(a) $a(z,\mathbf{n},\Xi_o) + b(z,\mathbf{n},\Xi_o)$ is the *attenuation function* for $\overline{H}(z,\mathbf{n},\Xi_o)$,

(b) $\overline{H}_q(z,\mathbf{n},\Xi_o)$ is the *equilibrium function* for $\overline{H}(z,\mathbf{n},\Xi_o)$.

The transport equation (10) is a generalization of the standard two-flow equations for $H(z,+)$ and $H(z,-)$ (Chapter 8). (In the latter case, for example, $\Xi_o = \Xi_-$, the downward hemisphere, and $\mathbf{n} = -\mathbf{k}$, where $\mathbf{k}$ is the unit outward normal to the plane-parallel medium.)

Each of the four preceding transport equations may be cast into a *canonical form* (see (2) of Sec. 11.2) by introducing the appropriate K - function for the associated radiometric quantity (see general definition (1) of Sec. 11.4). Therefore, a transport equation for each of these K - function exists, and is of the form (1) of Sec. 11.3. The equilibrium principle holds for N^n, N_*, $\mathbf{H}$, and U^n.

Summary and Conclusion

To summarize, the domain of applicability of the universal transport equation (1) of Sec. 11.3 is quite wide. In fact its domain covers the totality of radiometric functions used and known to date in radiative transfer theory (the 17 distinct types of radiometric concepts and their corresponding K - functions discussed above--34 concepts in all). By means of it, the general mathematical structure of the light field in plane-parallel media can be contained in a single unifying framework, and the necessity of invoking individual discussions and principles for each of the many radiometric quantities, at least on a logical level, is now obviated. All this from the interaction principle. Hence:

*Frustra fit per plura quod potest fieri per pauciora.**

William of Ockham (ca. 1300-1347)

11.5 Bibliographic Notes for Chapter 11

The concept of a universal radiative transport equation was introduced in [240]. The mathematical vehicle of the universal radiative transport equation is that of a Riccati differential equation in factored form:

$$\frac{df(x)}{dx} = \mu(x)[\delta f(x) - f_\alpha(x)][f(x) - f_q(x)]$$

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*It will be futile to employ more [principles] when it is possible to employ fewer.

and with each radiometric quantity f (or K - function, or reflectance function, etc.) the equation associates two auxiliary functions f_α, f_q which, as was seen in the text, play the roles of attenuation and quilibrium functions, respectively. For an elementary discussion of the nondegenerate Riccati equation, see, e.g., [116]. For modern developments of the theory of nondegenerate Riccati equations pertinent to possible radiative transfer applications, see the work of Redheffer [254], [255], [256], [257], and also Reid [261] and [262]. These mathematical studies are also of potential applicability to the operator equations for the R and T operators in Chapter 7, as noted in Sec. 7.15. In view of the work of this chapter and that of Chapter 7, along with the results of the work of Redheffer and Reid, it is clear that the Riccati differential equation enters a productive new phase in mathematical physics.

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