

where we have written

$$" \hat{\mathcal{T}}_- " \quad \text{for} \quad \mathcal{D}_-^0(\hat{S}, \hat{X}) + \mathcal{Q}_-^0(\hat{S}) \mathcal{T}_-(\hat{S}, \hat{X}) [I - \mathcal{Q}_-(\hat{S}, \hat{X}) \mathcal{T}_-(\hat{S}, \hat{X})]^{-1} \quad (20)$$

$$" \hat{\mathcal{Q}}_- " \quad \text{for} \quad \mathcal{Q}_-^0(\hat{S}) + \hat{\mathcal{T}}_- \mathcal{Q}_-(\hat{S}, \hat{X}) \quad . \quad (21)$$

Equations (18) and (19) completely solve, in principle, the problem of the time-averaged radiance distribution of the dynamic air-water surface irradiated by skylight \hat{N}_-^0 . (Recall the n^2 -convention stated at the outset of Sec. 12.10.) The operator $\hat{\mathcal{T}}_-$ is a general time-averaged complete transmittance of \hat{S} , and $\hat{\mathcal{Q}}_-$ is a general time-averaged complete reflectance of \hat{S} , where the inner structure of $\hat{\mathcal{Q}}_-$ and $\hat{\mathcal{T}}_-$ is completely determinable by retracing the thread of reasoning beginning with (20) and (21) and working back to Sec. 12.10.

Comparison with the Static Case

In closing, it is of interest to compare (18) and (19) with the representation of the radiance of the static air-water case given in (13) and (14) of Sec. 12.2. (At this point the reader should recall the n^2 -convention for radiances stated at the outset of Sec. 12.10.) The operator $\mathcal{T}(-1, 0, z_1)$ in (16) of Sec. 12.2 is a special case of $\hat{\mathcal{T}}_-$ occurring in (18). Furthermore, the operator $\mathcal{Q}(-1, 0, z_1)$ in (17) of Sec. 12.2 is a special case of $\hat{\mathcal{Q}}_-^0$ occurring in (19). This operator, when applied to \hat{N}_-^0 , yields $\hat{N}_+(\hat{S})$. The similarity in the structure of these operators with their static counterparts is quite striking and is traceable, of course, to the interaction principle underlying all algebraic descriptions of radiative transfer phenomena. The reader will find it instructive to show that \mathcal{T}_- and \mathcal{Q}_- reduce exactly to (16) and (17), respectively, of Sec. 12.2 as the dynamic air-water surface S continuously approaches \hat{S} . This may be done for example by adopting the Neumann spectrum model for S (Sec. 12.8), letting $U_a \rightarrow 0$, (i.e., letting the equilibrium wind speed go to zero) and using the gaussian representations of the weighting functions Q , Q^0 , Q_+ , Q_- . Or it may be done by simple intuitive considerations on the necessary properties the Q -functions must have for any reasonable model of the air-water surface, as the air-water surface continuously approaches the static plane form.

12.14 Observations on the Theory of Time-Averaged Radiance Fields for Dynamic Air-Water Surfaces

The theory of time-averaged radiance fields developed in the preceding four sections contains a great variety of special cases of practical interest in applied hydrologic optics. It is the purpose of this section to classify and discuss the main set of these special cases and to indicate their use in practice.

A Hierarchy of Approximate Theories

Starting with the basic equations (12) of Sec. 12.10 and (40) of Sec. 12.11, we may construct a sequence of approximate theories of the time-averaged light fields for the dynamic air-water surface of natural hydrosols. As one proceeds along this sequence, the descriptive powers of the models decrease and the numerical tractability increases. The main sequence of approximations is as follows:

(i) *The Exact Time-Averaged Theory.* Starting with (12) of Sec. 12.10 and (40) of Sec. 12.11 with N^0 given along with the prescribed motion of the air-water surface S , the radiance $N(x, \xi, t)$ at point x on S in the direction ξ for every t in some interval $(0, T)$ of time may be computed. The resultant set of radiances may then be averaged to obtain a numerical estimate of $\hat{N}(\hat{x}, \xi)$ which is, in principle, exact in the sense that no special assumptions are used to obtain the averages.

