \quad \mathbf{n}(x)\cdot\mathbf{H}(x,\Xi_o(x)) \tag{35}$$

It is clear that $H(x,\mathbf{n},\Xi_o)$ is the quantity measured by a subtracting janus plate (Sec. 2.8) whose collecting surfaces are exposed to the set $\Xi_o(x)$ of directions and whose pointer is directed along $\mathbf{n}$ (cf. Figs. 8.11 and 2.21). Associated with $H(x,\mathbf{n},\Xi_o)$ is the scalar irradiance $h(x,\Xi_o)$, where we have written:

$$"h(x,\Xi_o)" \quad \text{for} \quad \int_{\Xi_o} N(x,\xi)\,d\Omega(\xi) \tag{36}$$

$h(x,\Xi_o)$ is measured in practice by a spherical irradiance collector exposed to the direction set Ξ_o.

We pause to observe that by suitable choice of Ξ_o, $H(x,\mathbf{n},\Xi_o)$ can generate the usual irradiances $H(x,\xi)$ and the radiances $N(x,\xi)$ (see Fig. 8.11). In the former case we need only set $\mathbf{n}(x) = \xi$ and $\Xi_o(x) = \Xi(\xi)$. In the latter case, we let $\Xi_o(x)$ be a variable circular conical set with central direction ξ. Then:

$$N(x,\xi) = \lim_{\Xi_o \to \{\xi\}} \frac{H(x,\xi,\Xi_o(x))}{\Omega(\Xi_o)} \tag{37}$$

