

developed in Ref. [219], is based on the canonical form of the equation of transfer. To illustrate the method, we consider an arbitrarily stratified plane-parallel medium. The associated canonical form of the equation of transfer is given by (21) of Sec. 4.5, and is repeated here:

$$N(z, \xi) = \frac{N_*(z, \xi)}{\alpha(z) + K(z, \xi) \cos \theta} .$$

Solving this for $N_*(z, \xi)$:

$$N_*(z, \xi) = [\alpha(z) + K(z, \xi) \cos \theta] N(z, \xi) . \quad (7)$$

Hence if all quantities on the right side of (7) are known, or determinable, then $N_*(z, \xi)$ is determinable. Examples of the use of (7) using real data are given in Ref. [219].

13.4 Operational Definition of Volume Attenuation Function

The volume attenuation function, as developed in Sec. 3.11 from first principles may be obtained in several alternate empirical procedures by performing measuring operations in natural waters which contain either natural or artificial sources of radiant energy. In this section we shall outline some of the more fundamental of these procedures.

We consider first the most direct operational means of defining α . From (1) of Sec. 3.11 we have:

$$\alpha(x, \xi) = \frac{1 - T_r(x, \xi)}{r} + O(r) \quad (1)$$

where $O(r)$ is a quantity which goes to zero with r , and is on the order of magnitude of the quantity αr . Thus, if beam transmittance measurements are available, we have, very nearly:

$$\alpha(x, \xi) = \frac{1 - T_r(x, \xi)}{r} \quad (2)$$

for relatively small r , that is for r on the order of 1/2 to 1 meter in most oceanic waters and for wavelengths in the vicinity of 500 m μ . Of course for more turbid waters, with respect to the given wavelength, r must be chosen correspondingly smaller in order for the term $O(r)$ to be negligible. In virtually all natural optical media α will be independent of ξ , and we henceforth drop " ξ " from the notation.

An alternate operational definition of α , one that is quite general and which we shall adopt here, is that based on (2) or (5) of Sec. 3.11. Thus:

$$\alpha(z) = - \frac{1}{T_r(x, \xi)} \frac{dT_r(x, \xi)}{dr} \quad (3)$$

in which we have written:

$$"z" \quad \text{for} \quad x + r\xi \quad . \quad (4)$$

The physical situation associated with (3) is depicted in Fig. 13.5. The important point to observe in (3) is that if the path $\mathcal{O}_r(x, \xi)$ is varied in X by holding x, ξ fixed and varying r (which is the intended meaning of the derivative in (3)), then the result of the indicated operations on $T_r(x, \xi)$ is precisely $\alpha(z)$, the value of α at *terminal point* z of $\mathcal{O}_r(x, \xi)$.

As a simple application of (3), consider an experimental arrangement such as that depicted in Fig. 13.4, and as summarized in (7) of Sec. 13.2. Then by using the results of this arrangement, (3) becomes:

$$\alpha(y) = - \frac{d}{dr} \ln \left[\frac{\Delta N(y, \xi)}{\Delta N(x, \xi)} \right] \quad (5)$$

where now $y = x + r\xi$, as shown in Fig. 13.4.

Further applications of (3) are possible. For example, the results of the one-path method of determining $T_r(x, \xi)$, as in (10) of Sec. 13.2, may be used in (3). Also suggested by (3) is the possibility of self luminous *moving probes* in the sea, that is probes with self contained light sources and radiance pickups. For further details in this direction, the reader is referred to [238], which contains a general theory for such advanced techniques.

An alternate method of measuring α to those considered above makes use of a frequently occurring regularity of the natural lighting conditions in homogeneous media, and is called the *dark target technique* (cf. Sec. 3.3). For suppose the conditions are just right in a medium so that the classical canonical equation for apparent radiance (2) of Sec. 4.4 holds:

$$N_r(z, \xi) = N_o(z, \xi) e^{-\alpha r} + \frac{N_*(z, \xi)}{\alpha} [1 - e^{-\alpha r}] \quad (6)$$

where ξ is a horizontal direction (so that $\cos \theta = 0$). Next, place a black target of some kind at a distance r from the observation point, so that $N_o(z, \xi) = 0$ in (6). Then under these conditions:

$$N_r(z, \xi) = N_r^*(z, \xi) = \frac{N_*(z, \xi)}{\alpha} [1 - e^{-\alpha r}] \quad . \quad (7)$$

Here we observe that $N_*(z, \xi)/\alpha$ is the equilibrium radiance $N_q(z, \xi)$ and which in this case is simply the measurable horizontal radiance $N(z, \xi)$ at depth z in direction ξ . Letting $r \rightarrow \infty$, we have:

$$\lim_{r \rightarrow \infty} N_r(z, \xi) = \frac{N_*(z, \xi)}{\alpha} = N(z, \xi)$$

an observable quantity, namely $N_q(z, \xi)$ (cf., (4) of Sec. 4.4). Hence (7) becomes:

$$N_r^*(z, \xi) = N_q(z, \xi) \left[1 - e^{-\alpha r} \right] \quad (8)$$

in which $N_r^*(z, \xi)$, $N_q(z, \xi)$ and r are in principle measurable. Hence if $N_q(z, \xi)$ is measured for z and ξ and then a small black target placed at a distance r on the same path from the observation point, (8) yields a means for obtaining α . Of course, care must be taken so that the $N_q(z, \xi)$ is not perturbed too greatly over the path segment between the target and observation point.

Observe that if one sets $r = \infty$, then (8) becomes:

$$N = N_q = \frac{N_*}{\alpha} \quad (9)$$

which determines α once N and N_* are determined.

A complementary procedure (the *bright target* technique) to that just described arranges matters so that N_* is effectively zero, with the result that (6) implies:

$$-\frac{1}{r} \ln \frac{N_r(z, \xi)}{N_o(z, \xi)} = \alpha \quad (10)$$

Finally, we observe that α may be determined from the relation:

$$\alpha = a + s \quad (11)$$

when the volume absorption and total scattering functions are known. Relation (11) follows from the definition in (4) of Sec. 4.2. Ways of independently measuring a and s will be considered in Sec. 13.7 and Sec. 13.8.

Canonical Equation Method

Using (21) of Sec. 4.5, rearranged in the following form:

$$\alpha(z) = \frac{N_*(z, \xi)}{N(z, \xi)} - K(z, \xi) \cos \theta, \quad (12)$$

we find that $\alpha(z)$ is determinable via direct light measurements of $N(z, \xi)$ and, say, a dark target technique for $N_*(z, \xi)$ as in Sec. 13.3.