(ii) *The Statistical Time-Averaged Theory.* The theory summarized in (18) and (44) of Sec. 12.11 and in the equations of Sec. 12.13 is based on certain observed or theoretical statistical regularity properties of the dynamic air-water surface such as the stationarity condition (5) of Sec. 12.11, the independence condition (10) of Sec. 12.11, and the ergodic hypothesis (31) of Sec. 12.11 (which allows space averages to be interchanged with time averages). This theory contains the effects of the radiometric interaction of S with itself and with the body of the hydrosol in addition to the hiding effects of finite wave slopes and finite wave heights from the sky radiance distribution.

(iii) *The Wave-Slope, Wave-Height Time-Averaged Theory.* The statistical time-averaged theory in (ii) is made somewhat more tractable but less descriptive by dropping the self-interaction term in (18) of Sec. 12.11. The result is:

$$\hat{N}(\hat{x}, \xi) = S(\hat{x}, \xi) \hat{A}_+(\hat{x}, \xi) \quad (1)$$

for all ξ in E , and which is a straightforward integral representation (and no longer an integral equation) of $\hat{N}(\hat{x}, \xi)$ which may be evaluated once \hat{N} , Q^0 , Q_+ , R_- , T_+ are prescribed. It is clear that the effects of finite wave heights and finite wave slopes are still included in this approximation. From (19) of Sec. 12.13, the upward time-averaged radiance of \hat{S} is given in the statistical theory by

$$\hat{N}_+(\hat{S}) = \hat{N}^0 \hat{\mathcal{R}}_- \quad (2)$$

In the present wave-slope, wave-height mode, $\hat{\mathcal{R}}_-$ takes the form:

$$\hat{\mathcal{R}}_- = R_-^0(\hat{S}) + T_-^0(\hat{S}) [I - R_-(\hat{X})R_+^0(\hat{S})]^{-1} R_-(\hat{X})T_+^0(\hat{S}) \quad (3)$$

That is, the effect of dropping the self-interaction term is to require:

$$\begin{aligned} R_-(\hat{S}) &= 0 \\ T_-(\hat{S}) &= 0 \end{aligned} \quad (4)$$

whence, in particular

$$\begin{aligned} Q_-(\hat{S}) &= R_-(\hat{S}) \\ \mathcal{T}_-(\hat{S}) &= T_-(\hat{S}) \\ \mathcal{T}_-(\hat{S}, \hat{X}) &= 0 \end{aligned} \quad (5)$$

(iv) *The Wave-Slope Time-Averaged Theory.* The wave-slope, wave-height time-averaged theory of (iii) is further simplified by neglecting effects of the wave heights of the dynamic air-water surface. Thus the immediate effect is to require $q_-(\hat{x}, \xi) = 0$, whence by (35a) of Sec. 12.11 $Q(\hat{x}, \xi) = 0$, and so that by (27) of Sec. 12.11 we now have:

$$Q^0(\hat{x}, \xi) + Q_+(\hat{x}, \xi) = 1$$

Furthermore, by the definition of $S(\hat{x}, \xi)$ in (18) of Sec. 12.11, and (1), $\hat{N}(\hat{x}, \xi) = 0$ for ξ in Ξ_- . Otherwise, the general form of $\hat{A}_+(\hat{x}, \xi)$ is the same. The resultant theory is of great importance to optical studies of the dynamic air-water surface since the distribution of *wave slopes* rather than *wave heights* is more often the dominant physical determinant of the time-averaged radiance of air-water surfaces. For example, as a fresh wind blows over an otherwise calm water surface on a clear day, a fine dark patina quickly overspreads the windblown surface in which the higher parts of the dark sky are mirrored. On the other hand, when the observer-sun geometry permits, a bright glitter pattern springs into existence as the wind produces a great range of wave slopes of capillary waves. In such transient settings the overriding importance of the wave slope state of the sea is clear.

