In the event that the air-water surface is to be viewed from a great height (aircraft and satellite heights) then one may use $\hat{N}(0,\xi)$, with ξ in Ξ_+, as determined in the manner shown in Sec. 12.13, as the time-averaged inherent radiance of the surface and then go on to compute its apparent radiance in the usual manner for a given path of sight.

Equations (19) and (20) are particularly designed to give the exact time-averaged radiance field in the immediate vicinity of the air-water surface, in particular over the depth region with $2m_{oo}$ above and below $\hat{S}$, and generally at depths in the hydrosol where bright moving beams of refracted sunlight are still observable.

One may generalize (19) and (20) from the vertical up-down case to the case where ξ is arbitrary nonhorizontal paths by merely replacing $y(t)$ in $T_{y(t)}$, and $G(y(t))$ by $-(\sec\theta)\cdot y(t)$ in (19) and (20) to account for the slant path attenuation (see Fig. 12.63). All other terms are unaffected by virtue of the assumed statistically stationary character of the motions of the hydrosol X and its boundary S. Hence (19) applies to all directions ξ in Ξ_- and (20) to ξ in Ξ_+. Furthermore, by invoking the ergodic hypothesis we may represent $\Phi(z-\zeta')$ for a path directed along ξ, by:

$$\Phi(\mathrm{sgn}(r)(z-\zeta')) = \int_0^\infty \overline{G}(r)T_{|r|}\ (\zeta',\xi)d\zeta' \tag{21}$$

where we have written:

$$\text{"}r\text{"} \quad \text{for} \quad -\sec\theta(z-\zeta')$$

and where $\xi\cdot\mathbf{k} = \cos\theta$. "sgn (r)" means the algebraic sign of r, namely + or -. The beam transmittance $T_{|r|}(\zeta',\xi)$ is readily evaluated in homogeneous media. In fact:

$$T_{|r|}(\zeta',\xi) = e^{-\alpha|r|} \tag{22}$$

and the time-average $\overline{G}(r)$ may be evaluated by means of (14) for seas with gaussian elevation distributions. Hence in such practical settings $\Phi(z-\zeta')$ is known. This completes the establishment of the connections between fixed-depth and cosurface averages of the radiance distributions beneath a dynamic air-water surface.

As a check on the connections (19) and (20) we can let the wind speed U_a go to zero, so that m_{oo}, σ_c, and σ_u go to zero. Consequently $\Phi(z-\zeta)$ goes to $T_{z-\zeta'}$, $\overline{G}(z)$ for $z > 0$ goes to 1, and $\overline{G}(-z)$ goes to zero for every positive z; and so (19) and (20) reduce to the integral forms of the equation of transfer for the static case (re: Sec. 12.2).

12.13 Synthesis of Time-Averaged Radiance Fields

The complete description of the time-averaged light field in a natural hydrosol with a wind-blown air-water surface will now be attained by gathering together the various pieces of the description fashioned in the preceding three sections.

The synthesis to be given is facilitated by casting equations (18) and (44) of Sec. 12.11 into operator form. Toward this end, and with (18) of Sec. 12.11 in mind, let us write:

$$\text{"}R^{o}_{-}(\hat{S})\text{"} \quad \text{for} \quad S(\xi) \int_{\Xi} [\ ] Q^{o}(\xi')R_{-}(\xi';\xi)\,d\Omega(\xi') \qquad (1)$$

$$\text{"}T^{o}_{+}(\hat{S})\text{"} \quad \text{for} \quad S(\xi) \int_{\Xi} [\ ]\, Q_{+}(\xi')T_{+}(\xi';\xi)\,d\Omega(\xi') \qquad (2)$$

where we have dropped reference to the point $\hat{x}$ in the mean surface $\hat{S}$. For example, "$S(\xi)$" is therefore a contracted name for $S(\hat{x},\xi)$, defined in (18) of Sec. 12.11. Furthermore, writing:

