13.6 Operational Definition of Volume Scattering Function

The operational definition of the volume scattering function, to which we now devote our attention, is second in importance only to that for beam transmittances (or equivalently the volume attenuation function). As we found in Chapter 9, the two concepts α and σ form a fundamental set for radiative transfer theory, from which all other optical properties may be deduced. It is therefore important that the operational definition for σ be given at least as much care in formulation and experimental realization as that given to α(or T_r). Thus we start with the basic relation (8) of Sec. 3.14 (or (4) of Sec. 3.17) which relates the observable radiances N_* and N at a point (and as measured by radiance meters without polarizers) with the values of σ at that point:

$$N_*(x,\xi) = \int_\Xi N(x,\xi')\sigma(x;\xi';\xi)d\Omega(\xi') \quad . \qquad (1)$$

We shall base our operational definition of σ on this relation in order that the practical numerical uses of σ, so found, will be consistent with the theoretical uses of σ.

In order to isolate the values $\sigma(x;\xi';\xi)$ of $\sigma(x;\cdot;\cdot)$ for distinct pairs ξ',ξ of directions, we choose $N(x,\cdot)$ so that it is zero over all of Ξ except a small conical subset Ξ' about ξ' of solid angle magnitude $\Omega(\xi')$. Over Ξ', $N(x,\cdot)$ is to be uniform of magnitude $N(x,\xi')$. Then (1) becomes.

$$N_*(x,\xi) = \int_{\Xi'} N(x,\xi')\sigma(x;\xi';\xi)d\Omega(\xi')$$

$$= N(x,\xi')\sigma(x;\xi';\xi)\Omega(\xi') + o(\Omega(\xi')) \qquad (2)$$

where $o(\Omega(\xi'))$ is a quantity such that $o(\Omega(\xi'))/\Omega(\xi')$ goes to zero with $\Omega(\xi')$. Matters can usually be arranged in either natural or artificial light fields, so that (2) holds. It follows from (2) that, very nearly:

$$\sigma(x;\xi';\xi) = \frac{N_*(x,\xi)}{N(x,\xi')\Omega(\xi')} \qquad (3)$$

(i.e., to within $o(\Omega(\xi'))/\Omega(\xi')$).

Equation (3) suggests the experimental arrangement depicted in Fig. 13.11. An element of volume of the optical medium about point x is irradiated in the direction ξ' by a source S so that, at x, the irradiating flux is of radiance magnitude $N(x,\xi')$ and arrives in a solid angle of magnitude $\Omega(\xi')$. The incident radiant flux is scattered in all directions by the element of volume; some of the scattered flux being directed in particular along the direction ξ. At a distance r' from x a radiance meter G records the path radiances $N_\ell^*(x,\xi)$ generated throughout the element of volume, where ℓ is the length of the element of volume along the direction ξ. Matters are so arranged that the beam transmittance