(v) *Partial Time-Averaged Theories.* Occasionally it is of interest to study the reflected or transmitted radiance from a dynamic air-water surface S which is due only to the light of the sun or only to the light of the sky or only to the radiance incident on the underside of S from the hydrosol proper and in which self-interaction, the wave-height and wave-slope effects may or may not be included. Equations (5) and (9) of Sec. 12.13 summarize these component radiances in clear intuitive form. For example, $\hat{N}_-(\hat{S})$ in (5) of Sec. 12.13 describes the inherent radiance of the lower side of the time-averaged surface \hat{S} as being composed of transmitted time-averaged sky light \hat{N}_- , self-interaction light $\hat{N}_+(S)$, and reflected time-averaged hydrosol light, $\hat{N}_+(\hat{X})$, and is therefore, of the same general form as the static radiance of curved surfaces considered in Example 1 of Sec. 3.5. Similarly, $\hat{N}_+(\hat{S})$ in (10) of Sec. 12.13, namely the upward time-averaged radiance of \hat{S} , is seen to be composed of reflected sky

light and transmitted hydrosol light. Any one of the five terms on the right of (5) or (10) of Sec. 12.13 may be isolated for the study of $\hat{N}_\pm(\hat{S})$ in a partial time-averaged theory.

Illustrations of Some Classical Partial Theories

Some of the features of the theory of time-averaged light fields as developed above can be illustrated by selecting for examination each of the four terms in (18) and (19) of Sec. 12.13. By doing so we shall also deduce some of the classical instances of the theory of time-averaged reflected and transmitted radiance at the surface of natural hydrosols.

We begin with the component of the time-averaged upward radiance $\hat{N}_+(\hat{S})$ of S contributed by reflected sky and sunlight. Thus from (19) of Sec. 12.13 we consider $\hat{N}_+^{\circ} \hat{Q}_+^{\circ}(\hat{S})$. For simplicity of exposition we shall omit the self-interaction effect of S so that $\hat{Q}_+(\hat{S}) = R_+(\hat{S})$, which follows from (5). Hence from (1) of Sec. 12.13 and the present selection of effects:

$$\hat{N}(\xi) = \int_{E_-} \hat{N}^{\circ}(\xi') Q^{\circ}(\xi') R_-(\xi'; \xi) d\Omega(\xi') \quad ,$$

with ξ in E_+ , and where we have dropped reference to the point \hat{x} in \hat{S} , the uniformity of optical properties over all of \hat{S} being assumed.

The factor $Q^{\circ}(\xi')$ in the integrand of (6), as defined in (11) of Sec. 12.11, takes into account the shielding effect of the wave heights and wave slopes against the light of the sky and sun. $Q^{\circ}(\xi')$ may be evaluated using, e.g., (27), (29), and (35) of Sec. 12.11. Even when wave heights are considered negligible (i.e., $q_-(\hat{x}, \xi) = 0$), so that $Q(\xi) = 0$, we see from (29b) and (35b) of Sec. 12.11 and (6) that *wave slope* shielding may be in effect. By means of (23) of Sec. 12.11, we may represent $\hat{N}(\xi)$ in greater detail as follows:

$$\hat{N}(\xi) = \int_{E_-} \hat{N}^{\circ}(\xi') Q^{\circ}(\xi') r(\xi', \xi) p(\xi', \xi) d\Omega(\xi') \quad (7)$$

where $r(\xi', \xi)$ is the value of the Fresnel reflectance function for incident and reflected directions ξ', ξ , respectively, in E_- and E_+ . The function $p(\xi', \xi)$ is the probability that the wave normal to S is such that the incident and reflected directions ξ' and ξ occur for Fresnel reflection. If the sea surface is statistically gaussian, then either (24) of Sec. 12.11 may be used or its anisotropic generalization (28) of Sec. 12.5, after an appropriate change of variables in (7), if desired, from direction space E to slope space E_2 (i.e., the two-dimensional Euclidean plane).

