light field at great depths in natural waters has been completely solved only recently. As a result of having a theoretical basis for the existence of the regular behavior of the light field at great depths (for most practical purposes, beyond 10-20 attenuation lengths), we can justify certain simplifications of the models for light fields below such depth intervals. In particular, in such regions the classical canonical form of the equation of transfer (Chapter 4) for radiance, and the two-D model for irradiance (Chapter 8) can be demonstrated to serve as accurate tools with which to quantitatively predict the magnitudes of the natural light fields.

10.1 On the Structure of the Light Field at Shallow Depths: Introductory Discussion

In this section experimental determinations of the upwelling and downwelling irradiances are studied with the purpose of explaining certain observed regular nonlinear trends in the semilog plots of these irradiances, principally at shallow depths in media with flat calm surfaces. We shall develop a mathematical model from the general equation of transfer which describes these irradiances in great detail over the shallow-depth range. The model explains the observed phenomena in terms of the inherent optical properties of the medium and its external lighting conditions. On the basis of experimental evidence, cited below, and on the basis of supporting theory, the following hypothesis about light fields in all homogeneous natural hydrosols is proposed: (a) The ratio of the upwelling irradiance to the downwelling irradiance, i.e., the observable reflectance function $R(z,-)$, is invariably monotonic increasing or decreasing at shallow depths with increasing depth (depending on the medium) and approaches a limit which is independent of the external lighting conditions and which depends only on the inherent optical properties of the medium. (b) The logarithmic derivatives, i.e., the K-functions $K(z,\pm)$, of the upwelling and downwelling irradiance at shallow depths are monotonic increasing or decreasing with increasing depth (depending on the medium) and approach a common limit which is independent of the external lighting conditions and which depends only on the inherent optical properties of the medium. In this way we arrive at a fairly detailed understanding of the light field at extreme depths (shallow and deep) in all homogeneous natural hydrosols.

To set the stage for the general reader, the following observations on the empirical roles of the K and R functions will be helpful. For many practical purposes in applied hydrologic optics the downwelling irradiance $H(z,-)$ at a depth z in a natural hydrosol can be represented by the following simple formula

$$H(z,-) = H(0,-)e^{-Kz} \quad , \qquad (1)$$

where K is a fixed number which characterizes the overall flux transmitting properties of the hydrosol. A similar formula may be used to determine the upwelling irradiance $H(z,+)$ at any depth z :

$$H(z,+) = H(0,+)e^{-Kz} \quad , \tag{2}$$

where--again for many practical purposes--K is a fixed number and in fact identical to the one appearing in (1). Simple models of the light field, such as the two-D and one-D models of Secs. 8.5 and 8.6 supply detailed formulas of the kind (1) and (2).

Still another practical formula is the one which describes the depth dependence of scalar irradiance $h(z)$ at each depth z :

$$h(z) = h(0)e^{-Kz} \quad , \tag{3}$$

where K is the same number as that appearing in (1) and (2).

In practice $H(z,+)$ and $H(z,-)$ are measured by suitably designed horizontal flat plate collectors exposed to the appropriate hemisphere, and $h(z)$ is measured by a suitably designed spherical collector. In view of (1), (2), and (3), relatively quick estimates of a K for a particular natural body of water can be obtained by measuring any one of these three radiometric quantities at two distinct depths, and using the formula:

$$K = \frac{1}{(z_2 - z_1)} \ln \frac{A(z_1)}{A(z_2)} \quad , \tag{4}$$

where "$A(z)$" stands for any one of the three quantities: $H(z,+)$, $H(z,-)$, or $h(z)$ at depth z .

The practical procedures of hydrologic optics, summarized in formulas (1)-(4), are quite analogous to the following well-known procedure used in applied heat conduction studies to estimate the temperature $T(t)$ of a cooling spherical body at time t immersed in an infinite bath of zero temperature:

$$T(t) = T(0)e^{-kt} \quad , \tag{5}$$

where k is a known fixed number which characterizes the overall heat conducting properties of the material comprising the spherical body. Conversely, (5) may be used to estimate k by measuring $T(t)$ at two distinct times and using a formula exactly analogous to (4).

The specialists who use (5) are aware of the fact that it is a useful approximate formula which becomes an exact formula for $T(t)$ in the limit as $t \to \infty$. They also realize that (5) becomes quite inadequate for relatively accurate estimates of $T(t)$ whenever t is small, and must resort in such estimates to more general forms representing $T(t)$. These more general forms, of which (5) is a special limiting case, are well known and are solidly founded in the general theory of heat conduction, and experimental fact, and may be found in standard treatises on heat conduction.