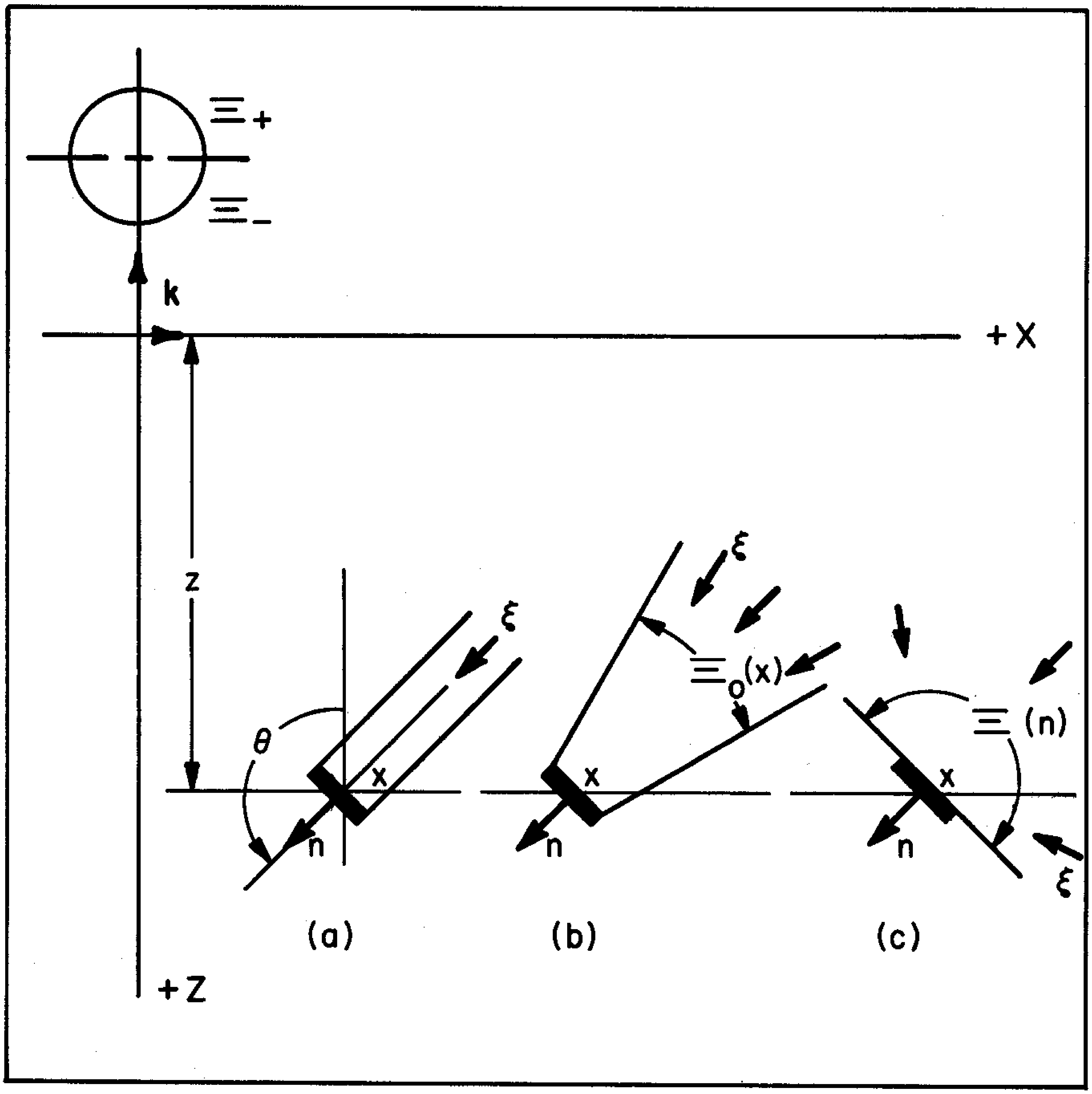

FIG. 8.11 The radiometric constructs, as defined operationally, which are used in the canonical equation for irradiance.

The mathematical basis for this rests in (4) of Sec. 2.5.

Next we define the distribution function associated with the set Ξ_o. We write:

$$\text{"}D(x,\mathbf{n}(x),\Xi_o(x)\text{"} \quad \text{for} \quad \frac{h(x,\Xi_o(x))}{H(x,\mathbf{n}(x),\Xi_o(x))} \tag{38}$$

This is a generalization of the distribution functions introduced in (5) of Sec. 8.3. For example, $D(x,+)$ is obtained from (38) by letting $\mathbf{k} = \mathbf{n}(x)$ and $\Xi_o(x) = \Xi_+$.

We are now ready to cast the equation of transfer into the canonical form for $H(x,\mathbf{n},\Xi_o)$. The derivation model we shall follow is that of (1)-(5) of Sec. 4.5. Thus, let us write:

$$"K(x,\mathbf{n}(x),\Xi_o(x))" \quad \text{for} \quad -\frac{1}{H(x,\mathbf{n},\Xi_o)}\nabla\cdot\mathbf{H}(x,\Xi_o) \tag{39}$$

Integrating the equation of transfer:

$$\xi\cdot\nabla N(x,\xi) = -\alpha(x)N(x,\xi) + \int_{\Xi} N(x,\xi')\sigma(x;\xi';\xi)\,d\Omega(\xi')$$

over $\Xi_o(x)$, we have:

$$\nabla\cdot\mathbf{H}(x,\Xi_o) = -\alpha(x)h(x,\Xi_o) + \int_{\Xi_o} N_*(x,\xi)\,d\Omega(\xi)$$

and using the preceding definitions, we have:

$$\boxed{K(x,\mathbf{n},\Xi_o) = \alpha(x)D(x,\mathbf{n},\Xi_o) - \frac{1}{H(x,\mathbf{n},\Xi_o)}\int_{\Xi_o} N_*(x,\xi)\,d\Omega(\xi)} \tag{40}$$

A final arrangement yields:

$$\boxed{D(x,\mathbf{n},\Xi_o)H(x,\mathbf{n},\Xi_o) = \frac{\int_{\Xi_o} N_*(x,\xi)\,d\Omega(\xi)}{\left[\alpha(x) - \frac{K(x,\mathbf{n},\Xi_o)}{D(x,\mathbf{n},\Xi_o)}\right]}} \tag{41}$$

which is the desired canonical representation of $H(x,\mathbf{n},\Xi_o)$. We readily verify that (41) is a proper generalization of (5) of Sec. 4.5 by recalling (37) and observing that:

$$\lim_{\Xi_o\to\{\xi\}} \frac{\int_{\Xi_o} N_*(x,\xi)\,d\Omega(\xi)}{\Omega(\Xi_o)} = N_*(x,\xi)$$

$$\lim_{\Xi_o\to\{\xi\}} D(x,\xi,\Xi_o) = 1 \quad .$$

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