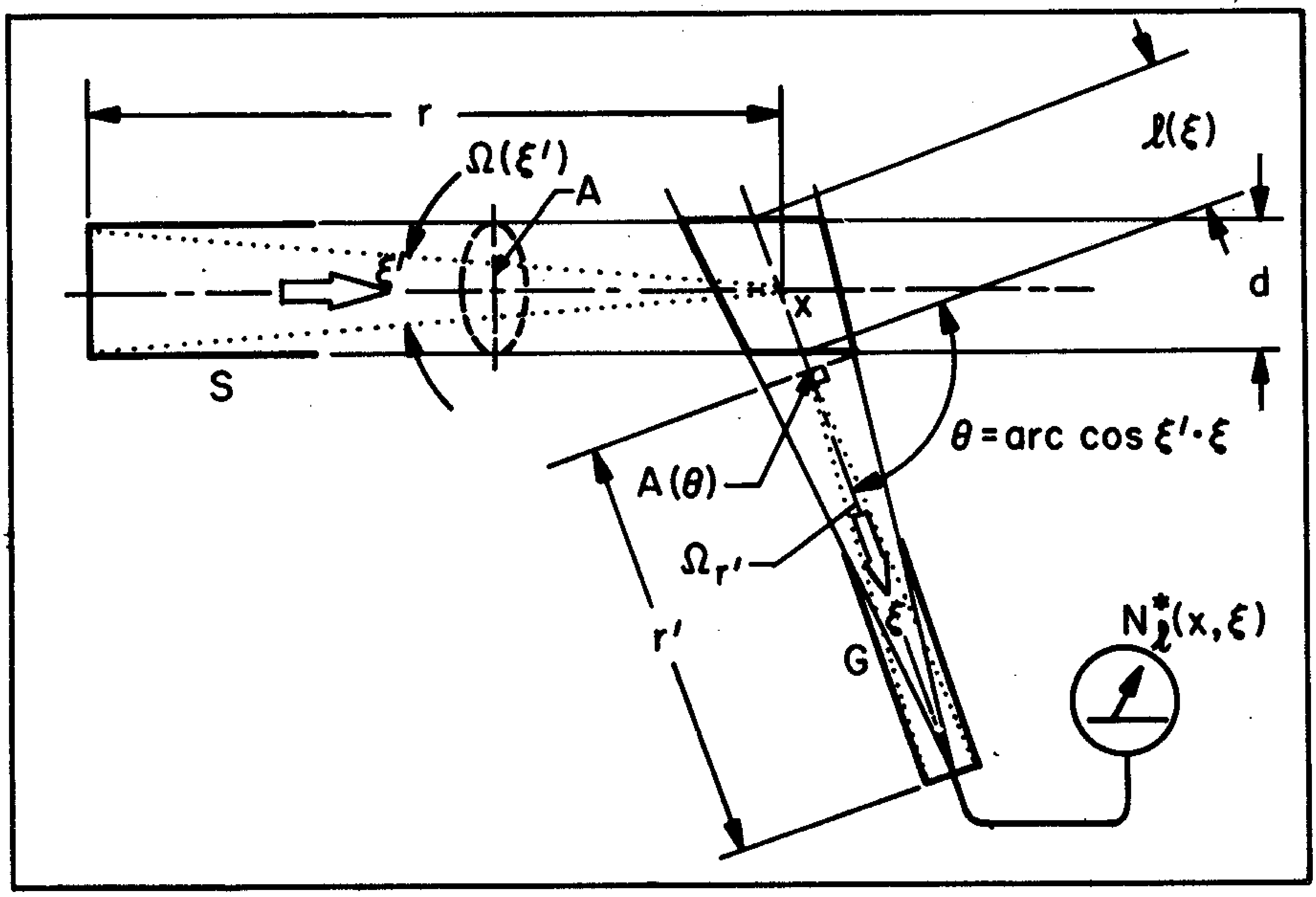

FIG. 13.11 Arrangement for measuring volume scattering function.

$T_{r'}(x,\xi)$ is essentially 1, and that the path of sight from G to x is essentially dark so that no further flux is added to that comprising $N_\ell^*(x,\xi)$. From (2) of Sec. 3.12, we may approximate $N_*(x,\xi)$ by:

$$\frac{N_\ell^*(x,\xi)}{\ell(\xi)} ,$$

so that (3) above yields:

$$\boxed{\sigma(x;\xi';\xi) = \frac{N_\ell^*(x,\xi)}{N(x,\xi')\,\ell(\xi)\,\Omega(\xi')}} \qquad (4)$$

which we adopt as the operational definition of $\sigma(x;\xi';\xi)$.

Most natural optical media are isotropic, so that the values $\sigma(x;\xi';\xi)$ depend only on $\xi'\cdot\xi$ (= arc cos θ, in Fig. 13.11). Further, since the σ measuring experiment takes place at a fixed location x with irradiation fixed along ξ', we can usually drop references to x and ξ' in discussions of experimental results. Thus let us write *ad hoc:*

"$\sigma(\theta)$" for $\sigma(x;\xi';\xi)$

"N_r" for $N(x,\xi')$

$$\text{"}N_\ell^*(\theta)\text{"} \quad \text{for} \quad N_\ell^*(x,\xi)$$

$$\text{"}\ell(\theta)\text{"} \quad \text{for} \quad \ell(\xi')$$

and:

$$\text{"}\Omega_r\text{"} \quad \text{for} \quad \Omega(\xi') \quad .$$

We now may inquire about the directional distribution of the scattered flux. First we rewrite (4) as:

$$\sigma(\theta) = \frac{1}{N_r\Omega_r} \cdot \frac{N_\ell^*(\theta)}{\ell(\theta)} \quad . \tag{5}$$

When this operation on the observable quantities N_r, Ω_r, $N_\ell^*(\theta)$ and $\ell(\theta)$ is examined in detail, we uncover the following set of experimental facts:

(i) $\sigma(\theta)$ is found to be independent of the amount of irradiation $N_r\Omega_r$.

(ii) $\sigma(\theta)$ is independent of the magnitude of $\ell(\theta)$.