As a specific illustration of (7), suppose that the sun is of uniform radiance \hat{N}^0 and positioned such that the radiance of its center is directed along ξ_0 , and that its direction set is $\Xi_0 \subset \Xi_-$. Then since $\Omega(\Xi_0)$ is quite small, we may approximate the contribution to $\hat{N}(\xi)$ by sunlight, by:

$$\hat{N}(\xi) = \hat{N}^0 Q^0(\xi_0) r(\xi_0, \xi) p(\xi_0, \xi) \Omega(\Xi_0) \quad (8)$$

A plot of $\hat{N}(\xi)$ as a function of ξ will, therefore, yield a radiometric map of the sun's glitter pattern over Ξ_+ . If $p(\xi_0, \xi)$ is given by (24) of Sec. 12.11 or (28) of Sec. 12.5, then the glitter pattern is seen to be a function of the root mean slope σ , and, owing to the general presence of $Q^0(\xi_0)$, to the root mean elevation m_{00} . It is, in fact, quite possible to use (8) to form a systematic tabulation of the values:

$$R_j(\xi) \equiv Q^0(\xi_j) r(\xi_j, \xi) p(\xi_j, \xi) \Omega(\Xi_j) \quad (9)$$

as a function of ξ_j, ξ and the statistical sea surface parameters σ and m_{00} . Then, when the radiance distribution of the sky hemisphere Ξ_- is known over a partition $\{\Xi_1, \Xi_2, \dots, \Xi_n\}$ of Ξ_- with central direction $\xi_j, j = 1, \dots, n$, for each Ξ_j , we can compute:

$$\hat{N}(\xi) = \sum_{j=1}^n \hat{N}_j^0 R_j(\xi) \quad (10)$$

which is a discrete version of (7). A different kind of approximate formula for computing reflected sunlight from a roughened water surface is given by Cox and Munk in [56]. Equation (10) also may be used to estimate the time-averaged radiance of an air-water surface which has an extraordinary ground swell under the usual statistically stationary jumbled surface. This would be accomplished by literally tilting the coordinate axes used throughout the preceding derivations so that the previously horizontal reference frame now lies in the rising or descending part of the swell. Computations then proceed as usual in this new frame of reference. With some care, the additional shielding effect due to the swell's geometry can be included to supplement the Q^0 factor shielding effect.

As another example of a classical representation of $\hat{N}_+(\hat{S})$ we consider the upward transmitted component $\hat{N}_+(\hat{X}) \cdot \mathcal{J}_+^0(\hat{S})$ in (10) of Sec. 12.13. Once again, for simplicity, the self-interaction effect is omitted so that $\mathcal{J}_+^0(\hat{S}) = T_+^0(\hat{S})$ and we have for the case at hand:

$$\hat{N}(\xi) = \int_{\Xi_+} \left[\frac{N^0(\xi')}{n^2} \right] Q_+(\xi') T_+(\xi', \xi) d\Omega(\xi') \quad (11)$$

where ξ is in Ξ and where $\hat{N}^0(\xi')$ is the radiance of the hydrosol incident on the lower side of \hat{S} in the direction ξ' .

