relatively long tube, the radiance meter should not be rotated around its detector end, but rather it should be rotated about the forward end of the radiance tube. If, as is usual, the radiance meter is rotated around an axis at the detector end of the radiance tube, then the radiance measured in each direction will be effectively for a different depth, namely the depth of the forward end of the tube. Since the variation of radiance with depth in relatively turbid media is quite marked over short distances and since it is nonlinear and different in every direction, correction of the data to a single depth under such circumstances would be very difficult. The use of a carefully designed optical system for restricting the solid angle of acceptance makes it possible to obtain the same resolution with a shorter tube, thereby alleviating this difficulty. In this connection, appropriate internal baffling and black finish are just as important for an optical system as for the baffle tube. In any case, for very accurate results in water having relatively large volume attenuation function α, measurements with an optical system may possibly have to be corrected for significant depth changes at the end of the measuring tube, as the tube scans its environment.

In closing it may be observed that radical new radiance distribution measurement techniques may be possible by adopting some of the observations of Examples 14 and 15, Sec. 2.11, built around novel collection devices and Legendre polynomial analysis and synthesis. Another interesting possibility is the use of Fourier analysis techniques on the radiance distribution $N(x,\cdot)$ over the compact set Ξ of the directions ξ. In particular, the sampling theory [29] coupled with resolution requirements may obviate the need for scanning *each* point of Ξ, but only a discrete set of appropriately placed points in Ξ. This of course holds also for the Legendre polynomial analysis, and indeed, for any orthogonal family of functions defined on Ξ.

Finally, it may be noted that the time-consuming sweep over all directions in order to determine radiance distributions may be altogether eliminated by employing a "fisheye" lens which maps each ray in a fixed incoming 2π solid angle into a unique point in a small circular area at the base of the lens. This lens would allow an instantaneous photorecording of a hemispherical region of radiance values, which may be analyzed by a specially programmed computer. Such a device has been studied by John Tyler and Raymond Smith at Scripps Institution of Oceanography.

13.2 Operational Definition of Beam Transmittance

We consider next some possible experimental means of determining the beam transmittance of a general path of sight in a natural hydrosol. The development in Sec. 3.10 of the concept of beam transmittance starting from the interaction principle exhibits the physical foundations and basic meaning of the beam transmittance concept, and we now show how we can build on that development in several ways, so that beam transmittances can be obtained using standard radiance measuring equipment.

General Two-Path Method

Let $\mathcal{P}_r(x_1,\xi)$ and $\mathcal{P}_r(x_2,\xi)$ be two parallel paths of length r as depicted in Fig. 13.4. We now show how measurements of the radiances at the extremities of these paths can lead to a determination of $T_r(x,\xi)$, the beam transmittance of the path $\mathcal{P}_r(x,\xi)$. We need only arrange matters so that all three paths are in a regular neighborhood of paths (Def. 2, Sec. 9.5), i.e., so that:

$$T_r(x_1,\xi) = T_r(x_2,\xi) = T_r(x,\xi) \tag{1}$$

and

$$N_r^*(y_1,\xi) = N_r^*(y_2,\xi) = N_r^*(y,\xi) \quad . \tag{2}$$

Such paths are often encountered in real optical media, so that what follows is of more than academic interest. By means of (5) of Sec. 3.13, the apparent radiances at the terminal points of the paths $\mathcal{P}_r(x_1,\xi)$, $\mathcal{P}_r(x_2,\xi)$ are expressible as:

