

13.3 Operational Definitions of Path Radiances and Path Functions

The path radiance and path function were conceptually formulated by means of the interaction principle in Sec. 3.12, and so are firmly grounded in the basic principles of radiative transfer theory. We now turn those conceptual formulations into useful operational definitions of the path radiance and path function.

Operational Formulation of Path Radiance

There are two general ways in which we may obtain the path radiance $N_R^*(z, \xi)$ of a path $\mathcal{P}_R(x, \xi)$ in an optical medium, as depicted, e.g., in Fig. 13.5. Each method is based on the fundamental relation:

$$N(z, \xi) = N(x, \xi)T_R(x, \xi) + N_R^*(z, \xi) \quad (1)$$

which is an instance of (5) of Sec. 3.13.

Suppose that it is possible to set:

$$N(x, \xi) = 0 \quad ,$$

then (1) yields $N_R^*(z, \xi)$ immediately. This is the *dark target* method of determining $N_R^*(z, \xi)$. Workable equivalents to the

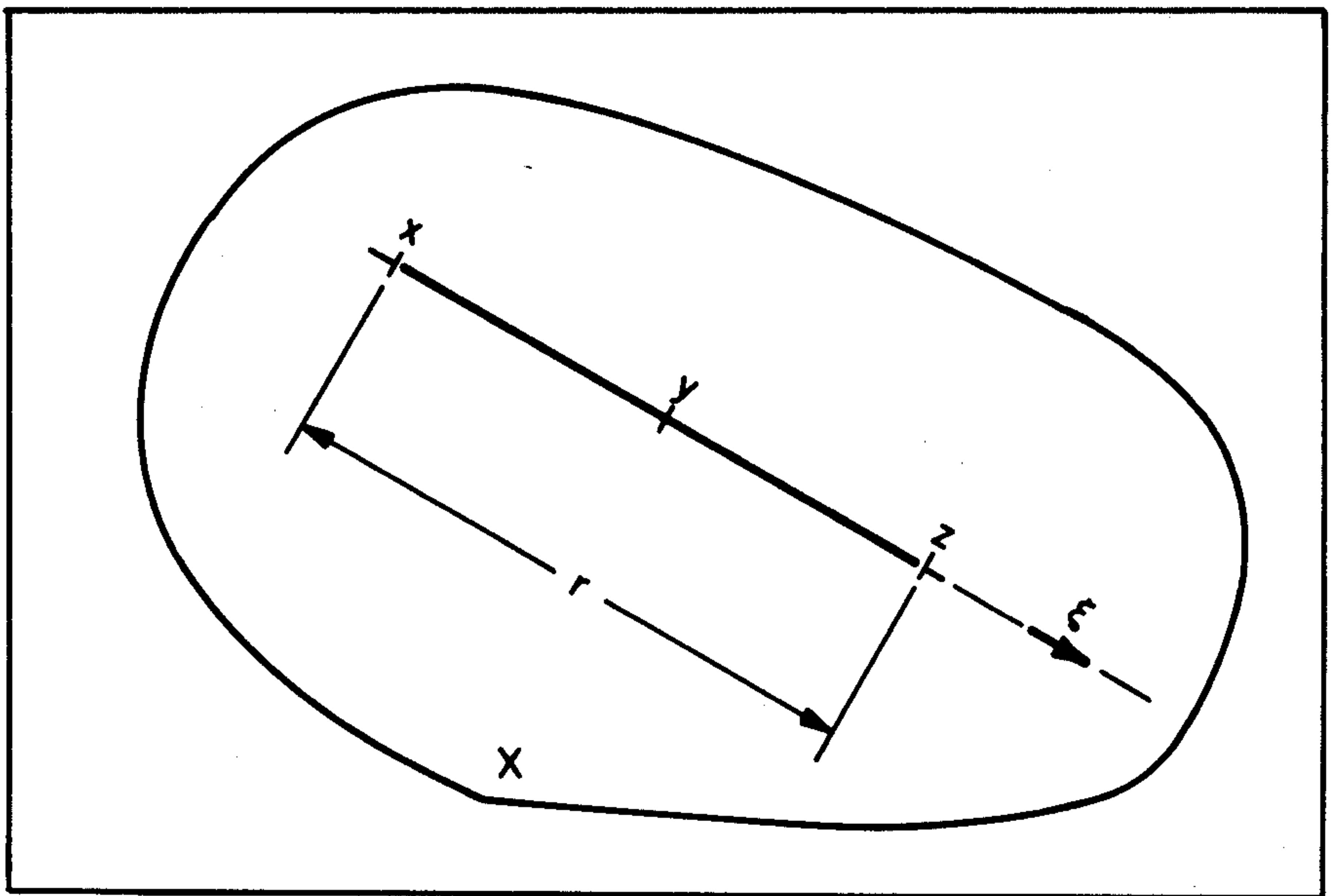


FIG. 13.5 A path $\mathcal{P}_R(x, \xi)$ with initial point x , internal point y , and end point z .

condition $N(x, \xi) = 0$ may be obtained by actually viewing a small black light trap placed at x .