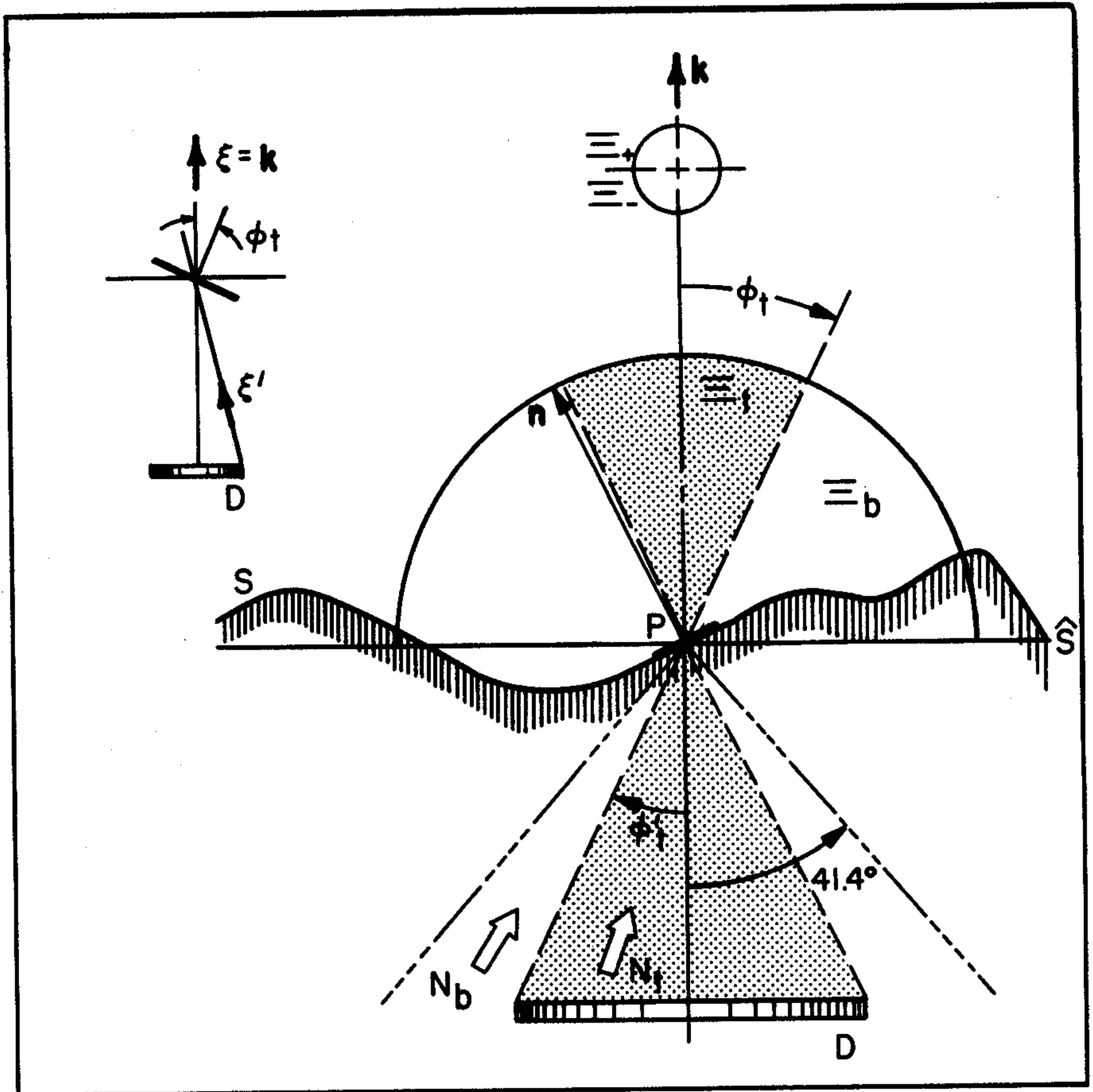


FIG. 12.65 Setting for the classical apparent contrast reduction formula by time-averaged refraction at the air-water surface.

We have momentarily set aside the n^2 -convention, stated at the outset in Sec. 12.10, so as to avoid an accidental misapplication of (11) by readers using only this discussion as a reference source (n for air is taken as $n = 1$).

Equation (11) can be unfolded layer by layer for particular applications just as we did with (6). Thus, by (26) of Sec. 12.11, we can write:

$$\hat{N}(\xi) = \int_{E_+} \left[\frac{\hat{N}^0(\xi')}{n^2} \right] Q_+(\xi') t(\xi', \xi) p(\xi', \xi) d\Omega(\xi') \quad (12)$$

where ξ is in E .