$$\text{"}R_{-}(\hat{S}) \quad \text{for} \quad S(\xi) \int_{\Xi} [\ ] S(\xi')R_{-}(\xi';\xi)\,d\Omega(\xi') \quad \text{m} \qquad (3)$$

we can then write (18) of Sec. 12.11 as:

$$\boxed{\hat{N}_{+}(\hat{S}) = \hat{N}^{o}_{-}R_{-}(\hat{S}) + \hat{N}_{+}(\hat{X})T^{o}_{+}(\hat{S}) + \hat{N}_{+}(\hat{S})R_{-}(\hat{S})} \qquad (4)$$

where "$\hat{N}_{+}(\hat{S})$" denotes the function $\hat{N}$ governed by (18) of Sec. 12.11, with the subscript "+" as a reminder that $\hat{N}$ is the averaged response of S over $\Xi_{+}(x,t)$. The surface radiance superscript is now dropped as being understood. N^{o}_{-} is the time-averaged sky radiance distribution (defined over Ξ_{-}) and $\hat{N}_{+}(\hat{X})$ is the time-averaged response of the body $\hat{X}$ of the averaged hydrosol whose boundary is $\hat{S}$. Thus $\hat{N}_{+}(\hat{X})$ is the time-averaged radiance function N^{o} over $\Xi_{+}(x,t)$ occurring in (12) of Sec. 12.10 and in (40) of Sec. 12.11. (Recall the n^2-convention stated at the outset of Sec. 12.10.)

In a similar manner (44) of Sec. 12.11 can be cast into the form (again all radiances being *surface type* radiances, the superscript "+" may now be dropped):

$$\boxed{\hat{N}_{-}(\hat{S}) = \hat{N}^{o}_{-}T^{o}_{-}(\hat{S}) + \hat{N}_{+}(\hat{S})T_{-}(\hat{S}) + \hat{N}_{+}(\hat{X})R^{o}_{+}(\hat{S})} \qquad (5)$$

where we have written:

$$\text{"}R^{o}_{+}(\hat{S}) \quad \text{for} \quad \int_{\Xi} [\ ] Q_{+}(\xi')R_{+}(\xi';\xi)\,d\Omega(\xi') \qquad (6)$$

$$\text{"}T^o_-(\hat{S})\text{"} \quad \text{for} \quad \int_\Xi [\ ]Q^o(\xi')T_-(\xi';\xi)d\Omega(\xi') \tag{7}$$

$$\text{"}T_-(\hat{S})\text{"} \quad \text{for} \quad \int_\Xi [\ ]Q(\xi')T_-(\xi';\xi)d\Omega(\xi') \quad . \tag{8}$$

The functions $R_\pm$, $T_\pm$ are defined in (23), (26), (45), and (46) of Sec. 12.11.

By virtue of the conclusions following (11) of Sec. 12.12 we have:

$$\boxed{\hat{N}_+(\hat{X}) = \hat{N}_-(\hat{S})R_-(\hat{X})} \tag{9}$$

where $R_-(\hat{X})$ is the time-averaged reflectance operator for the medium $\hat{X}$. This operator equation is obtained by applying the steady state theory of radiative transfer in plane-parallel media, with appropriate modifications, to the time-averaged equation of transfer (11) of Sec. 12.12, defined in the medium with upper boundary $\hat{S}$. The salient modification relative to the static case is that $R_-(\hat{X})$ is an integral operator with a representation of the form:

$$R_-(\hat{X}) = \int_\Xi [\ ]R(\hat{X};\xi';\xi)d\Omega(\xi') \tag{9a}$$

with ξ in Ξ. In other words the integration is over all of Ξ rather than just Ξ_- (as is the case for $R(a,b)$ in the static theory of Chapters 3 and 7). Furthermore, the response of $\hat{X}$ can be along every ξ in Ξ. The visualization of this new situation is quite easy, after the derivations of Secs. 12.10-12.12 have been thoroughly assimilated. Otherwise, the theory of $R_-(\hat{X})$ (and the other three time-averaged operators $R_+(\hat{X})$, $T_\pm(\hat{X})$) is exactly analogous to the static plane-parallel case and is essentially as developed in Chapters 3 and 7. The further exploration of this aspect of the time-averaged theory will be left to future students of the discipline of radiative transfer.