The alternative method of finding $N_r^*(z, \xi)$ is to first determine the beam transmittance of $\mathcal{P}_r(x, \xi)$, as in Sec. 13.2. Then, (1) allows the rigorous computation of $N_r^*(z, \xi)$ in the form:

$$N_r^*(z, \xi) = N(z, \xi) - N(x, \xi)T_r(x, \xi) \quad (2)$$

assuming all three quantities on the right in (2) are measurable. The two radiances on the right in (2) may be as they are found in nature, or as generated by artificial sources.

Operational Formulation of Path Function

An operational formulation of $N_*(x, \xi)$ can be based on (15) of Sec. 3.12. For suppose $\mathcal{P}_r(x, \xi)$ is a path so short that the radiance distributions over it are independent of location and that for all practical purposes $T_r(x, \xi) = 1$. Then, very nearly:

$$N_*(x, \xi) = \frac{N_r^*(x, \xi)}{r} \quad (3)$$

in which $N_r^*(x, \xi)$ may be determined by means of the preceding dark target method or by means of (2). In general, (15) of Sec. 3.12 may be written rigorously as:

$$N_r^*(z, \xi) = N_*(x, \xi)r + o(r) \quad (4)$$

where $o(r)/r \rightarrow 0$ as $r \rightarrow 0$ (cf., (2) of Sec. 3.12). From this, (3) follows for small r .

An alternative means of finding $N_*(z, \xi)$ under special lighting conditions and in homogeneous media is given by (1) of Sec. 4.3. For, then:

$$N_r^*(z, \xi) = \frac{N_*(x, \xi)}{\alpha} \left[1 - e^{-\alpha r} \right] \quad (5)$$

over a path $\mathcal{P}_r(x, \xi)$ (as in Fig. 3.33) which lies in a homogeneous, uniformly lighted stretch of optical medium. Knowing $N_r^*(z, \xi)$ and α , then yields $N_*(x, \xi)$. Formulation (5) casts light on what is meant in (3) by "small r ." We shall consider r "small" when $\alpha r < 1$, i.e., when $r < 1/\alpha = L_\alpha$, where L_α is the attenuation length of the medium.

Actual measurement procedures leading to $N_*(x, \xi)$ can be based on a device of the kind depicted in Fig. 13.6. The device consists of a radiance meter directed into a black target. The ambient radiance distribution $N(x, \cdot)$ at a general