As a particular illustration of (12) let us estimate the time-averaged radiance in the vertical direction \mathbf{k} for the case where $\hat{N}^0(\xi')$ is of the form:

$$\hat{N}^0(\xi') = \begin{cases} N_t & \text{if } \xi' \text{ is in } E_t \\ N_b & \text{if } \xi' \text{ is in } E_b \\ 0 & \text{if } \xi' \text{ is in } E_- \end{cases}$$

where E_t and E_b are the sets of directions which partition E_+ , shown in Fig. 12.65. The set of directions E_t is subtended by a circular disk D of uniform apparent time-averaged radiance N_t and E_b is the set of directions of its background. As a result of this choice of direction \mathbf{k} , and form of \hat{N}^0 , (12) becomes:

$$\begin{aligned} \hat{N}(\mathbf{k}) &= \frac{N_t}{n^2} \int_{E_t} Q_+(\xi') t(\xi', \mathbf{k}) p(\xi', \mathbf{k}) d\Omega(\xi') \\ &+ \frac{N_b}{n^2} \int_{E_b} Q_+(\xi') t(\xi', \mathbf{k}) p(\xi', \mathbf{k}) d\Omega(\xi') \dots \quad (13) \end{aligned}$$

Now the presence of the Fresnel transmittance factor in (13) is such as to limit the range of integration of ξ' in E_+ to a solid angle E_f of half angle 41.4° within which $t(\xi', \mathbf{k}) > 0$, and outside of which $t(\xi', \mathbf{k}) = 0$ owing to total internal reflection. Hence the range of integration may be limited to $E_t \cap E_+$ and $E_b \cap E_f$, when working toward actual numerical estimates of $\hat{N}(\mathbf{k})$. To see the general order of magnitude of $\hat{N}(\mathbf{k})$, let us set:

$$Q_+(\xi') t(\xi', \mathbf{k}) = 1$$

for all ξ' over E_+ , so that (13) becomes:

$$\hat{N}(\mathbf{k}) = \frac{N_t}{n^2} \int_{E_t} p(\xi', \mathbf{k}) d\Omega(\xi') + \frac{N_b}{n^2} \int_{E_b} p(\xi', \mathbf{k}) d\Omega(\xi') \dots \quad (14)$$

The integrals in this expression have very simple interpretations: the integral over E_t , for example, is the probability that the normals to the wave slopes at point P in Fig. 12.65 are tilted at less than the angle ϕ_t from \mathbf{k} , the angle for which the refracted rays from the rim of the submerged disk D emerge from the hydrosol along the direction \mathbf{k} . (See insert, Fig. 12.65.) Using the gaussian model of the sea surface (24) of Sec. 12.11, we have at once from (17) of Sec. 12.5:

$$\int_{\Xi_t} p(\xi', \mathbf{k}) d\Omega(\xi') = 1 - e^{-\frac{\tan^2 \phi t}{2\sigma^2}} \quad (15)$$

and

$$\int_{\Xi_b} p(\xi', \mathbf{k}) d\Omega(\xi') = e^{-\frac{\tan^2 \phi t}{2\sigma^2}} \quad (16)$$

With these representations, (14) may take the form:

$$N(\mathbf{k}) = \frac{N_t}{n^2} \left(1 - e^{-\frac{\tan^2 \phi t}{2\sigma^2}} \right) + \frac{N_b}{n^2} e^{-\frac{\tan^2 \phi t}{2\sigma^2}} \quad (17)$$

This approximate representation of $\hat{N}(\mathbf{k})$ is of historic interest, being essentially the result of the first calculation of a time-averaged radiance viewed through a moving air-water surface (cf. [82]). Equation (17) may be rewritten to use the observed time-averaged radiances \hat{N}_t, \hat{N}_b seen just above the surface \hat{S} . For this purpose we use (18) of Sec. 12.1 but retain the present assumption that $t(\xi', \xi) = 1$, so that (17) becomes:

$$\hat{N}(\mathbf{k}) = \hat{N}_t \left(1 - e^{-\frac{\tan \phi t}{2\sigma^2}} \right) + \hat{N}_b e^{-\frac{\tan \phi t}{2\sigma^2}} \quad (18)$$

$$\hat{N}_t = N_t/n^2, \quad \hat{N}_b = N_b/n^2$$

One of the first calculations of contrast reduction by time varying refraction was based on (18). Thus, as in [82], let us define the *time-averaged apparent contrast* of the apparent radiance \hat{N}_t of the center of the submerged disk with respect to the background radiance \hat{N}_b by writing:

$$"C" \quad \text{for} \quad \frac{\hat{N}(\mathbf{k}) - \hat{N}_b}{\hat{N}_b} \quad (19)$$