We continue the present synthesis by showing that the three equations (4), (5), and (9) may be formally solved to yield the three radiance fields $\hat{N}_\pm(\hat{S})$ and $\hat{N}_+(\hat{X})$. By (4) we have:

$$\boxed{\hat{N}_+(\hat{S}) = \hat{N}^o_-\mathcal{Q}^o_-(\hat{S}) + \hat{N}_+(\hat{X})\mathcal{T}^o_+(\hat{S})} \tag{10}$$

where we have written:

$$\text{"}\mathcal{R}_-^{\circ}(\hat{S}) \quad \text{for} \quad R_-^{\circ}(\hat{S})[I - R_-(\hat{S})]^{-1} \tag{11}$$

$$\text{"}\mathcal{T}_+^{\circ}(\hat{S})\text{"} \quad \text{for} \quad T_+^{\circ}(\hat{S})[I - R_-(\hat{S})]^{-1} \tag{12}$$

Furthermore, (5) and (9) may be combined to yield:

$$\hat{N}_-(\hat{S}) = \hat{N}_-^{\circ}T_-^{\circ}(\hat{S}) + \hat{N}_+(\hat{S})T_-(\hat{S}) + \hat{N}_-(\hat{S})R_-(\hat{X})R_+^{\circ}(\hat{S})$$

which in turn yields the following formal solution for $\hat{N}_-(\hat{S})$: in terms of $\hat{N}_+(\hat{S})$ and $\hat{N}_-^{\circ}$:

$$\boxed{\hat{N}_-(\hat{S}) = \hat{N}_-^{\circ}\mathcal{T}_-^{\circ}(\hat{S},\hat{X}) + \hat{N}_+(\hat{S})\mathcal{T}_-(\hat{S},\hat{X})} \tag{13}$$

where we have written

$$\text{"}\mathcal{T}_-^{\circ}(\hat{S},\hat{X})\text{"} \quad \text{for} \quad T_-^{\circ}(\hat{S})[I - R_-(\hat{X})R_+^{\circ}(\hat{S})]^{-1} \tag{14}$$

$$\text{"}\mathcal{T}_-(\hat{S},\hat{X})\text{"} \quad \text{for} \quad T_-(\hat{S})[I - R_-(\hat{X})R_+^{\circ}(\hat{S})]^{-1} \, . \tag{15}$$

The operator (15) acts like a transmittance operator for $\hat{N}_+(\hat{S})$ because $\hat{N}_+(\hat{S})$ is the averaged surface radiance of S leaving S and descending down onto S to be transmitted through S and hence to contribute to $N_-^{+}(S)$. With the help of (9), we can write (10) so that, like (13), it involves only the inputs $\hat{N}_-^{\circ}$ and $\hat{N}_-(S)$ to $\hat{S}$:

$$\boxed{\hat{N}_+(\hat{S}) = \hat{N}_-^{\circ}\mathcal{R}_-^{\circ}(\hat{S}) + \hat{N}_-(\hat{S})\mathcal{R}_-(\hat{S},\hat{X})} \tag{16}$$

where we have written

$$\text{"}\mathcal{R}_-(\hat{S},\hat{X})\text{"} \quad \text{for} \quad R_-(\hat{X})\mathcal{T}_+^{\circ}(\hat{S}) \tag{17}$$