(iii) If θ, r, r', d, and N_o are all held fixed and G is swung around the beam, $\sigma(\theta)$ remains fixed.

(iv) $\sigma(\theta)$ is independent of the *absolute* orientation of S and G about x (medium is isotropic).

These four experimental findings form the empirical basis for the conclusion that $\sigma(\theta)$ is an inherent optical property of the medium. (The theoretical basis for this property of σ is developed in Example 1 of Sec. 3.17.) Clearly, on the basis of (i), $\sigma(\theta)$ does not depend on the absolute amount of irradiation on the element of volume of the medium. Furthermore, on the basis of (ii), the *relative amount* of flux observed to be scattered at a given angle θ by a *small* irradiated volume does not depend on the length of the path of sight through that small volume. Finally, according to (iii) and (iv), $\sigma(\theta)$ does not depend on the spatial orientation of the plane formed by the direction ξ' irradiating beam and the direction ξ of flow of the scattered flux. The function σ, which thus depends only on θ at point x is called the *volume scattering function* at x. Its operational definition is given by (4) or (5), or succinctly by the equivalent form:

$$\sigma(\theta) = \frac{N_*(\theta)}{N_r\Omega_r} \quad , \tag{6}$$

where we have written:

$$\text{"}N_*(\theta)\text{"} \quad \text{for} \quad \frac{N_\ell^*(\theta)}{\ell(\theta)} \quad , \tag{7}$$

$N_*(\theta)$ is a quantity independent of the length $\ell(\theta)$ (fact (ii)). N_r and Ω_r refer to the radiance and solid angle subtense (at the point x) of the irradiating source. The dimensions of σ

are: *per unit length per unit solid angle.* Both the unit of length and solid angle are clearly in the *direction* ξ of observation of the irradiated volume.

An alternate form of (4) may be obtained by observing that, by the definition of surface radiance:

$$N_\ell^*(\theta) = \frac{P_\ell^*(\theta)}{A(\theta)\Omega_{r'}} \quad , \tag{8}$$

where $A(\theta)$ is the area of the projection of the element of volume on a plane perpendicular to ξ (Fig. 13.11), and $\Omega_{r'}$ is the solid angle subtense of the radiance meter's collecting surface at x. Using (8) in (4) we have:

$$\sigma(\theta) = \frac{1}{N_r\Omega_r} \cdot \frac{P_\ell^*(\theta)}{\ell(\theta)A(\theta)\Omega_{r'}} \quad . \tag{9}$$

We next observe that (to within second order effects)

$$H_r = N_r\Omega_r$$

and that:

$$V(\theta) = \ell(\theta)A(\theta)$$

$$J_\ell^*(\theta) = P_\ell^*(\theta)/\Omega_r$$

where H_r is irradiance produced by N_r over Ω_r, $V(\theta)$ is the volume of the irradiated element, and $J_\ell^*(\theta)$ is the intensity of the radiant flux scattered in the direction ξ by the element of volume (cf. Sec. 2.9 for a detailed study of the concept of radiant intensity). With these observations, (6) becomes:

$$\sigma(\theta) = \frac{J_\ell^*(\theta)}{H_rV(\theta)} \quad . \tag{10}$$

Some experimental arrangements may favor (10) over (4) or (5); but actually, to measure $J_\ell^*(\theta)$, one must in essence measure $N_\ell^*(\theta)$, so that (4) (or (6)) is in the last analysis operationally more basic than (10). It is of interest to note that the alternate form (10) can be anticipated by a purely dimensional analysis of the path function (cf. note (h) following Table 3 of Sec. 2.12).

σ-Recovery Procedures

We consider next the problem of determining the volume scattering function at a point x in a medium when it is not possible or not convenient to control the light field about x in the very special way which led the associated theory from (1) to (4). Thus, suppose that we know both $N_*(x,\cdot)$ and $N(x,\cdot)$ at point x. Is it possible to recover the information about $\sigma(x;\cdot;\cdot)$? We are led by the query to rewrite (1) as:

$$N_*(x,\xi) = \sum_{i=1}^{n} \int_{\Xi_i} N(x,\xi')\sigma(x;\xi';\xi)d\Omega(\xi') \qquad (11)$$

where the n direction sets Ξ_i partition Ξ (see, e.g., Fig. 13.1). If the partition is fine enough so that $N_*(x,\xi_i)$, with ξ_i in Ξ_i, is representative of all the values $N_*(x,\xi)$ with ξ in Ξ_i, and similarly with the values $N(x,\xi_i)$ and $\sigma(x;\xi_i;\xi_j)$, then writing:

$$\text{"}N_{*j}\text{"} \quad \text{for} \quad N_*(x,\xi_j) \qquad (12)$$

$$\text{"}N_i\text{"} \quad \text{for} \quad N(x,\xi_i) \qquad (13)$$

$$\sigma'ij \quad \text{for} \quad \sigma(x;\xi_i;\xi_j)\Omega(\Xi_i) \qquad (14)$$