This is an extension of the standard notion of contrast, defined in Sec. 9.5, to the time-averaged case. The *apparent contrast C for the static case* is, by definition of Sec. 9.5, simply:

$$C = \frac{\hat{N}_t - \hat{N}_b}{\hat{N}_b} \quad (20)$$

Here we are implicitly assuming that the time-averaged radiances \hat{N}_t and \hat{N}_b are the same as would be measured in the static case, i.e., we are assuming that $\hat{N}_t = \hat{N}_t/n^2$, $\hat{N}_b = \hat{N}_b/n^2$. In other words, we are hypothesizing for the purposes of the present discussion that the movements of the surface S do not change the *average* values of the apparent radiances over the disk D and over its background. With this assumption in mind we can obtain a simple and useful connection between \bar{C} and C using (18):

$$\bar{C} = C \left[1 - e^{-\frac{\tan^2 \phi_t}{2\sigma^2}} \right] \quad (21)$$

From this representation of \bar{C} , despite the multitude of assumptions on which it has been based, we gain valuable quantitative information about the effect of the movement of an air-water surface on the apparent contrast of submerged objects viewed through that surface. Equation (21) is particularly useful when the submerged target is angularly small and the water surface is freshly crinkled with capillary waves. Then we can readily see how \bar{C} varies with ϕ_t and σ : the larger ϕ_t , for a given σ , the larger \bar{C} is, meaning the less the diminution of contrast by wave action. Thus small objects tend to be readily blended into their background by the water surface disturbances. On the other hand for a given target size, \bar{C} goes down as σ increases. For example, if we use the relation

$$\sigma^2 = \gamma U_a$$

for some constant γ (cf. (28) and (29) of Sec. 12.5 and (11) of Sec. 12.8), (21) can be written as:

$$\bar{C} = C \left[1 - e^{-\frac{\tan^2 \phi_t}{2\gamma U_a}} \right] \quad (22)$$

which for small targets and fresh breezes over an otherwise calm surface can be reduced to the rule of thumb:

$$\bar{C} = C \left(\frac{\tan^2 \phi_t}{2\gamma U_a} \right) \quad (23)$$

This equation allows one to readily see the *direct* variation between ϕ_t and \bar{C} , and the *inverse* variation between U_a and \bar{C} .

As a final illustration of some partial representations of the time-averaged radiance $\hat{N}_+(\hat{S})$, we consider (19) of Sec. 12.13. Thus we are to consider (compare with (2)):

$$\hat{N}_+(\hat{S}) = \hat{N}_-^{\circ} \hat{\mathcal{A}}_-$$

Once again let us omit self-interaction effects over S so that (4) holds. Furthermore we concentrate on the transmitted upward-flux term in the preceding relation, thus we consider the second term of

$$\hat{N}_+(\hat{S}) = \hat{N}_-^{\circ} R_-^{\circ}(\hat{S}) + \hat{N}_-^{\circ} T_-^{\circ}(\hat{S}) [I - R_-(\hat{X}) R_+^{\circ}(\hat{S})]^{-1} R_-(\hat{X}) T_+^{\circ}(\hat{S}) . \quad (24)$$