$$N(y_1,\xi) = N(x_1,\xi)T_r(x_1,\xi) + N_r^*(y_1,\xi) \tag{3}$$

$$N(y_2,\xi) = N(x_2,\xi)T_r(x_2,\xi) + N_r^*(y_2,\xi) \quad . \tag{4}$$

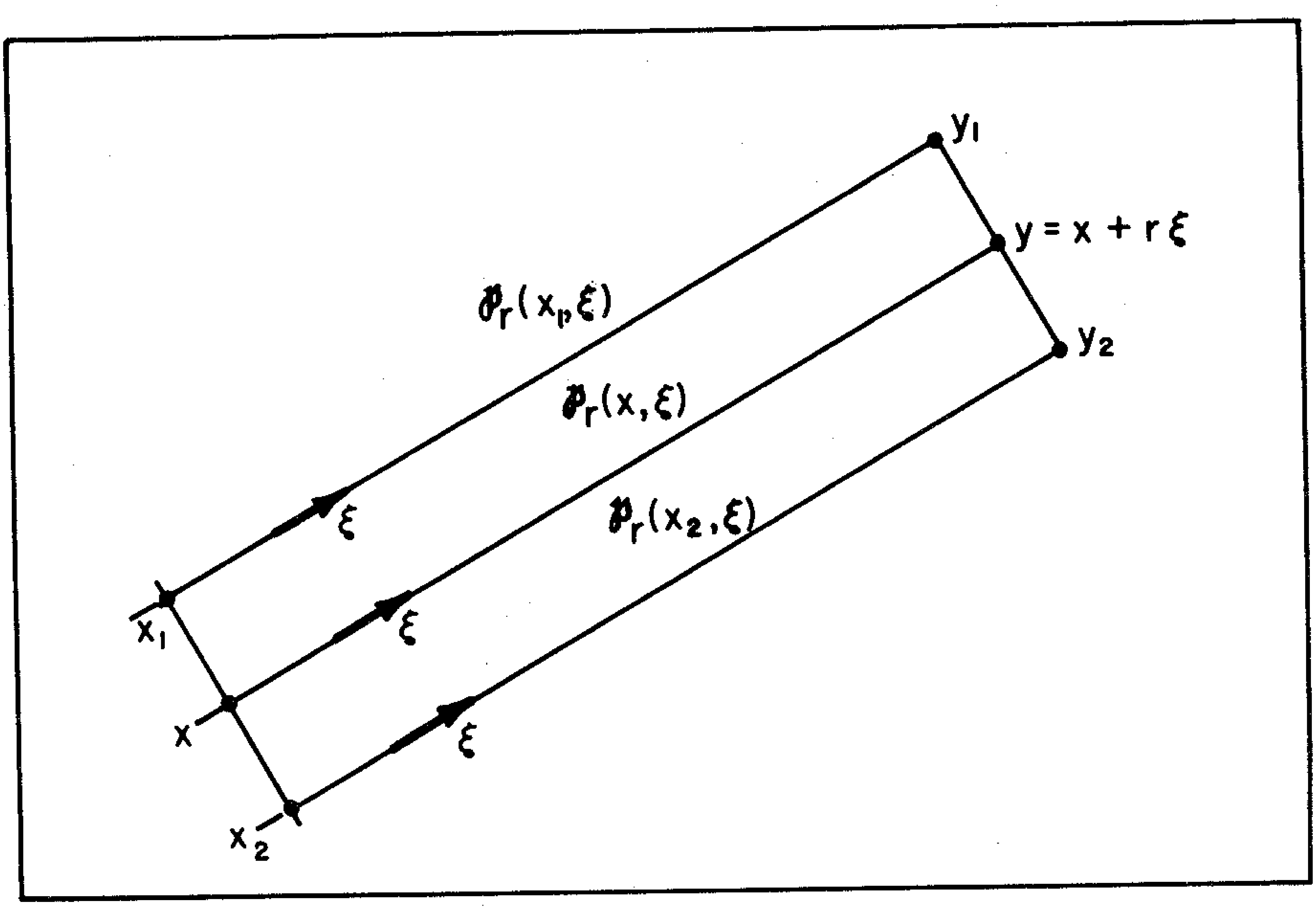

FIG. 13.4 The two-path method for beam transmittance.

Writing:

"$\Delta N(y,\xi)$" for $N(y_1,\xi) - N(y_2,\xi)$ (5)

"$\Delta N(x,\xi)$" for $N(x_1,\xi) - N(x_2,\xi)$ (6)

and subtracting (4) from (3), we have:

$$T_r(x,\xi) = \frac{\Delta N(y,\xi)}{\Delta N(x,\xi)} \quad . \tag{7}$$

Now for theoretical or practical purposes it is possible to use either the natural radiances occurring at x_1, x_2, y_1, y_2 for computations with (7) or to place artificial sources at the points x_1, x_2. In either case the method proceeds by measuring the four radiances under such conditions, and then using (7) to determine $T_r(x,\xi)$. In either case therefore, (7) will yield up numerical determinations of $T_r(x,\xi)$ so long as (1) and (2) hold. Equation (7) may thus provide an operational definition of $T_r(x,\xi)$.

General One-Path Method

The preceding development from (1) to (7) may be reinterpreted so that there is only one path, namely $\mathcal{P}_r(x,\xi)$, over which at time t_1 there is an artificial or natural radiance distribution such that (3) holds and a small time later at t_2, (4) holds. For example, a light beam along the path $\mathcal{P}_r(x,\xi)$ may change radiance arbitrarily or may blink periodically so that in any case it has two distinct radiances $N(x,\xi,t_1)$ and $N(x,\xi,t_2)$ with corresponding observable radiances $N(y,\xi,t_1)$, $N(y,\xi,t_2)$. If (1) and (2) hold in the present case, i.e., if:

$$T_r(x,\xi,t_1) = T_r(x,\xi,t_2) = T_r(x,\xi) \tag{8}$$

$$N_r^*(y,\xi,t_1) = N_r^*(y,\xi,t_2) = N_r^*(y,\xi) \tag{9}$$

then:

$$T_r(x,\xi) = \frac{\Delta N(y,\xi,t)}{\Delta N(x,\xi,t)} \quad , \tag{10}$$

where now we have written:

"$\Delta N(y,\xi,t)$" for $N(y,\xi,t_1) - N(y,\xi,t_2)$ (11)

"$\Delta N(x,\xi,t)$" for $N(x,\xi,t_1) - N(x,\xi,t_2)$. (12)

Thus by either spatial modulation of radiances, as in (7), or temporal modulation of radiances, as in (10), the beam transmittance over the corresponding regular neighborhood of paths may be operationally defined.