(11) is approximated by:

$$\boxed{\mathbf{N}_* = \mathbf{N}\boldsymbol{\sigma}'} \qquad (15)$$

where $\mathbf{N}_*$ and $\mathbf{N}$ are n component vectors defined by their n components in (12) and (13). Furthermore, $\boldsymbol{\sigma}'$ is an $n \times n$ matrix whose entries are defined in (14).

Equation (15) may be used as a basis for an operational determination of σ. For, apparently what we must do in general is to find n linearly independent vectors, $\mathbf{N}_i$, each of the kind appearing in front of $\boldsymbol{\sigma}'$ in (15). If N_{*i} are the corresponding path function values to these n radiance vectors, then we can make an invertible $n \times n$ matrix η whose rows are the $\mathbf{N}_i$. As a result, we have from (15):

$$\boxed{\sigma' = \eta^{-1}\,\eta_*} \qquad (16)$$

where η_* is the $n \times n$ matrix whose rows are the $\mathbf{N}_{*i}$. In order to obtain the n $\mathbf{N}_i$, one may, for example, select n different locations in a part of the medium over which σ is known not to change. The n linearly independent vectors are also very readily obtained by means of artificial lighting arrangements introduced into the natural light field if necessary.

An alternate means of finding σ using natural light fields makes explicit use of the generally available property of σ that $\sigma(x;\xi';\xi)$ depends only on $\xi' \cdot \xi$. For then (1) can be rewritten over again in the form (11), but now each $\Xi_i(\xi')$ is a spherical zone on Ξ symmetric about ξ', as in Fig. 13.12. If we write, *ad hoc*

$$"\sigma_i" \quad \text{for} \quad \sigma(x;\xi';\xi_i) \tag{17}$$

and

$$"h_i(\xi')" \quad \text{for} \quad \int_{\Xi_i(\xi')} N(x,\xi)\,d\Omega(\xi) \tag{18}$$

$$"N_*(\xi')" \quad \text{for} \quad N_*(x,\xi') \tag{19}$$

then (11) may be approximated by:

$$N_*(\xi') = \sum_{i=1}^{n} h_i(\xi')\sigma_i \quad . \tag{20}$$

Now suppose n different orientations ξ_i' of ξ' are chosen and let **h** be the matrix made up of rows, the ith of which is:

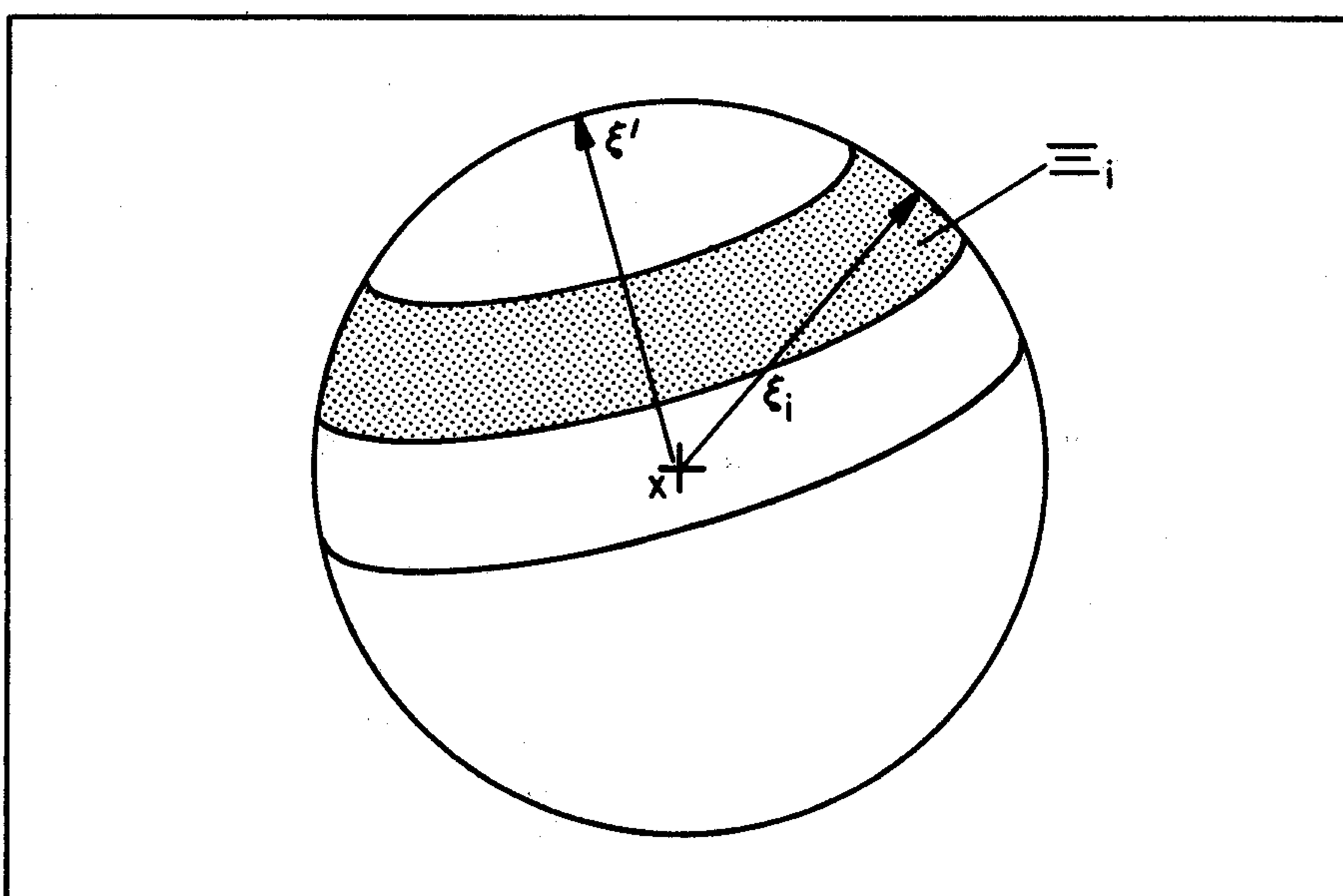

FIG. 13.12 General zone of direction space used in sigma-recovery technique.

$$(h_1(\xi_i'),\ h_2(\xi_i'),\ \ldots,\ h_n(\xi_i'))$$

and arrange matters, if possible, so that **h** is invertible. Then the system of equation (20) takes the form:

$$\mathbf{N}_* = \mathbf{h}\boldsymbol{\sigma}''$$

where $\mathbf{N}_*$ is the vector whose ith component is $N_*(\xi_i')$, and $\boldsymbol{\sigma}''$ is the n component vector whose ith component is σ_i. From this we have:

$$\boxed{\boldsymbol{\sigma}'' = \mathbf{h}^{-1}\,\mathbf{N}_*} \quad . \tag{21}$$

Detailed calculations based on (21) may be found in Ref. [214], along with a survey of practical schemes for inverting **h**.

Determining the Volume Scattering Matrix in the Polarized Case

When the radiance meter is fitted with analyzers for polarized radiance, as described in Sec. 2.10, then a whole new order of reality is opened to the empirical exploration of polarized natural light fields. All unpolarized or polarized radiative transfer phenomena previously viewed with an ordinary radiance tube now spring from a one-dimensional simplicity to a rich four-dimensional complexity. In the case of the volume scattering function, the simple scalar operational definition (4) is raised to a matricial operational definition in a manner now to be explained.

Irradiate an element of volume about point x by radiant flux such that the associated incident polarized radiances are in turn four linearly independent vectors:

$$\left.\begin{array}{l} {}^1\mathbf{N}(x,\xi') \\ {}^2\mathbf{N}(x,\xi') \\ {}^3\mathbf{N}(x,\xi') \\ {}^4\mathbf{N}(x,\xi') \end{array}\right\} \tag{22}$$

These radiance vectors are *local observable vectors*, as defined in Sec. 2.10. The geometric aspects of the scattering situation in the present case are described once again by Fig. 13.11. Let the path radiances generated by the irradiated volume be:

$$\left.\begin{array}{l} {}^1\mathbf{N}_\ell^*(x,\xi) \\ {}^2\mathbf{N}_\ell^*(x,\xi) \\ {}^3\mathbf{N}_\ell^*(x,\xi) \\ {}^4\mathbf{N}_\ell^*(x,\xi) \end{array}\right\} \tag{23}$$

respectively. Thus ${}^{i}\mathbf{N}(x,\xi')$ generates ${}^{i}\mathbf{N}^{*}_{\ell}(x,\xi)$, i = 1, 2, 3, 4, where ℓ is the length of the path of sight through the irradiated scattering volume.