We have reached the penultimate step in the formal solution of the time-averaged surface radiances $\hat{N}_+(\hat{S})$, $\hat{N}_-(\hat{S})$. Equations (13) and (16) are a system of operator equations in the requisite unknowns $\hat{N}_\pm(S)$ with given $\mathcal{R}$ and $\mathcal{T}$ operators and given input radiance $\hat{N}_-^{\circ}$ associated with the sky radiance distribution. These equations can be solved formally on the operator level in the manner illustrated repeatedly throughout Chapters 3 and 7. The results are:

$$\boxed{\begin{aligned} \hat{N}_-(\hat{S}) &= \hat{N}_-^{\circ}\mathcal{T}_- && (18) \\ \hat{N}_+(\hat{S}) &= \hat{N}_-^{\circ}\hat{\mathcal{R}}_- && (19) \end{aligned}}$$

where we have written

$$\text{"}\hat{\mathcal{T}}_{-}\text{"} \quad \text{for} \quad [\mathcal{T}^{\circ}_{-}(\hat{S},\hat{X}) + \mathcal{R}^{\circ}_{-}(\hat{S})\mathcal{T}_{-}(\hat{S},\hat{X})][I - \mathcal{R}_{-}(\hat{S},\hat{X})\mathcal{T}_{-}(\hat{S},\hat{X})]^{-1} \qquad (20)$$

$$\text{"}\hat{\mathcal{R}}_{-}\text{"} \quad \text{for} \quad \mathcal{R}^{\circ}_{-}(\hat{S}) + \hat{\mathcal{T}}_{-}\mathcal{R}_{-}(\hat{S},\hat{X}) \quad . \qquad (21)$$

Equations (18) and (19)completely solve, in principle, the problem of the time-averaged radiance distribution of the dynamic air-water surface irradiated by skylight $\hat{N}^{\circ}$. (Recall the n^2-convention stated at the outset of Sec. 12.10.) The operator $\hat{\mathcal{T}}_{-}$ is a general time-averaged complete transmittance of $\hat{S}$, and $\hat{\mathcal{R}}_{-}$ is a general time-averaged complete reflectance of $\hat{S}$, where the inner structure of $\hat{\mathcal{R}}_{-}$ and $\hat{\mathcal{T}}_{-}$ is completely determinable by retracing the thread of reasoning beginning with (20) and (21) and working back to Sec. 12.10.

Comparison with the Static Case

In closing, it is of inerest to compare (18) and (19) with the representation of the radiance of the static air-water case given in (13) and (14) of Sec. 12.2. (At this point the reader should recall the n^2-convention for radiances stated at the outset of Sec. 12.10.) The operator $\mathcal{T}(-1,0,z_1)$ in (16) of Sec. 12.2 is a special case of $\hat{\mathcal{T}}_{-}$ occurring in (18). Furthermore, the operator $\mathcal{R}(-1,0,z_1)$ in (17) of Sec. 12.2 is a special case of $\hat{\mathcal{R}}^{\circ}_{-}$ occurring in (19). This operator, when applied to $\hat{N}^{\circ}$, yields $\hat{N}_{+}(\hat{S})$. The similarity in the structure of these operators with their static counterparts is quite striking and is traceable, of course, to the interaction principle underlying all algebraic descriptions of radiative transfer phenomena. The reader will find it instructive to show that $\hat{\mathcal{T}}_{-}$ and $\hat{\mathcal{R}}_{-}$ reduce exactly to (16) and (17), respectively, of Sec. 12.2 as the dynamic air-water surface S continuously approaches $\hat{S}$. This may be done for example by adopting the Neumann spectrum model for S (Sec. 12.8), letting $U_a \to 0$, (i.e., letting the equilibrium wind speed go to zero) and using the gaussian representations of the weighting functions Q, Q°, Q_{+}, Q_{-}. Or it may be done by simple intuitive considerations on the necessary properties the Q-functions must have for any reasonable model of the air-water surface, as the air-water surface continuously appraches the static plane form.

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12.14 Observations on the Theory of Time-Averaged Radiance Fields for Dynamic Air-Water Surfaces

The theory of time-averaged radiance fields developed in the preceding four sections contains a great variety of special cases of practical interest in applied hydrologic optics. It is the purpose of this section to classify and discuss the main set of these special cases and to indicate their use in practice.