Equations (1)-(3) are regarded by the specialists in hydrologic optics in much the same way as (5) is regarded in its own discipline: They are useful approximate relations which can be shown to become exact formulas for $H(z,\pm)$ and $h(z)$ in the limit as $z \to \infty$ in deep homogeneous plane parallel optical media (see, e.g., Sec. 10.7). Perhaps what is not well known--or at any rate not fully realized--is that, like (5) these equations do not exactly represent $H(z,\pm)$ or $h(z)$ for small values of z, even in homogeneous hydrosols with uniform external lighting conditions and perfectly calm air-water surfaces. Thus, for relatively accurate estimates of $H(z,\pm)$ and $h(z)$ such as those required in basic scientific studies of the light fields in natural hydrosols, (1)-(3) are quite inadequate. They do not represent the small but experimentally demonstrable departures from linearity of the semilog plots of $H(z,\pm)$ and $h(z)$.

What is required at present in the discipline of hydrologic optics is a set of more general formulas which can accurately represent the quantities $H(z,\pm)$ and $h(z)$ in the small z ranges and which reduce to these simpler formulas in the limit as $z \to \infty$.

One of the two purposes of the following sections is to present a set of formulas for $H(z,\pm)$ which yield a closer approximation to reality than (1) and (2). The search for these formulas was motivated by the results of recently performed accurate measurements in the light field in real natural hydrosols, and their derivations are founded on the tenets of general radiative transfer theory.

The second purpose of the present discussion is to examine the resulting formulas for indications of possible general qualitative rules that may be hypothesized about the fine structure of shallow-depth light fields, and to put the hypotheses into forms which will be amenable to further theoretical study or experimental verification. On the basis of the models constructed below it was possible to formulate three such hypotheses about the quantities:

$$K(z,\pm) = -\frac{1}{H(z,\pm)}\frac{dH(z,\pm)}{dz} \tag{6}$$

and

$$R(z,-) = \frac{H(z,+)}{H(z,-)} \quad , \tag{7}$$

introduced in Chapter 9. These hypotheses governing the functions $K(z,\pm)$ and $R(z,-)$ are presented in detail below in Sec. 10.3, but for the present we shall undertake some preliminary discussion of the roles of these functions in the study of natural light fields.

As noted in Sec. 9.3, the quantities $K(z,+)$ and $K(z,-)$ are simply the slopes of the semilog plots of $H(z,+)$ and $H(z,-)$. According to the simple formulas (1) and (2), these slopes do not change with depth and in fact are of the form:

$$K(z,+) = K(z,-) = K \quad ,$$

where K is defined in (1) and (2). Careful experiments show, however, that $K(z,+)$ and $K(z,-)$ are generally distinct numbers that do change with depth. Furthermore, we shall see in Sec. 10.7 that, in homogeneous media,

$$\lim_{z\to\infty} K(z,+) = \lim_{z\to\infty} K(z,-) = k_\infty$$

where k_∞ is a number which depends only on the inherent optical properties of the medium and is completely independent of the external lighting conditions on the upper boundary of the medium. One of the goals of the present chapter is to find out something about the nonlinear behavior of $K(z,\pm)$ at relatively *small* depths.

The quantity $R(z,-)$ summarizes the flux transmitting and reflecting properties of the medium both above and below the hypothetical plane at depth z. According to the simple formulas (1) and (2),

$$R(z,-) = \frac{H(0,+)}{H(0,-)} \quad ,$$

a fixed number for all z. Careful experiments show, however, that $R(z,-)$ changes with depth; and in all homogeneous media it will be shown (Sec. 10.7), to approach a well-defined limit as $z\to\infty$:

$$\lim_{z\to\infty} R(z,-) = R_\infty \quad ,$$

where R_∞ is a number which depends only on the inherent optical properties of the hydrosol and is completely independent of the external lighting conditions on the upper boundary of the medium. Another of our present goals is to understand the nonlinear behavior of $R(z,-)$ for relatively small values of z.

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10.2 Experimental Basis for the Shallow Depth Theory

To prepare the groundwork leading to the theory of the light field at shallow depths, we now consider some experimental data which supplies graphic evidence of the nonlinear depth behavior of $K(z,\pm)$ and $R(z,-)$ in near-surface regions of a specific hydrosol. The experimental evidence presented in this and the following sections has been computed from the data obtained in Lake Pend Oreille, Idaho, by J. E. Tyler [298].

Figure 10.1 depicts the semilog plots of $H(z,+)$ and $H(z,-)$ over the range of depths $5 \le z \le 55$ meters; $H(z,\pm)$ are associated with a wavelength interval of width 64 mμ centered at 480 mμ. This depth range corresponds to a range of about 20 optical depths, so that the light field in the vicinity of 50 meters should have for all practical purposes attained the asymptotic limit--assuming complete homogeneity of the medium.