Let $\mathbf{N}(x,\xi')$ be the matrix formed by using the vectors (22) as rows; and let $\mathbf{N}^{*}_{\ell}(x,\xi)$ be the matrix whose rows are the vectors (23). Then, emulating (4) as closely as possible, we write:

$$\text{"}\sigma(x;\xi';\xi)\text{"} \quad \text{for} \quad \frac{[\mathbf{N}(x,\xi')]^{-1}\,\mathbf{N}^{*}_{\ell}(x,\xi)}{\ell(\xi)\Omega(\xi')} \quad . \tag{24}$$

Thus, by measuring the four linearly independent vectors (22) and their associated vectors (23) and by performing the indicated operations in (24), a 4 × 4 matrix $\sigma(x;\xi';\xi)$ is obtained for each choice of x, ξ', and ξ. This matrix is the *Local Observable Volume Scattering Matrix.*

It is not within the scope of the present work to pursue further this matter of the polarized version of the volume scattering function. A discussion of the operational definition of σ and of how it enters into the theory both on foundational and practical levels, may be found in Chapter 12 of Ref. [251].

However, before leaving this matter it is important to observe that there exists at least one set of linearly independent polarized radiance vectors that experimenters may use for their irradiation vectors. Thus consider the first three of the linearly polarized observable radiance vectors listed in Sec. 2.10, along with the right circularly polarized radiance vector. That is, consider:

$${}^{1}\mathbf{N} = \frac{1}{2}\,(2N,0,N,N) \qquad \text{(vertical linear)}$$

$${}^{2}\mathbf{N} = \frac{1}{2}\,(0,2N,N,N) \qquad \text{(horizontal linear)}$$

$${}^{3}\mathbf{N} = \frac{1}{2}\,(N,N,2N,N) \qquad \text{(45° linear)}$$

$${}^{4}\mathbf{N} = \frac{1}{2}\,(N,N,N,0) \qquad \text{(right circular)}$$

where N = 0. The determinant Δ of these four vectors is:

$$\Delta = \frac{N}{2}\begin{vmatrix} 2 & 0 & 1 & 1 \\ 0 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} = \frac{N}{2}\begin{vmatrix} 2 & 0 & 1 & 1 \\ 0 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 0 & 0 & -1 & -1 \end{vmatrix}$$

$$= \frac{N}{2}\begin{vmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 0 & 0 & -1 & -1 \end{vmatrix} = N\begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & -1 & -1 \end{vmatrix}$$

$$= N\begin{vmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ 0 & -1 & -1 \end{vmatrix} = 2N(-2+1) = -2N \neq 0 \qquad (25)$$

Hence the four given vectors above are linearly independent.

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13.7 Direct Measurement of the Volume Total Scattering Function

Our purpose in this section is to outline a method of direct measurement of the volume total scattering function s. We will be concerned principally with the derivation of the formula behind the method and a few words about the kinds of equipment that may be needed for the realization of the method. The general method outlined below will be applicable to any scattering medium, but it appears to be most useful in the experimental study of natural hydrosols. A few additional preliminary observations will help orient the reader and place the present discussion in the appropriate perspective.

The two fundamental inherent optical properties of scattering-absorbing media are the volume scattering function σ, and the volume attenuation function α. The pair (α,σ) is *fundamental* in the sense of Def. 3 of Sec. 9.1, i.e., that from these two, all other attenuation functions, either inherent or apparent, may effectively be found in accordance with certain well-defined rules of computation. However, the pair (α,σ), while being fundamental in the sense just explained, is by no means unique. Thus the pair (a,σ) is also fundamental, where a is the volume absorption function. The connecting link between these two fundamental pairs of optical functions is provided by the notion of the volume total scattering function s, defined by writing:

$$\text{"s"} \quad \text{for} \quad \int_{\Xi} \sigma \, d\Omega$$

(cf., (3) of Sec. 4.2). For then, by definition (i.e., (4) of Sec. 4.2) we have $\alpha = a + s$, so that the pair (α,σ) is known once (a,σ) is, and conversely.

By means of the work in Chapter 3, and also that of Secs. 13.4-13.6, the theory and practice of the direct measurement of α and σ, is now well established. As was shown, the measurement of α is accomplished by various beam transmittance procedures all of which, at their core, spring from