The interpretation of the second term of (24) is straightforward: given the incident radiance distribution \hat{N}_-° from sun and sky, operate on \hat{N}_-° with $T_-^{\circ}(\hat{S})$ as given in (7). The result is a time-averaged radiance distribution transmitted downward across \hat{S} . The latter distribution is operated on by the interreflection operator:

$$[I - R_-(\hat{X}) R_+^{\circ}(\hat{S})]^{-1}$$

which may be approximated arbitrarily closely by computing a sufficient number of terms of its representing infinite operator series (in practice two terms beyond the identity I should be sufficient):

$$[I - R_-(\hat{X}) R_+^{\circ}(\hat{S})]^{-1} = I + \sum_{j=1}^{\infty} T^j(\hat{S}, \hat{X}) \quad (25)$$

where we write:

$$"T^{j+1}(\hat{S}, \hat{X})" \quad \text{for} \quad T(\hat{S}, \hat{X}) T^j(\hat{S}, \hat{X}) \quad (26)$$

and:

$$T^1(\hat{S}, \hat{X})" \quad \text{or} \quad "T(\hat{S}, \hat{X})" \quad \text{for} \quad R_-(\hat{X}) R_+^{\circ}(\hat{S}) . \quad (27)$$

The result is a time-averaged downward radiance distribution to be operated on by $R_-(\hat{X}) T_+^{\circ}(\hat{S})$. The reader may deepen his understanding of (24) by referring back to a related but more readily visualized example in terms of irradiance. Thus, the exact irradiance counterpart to (19) of Sec. 12.13 is (2) of Sec. 12.2 and, in particular, the interacting counterpart to (24) is the second term (2) of Sec. 12.2. This shows that the algebraic structures of both the irradiance and radiance cases (in either the static or dynamic context) are identical. The practical difference between these cases is

that in the irradiance case one works with numbers; in the radiance case one works with functions and, in addition to extensive numerical details, the order of operations on functions in this case must be scrupulously observed.

Concluding Observations

One of the two main goals of this chapter has been the pair of equations (18) and (19) of Sec. 12.13. They describe the time-averaged radiance distributions $\hat{N}_+(\hat{S})$, $\hat{N}_-(\hat{S})$ directed into the aerosol and hydrosol, respectively, above and below the dynamic air-water surface. The main purpose of these equations is to form a solid mathematical and physical foundation for practical techniques of describing and predicting the average radiance distributions of wind-blown seas, lakes, and other natural hydrosols. The preceding discussion of the hierarchies of approximate theories leading from (18) and (19) of Sec. 12.13, shows that this purpose has been adequately fulfilled. Thus we were able to deduce the two classical formulas for the time-averaged apparent radiance and apparent contrasts of submerged objects as in (18) and (21). However, (18) and (19) of Sec. 12.13 allow us to go beyond these classical formulas and establish (7) and, more completely, (24) which can take into account the shielding effects of wave slopes and wave elevations from sky- and sunlight without the complications of radiometric self-interactions of the dynamic air-water surface.

The complete description of the averaged light field, with self-interactions, shieldings, and hydrosol light-fields, all in concert, as given in (18) and (19) of Sec. 12.13, has not yet been subject to numerical applications. Before this can be done, further study of the operator $R_-(X)$ in (9) of Sec. 12.13 must be made. To a first approximation we may use the $R(0, \infty)$ operators (with kernels $R(0, \infty; \xi'; \xi)$) in Sec. 7.6 instead of $R_-(\hat{X})$. However, interested students after careful study will see what more can be done here.

The theory of the optical properties of the air-water surface has thus been brought to a level of completion summarized by (18) and (19) of Sec. 12.13.

12.15 Simulation of the Reflectance of the Air-Water Surface by Mechanical Devices

We conclude this chapter on optical properties of the air-water surface with descriptions of some mechanical devices which may be used to predict, with relatively little advanced mathematical computations, the average sun and sky radiance reflected from wind-blown water surfaces. The theoretical discussions throughout this chapter have demonstrated that a virtually complete theory of radiative transfer across randomly moving water surfaces can be formulated so that the reflected and transmitted radiances can, in principle, be computed in great detail. However, it is not always possible or economical to solve daily problems of applied hydrologic optics with such relatively powerful batteries of tools. In the first and main part of this section we shall round out