

the curvature of a cup in the large-slope regions remains relatively constant with changing sea state. In short, the overall curvature of an ergodic cup varies directly with the sea-state parameters in such a way as to qualitatively reproduce the observed overall change of curvatures of real waves with changing sea state. It is not to be expected, however, that these depth and curvature features of an ergodic cup *quantitatively* represent the mean depths and mean curvatures in wave patterns which are to be found on a real sea.

### Sea Simulator Devices Beyond the Ergodic Cup

The concept of an ergodic cup as developed here is limited principally to the simulation of the *slope* properties of random sea surfaces. In view of the increasing numbers of optical properties of the sea surface required beyond slope properties (in particular mean height data are required) in order to make further progress in hydrologic optics, it is clear that further research is needed in the art of simulating the dynamic air-water surface by means of mechanical, electrical or optical devices.

A possible area of research in optical sea state simulation would be with laboratory (rather than natural) hydrosols on the surface of which is induced, by acoustic (or possibly mechanical) means, standing (or progressive) wave patterns with realistic directional spectra  $E(\mathbf{k})$ . A perusal of the linearized equations for surface waves shows that this is in principle quite feasible. By careful control of the induced wave patterns it may be possible to superimpose one standing wave pattern on another until a given  $E(\mathbf{k})$  is closely approximated by that of the dynamic test surface. In the initial stages of the study, the patterns should be analyzed harmonically to see the range of directional spectra possible over the test-controlled hydrosol. It may be possible to reproduce Neumann (or other gamma type) spectra with the standing wave patterns and obtain correct scale relations with respect to natural capillary and gravity wave spectra. Once sufficient techniques have been learned, the test-controlled hydrosol could perhaps be "tuned" to a given spectrum, and photometered under a prescribed radiance distribution, to obtain the requisite reflectance properties of its real counterpart. The advantages of such a tuned laboratory hydrosol are obvious: real water could be used in such a way that Fresnel reflectance properties would be realistically reproduced; furthermore, wave slope and wave height characteristics, if properly scaled, could simulate both slope and height shielding effects of natural waves.

### 12.16 Bibliographic Notes for Chapter 12

The Fresnel laws of reflection and refraction given in Sec. 12.1 may be found in any standard text on optics. However, the general connection between the electromagnetic field and the radiance distribution from which (7) of Sec. 12.1 is derived appears to be relatively new, and may be found in Sec. 124 of [251]. The results of exact reflectance calculations for cardioidal radiance distributions as presented in Example 2, were first given in [215].

The theory of radiative transfer across the static air-water surface in Sec. 12.2 is based mainly on the results of Chapter 8 of the present work. The formulas for the irradiance field are well-known classical results which may be traced through the bibliographic notes for Chapter 8. Their systematic generalization to the radiance case is new and was carried out in analogy with the irradiance case and with the formal help of the interaction principle of Chapter 3.

Section 12.3 on elementary hydrodynamics of the air-water surface is based in the main on the appropriate articles in Lamb's classic treatise [149]. However, the approach to hydrodynamics in the opening paragraphs (the fluid transfer process) is an abbreviated possible modern treatment to the subject based on semigroup formulations of physical processes. This mode of approach to hydrodynamics was chosen to point up the fact that both hydrodynamics and radiative transfer share a common conceptual root in the domain of semigroup formulations of physical processes imagined as taking place on differentiable manifolds. The notion of a semigroup served to unify the developments in Chapter 7 and, more basically, serves as the starting point for an axiomatic formulation of radiative transfer theory [251]. Group theoretic approaches to hydrodynamics may be found in [322] and [22].

The presence of Sec. 12.4 in this work on principles of hydrologic optics attests to the rapidly growing number of applications of the powerful mathematical methods of harmonic analysis (cf. [170], [321]) and second order probability theory [162] to the description of dynamic air-water surfaces. A detailed bibliography of the applications of stochastic methods to ocean wave studies may be built up from references in [191] and in various chapters of [109]. A recent promising approach to advanced mathematical descriptions of the dynamic air-water surface is made by Kampé de Fériet in [134]. Historical notes on harmonic analysis to physical problems are made, in passing, at appropriate points of the text in Sec. 12.4 above. However, special mention should be made of the pioneering efforts of Schuster [278] in early geophysical problems and to Wiener [320] in forming a mathematical foundation for harmonic analysis. The work of Pierson and Marks [202] (see also [199]) is one of the early definitive applications of general harmonic analysis to oceanographic problems. Further work on the statistics of the air-water surface may be found in [165]. Further problems are explored in [201].

The principal references consulted for the development of Sec. 12.5 on water wave slopes are those by Hulburt [113], Duntley [73], [82], Cox and Munk [56], and Schooley [274]. Some later experimental work on capillary waves by Cox may be found in [54] along with its relevance to Phillips' wave spectrum theory (papers following [54]). An alternate wave-slope measuring technique to those discussed in the text was considered by Culver [61]. A recent comprehensive survey of wind generated water waves was written by Kinsman [139]. See also [190].

References made in the wave generation and decay Sec. 12.6 and the wave spectrum data Sec. 12.7 can be supplemented

by the review articles of Pierson [199], Ursell [307], and the symposium reports [191]. The early pages of the Transactions of the American Geophysical Union are also a good source of papers on ocean wave data (volume 33 for example is particularly rich in results).

An excellent little text on probability theory, especially for the theoretically inclined worker in applied science, is that by Levy and Roth [158]; unfortunately it is now out of print; but it still can serve as an introduction to the relatively massive and abstract current standard [162]. In particular [158] was found a helpful reference in formulating some of the arguments of Sec. 12.9.

The sequence of Sec. 12.10 through Sec. 12.14 presents some results in the study of radiative transfer across dynamic air-water surfaces; and these results appear to be new. Some unpublished results in an attempt to extend (21) of Sec. 12.14 to inclined lines of sight are given in [317]. The theory of the ergodic cup in Sec. 12.15 first appeared in [232], and was revised and extended for the present exposition. The radiometric effects of the boundary surface between the atmosphere and the sea played an important part in the studies of Secs. 12.10-12.14. The reduction of the analysis of these effects to operator form, as in Sec. 12.13, materially simplifies thinking about the associated complex interactions. This approach was first used in [218], during a related study of the static air-water surface problem.