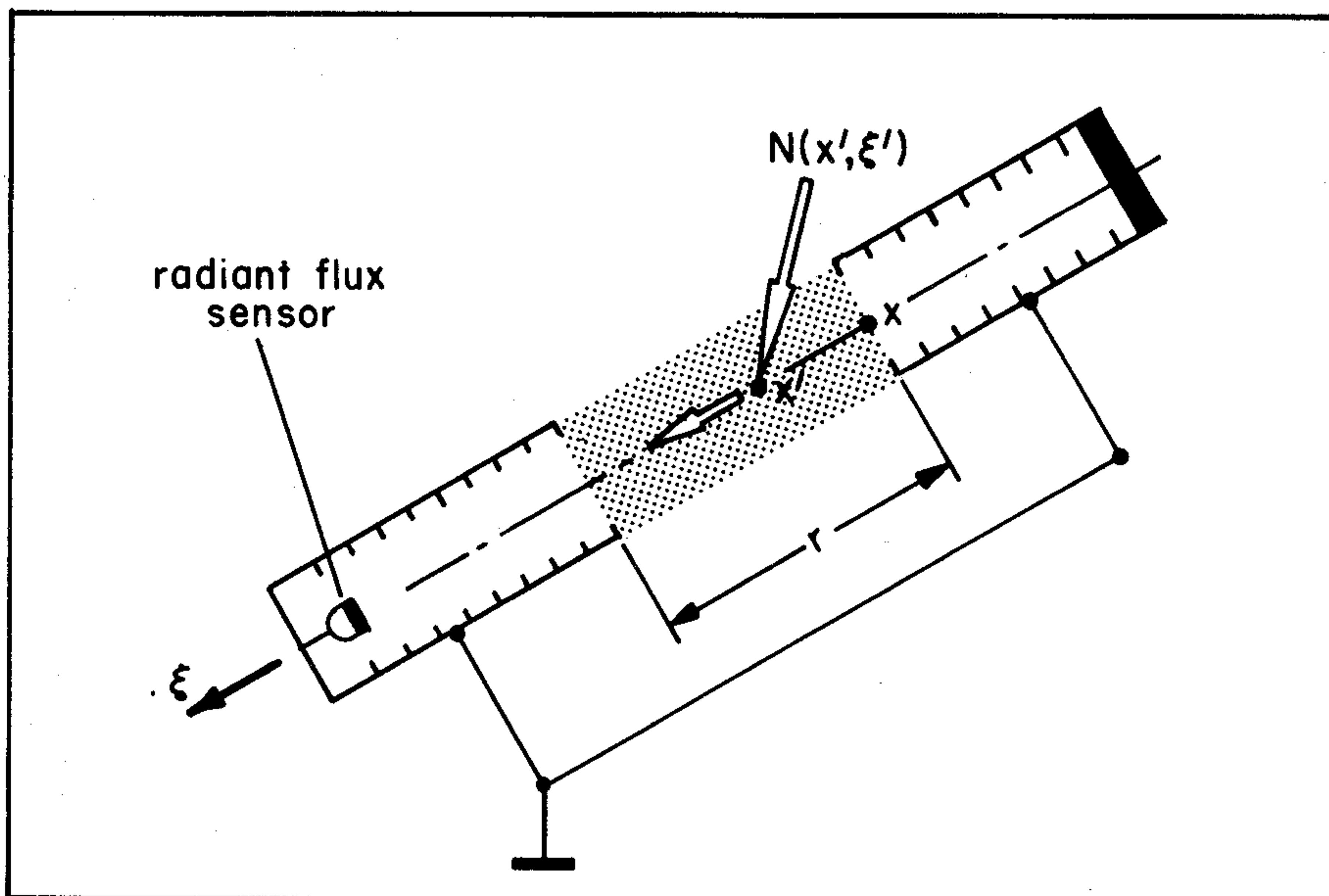


FIG. 13.6 Dark target arrangement for determining path function value $N_*(x, \xi)$.

point x is scattered into the direction ξ by the material comprising the cylinder around the short path segment $\mathcal{P}_r(x, \xi)$ (shown shaded) between the radiance meter and the black target. Conditions can usually be arranged so that the assumptions leading to (3) are satisfied with reasonable accuracy. An illustration of a *path function meter* of the kind described here may be found in Ref. [80].

Further, by combining (2) and (3) we have: very nearly:

$$N_*(x, \xi) = \frac{N(z, \xi) - N(x, \xi)T_r(x, \xi)}{r}, \quad (5)$$

the setting for which is depicted in Fig. 3.33. We shall adopt (3) as the operational definition of $N_*(x, \xi)$.

Still further, if the volume scattering function is known, then it follows at once from (8) of Sec. 3.14 that:

$$N_*(x, \xi_j) = \sum_{i=1}^n N(x, \xi_i) \sigma(x; \xi_i; \xi_j) \Delta\Omega_i \quad (6)$$

for each j , $j = 1, \dots, n$ over some suitable partition $\{E_1, \dots, E_n\}$ of E . Hence by measuring the functions $N(x, \cdot)$ and $\sigma(x; \cdot; \cdot)$, $N_*(x, \cdot)$ can be obtained by direct computation. Computations of this kind were explored in Ref. [213].

Finally, the *K-method* of determining the path function may be used in virtually all real media. The K-method, as

developed in Ref. [219], is based on the canonical form of the equation of transfer. To illustrate the method, we consider an arbitrarily stratified plane-parallel medium. The associated canonical form of the equation of transfer is given by (21) of Sec. 4.5, and is repeated here:

$$N(z, \xi) = \frac{N_*(z, \xi)}{\alpha(z) + K(z, \xi) \cos \theta} .$$

Solving this for $N_*(z, \xi)$:

$$N_*(z, \xi) = [\alpha(z) + K(z, \xi) \cos \theta] N(z, \xi) . \quad (7)$$

Hence if all quantities on the right side of (7) are known, or determinable, then $N_*(z, \xi)$ is determinable. Examples of the use of (7) using real data are given in Ref. [219].

13.4 Operational Definition of Volume Attenuation Function

The volume attenuation function, as developed in Sec. 3.11 from first principles may be obtained in several alternate empirical procedures by performing measuring operations in natural waters which contain either natural or artificial sources of radiant energy. In this section we shall outline some of the more fundamental of these procedures.

We consider first the most direct operational means of defining α . From (1) of Sec. 3.11 we have:

$$\alpha(x, \xi) = \frac{1 - T_r(x, \xi)}{r} + O(r) \quad (1)$$

where $O(r)$ is a quantity which goes to zero with r , and is on the order of magnitude of the quantity αr . Thus, if beam transmittance measurements are available, we have, very nearly:

$$\alpha(x, \xi) = \frac{1 - T_r(x, \xi)}{r} \quad (2)$$

for relatively small r , that is for r on the order of 1/2 to 1 meter in most oceanic waters and for wavelengths in the vicinity of 500 μ . Of course for more turbid waters, with respect to the given wavelength, r must be chosen correspondingly smaller in order for the term $O(r)$ to be negligible. In virtually all natural optical media α will be independent of ξ , and we henceforth drop " ξ " from the notation.

An alternate operational definition of α , one that is quite general and which we shall adopt here, is that based on (2) or (5) of Sec. 3.11. Thus:

$$\alpha(z) = - \frac{1}{T_r(x, \xi)} \frac{dT_r(x, \xi)}{dr} \quad (3)$$