Then, according to (13), if this varies linearly with r, the conditions of (13) are satisfied and the slope of the line is none other than the required value of s.

13.8 Operational Definition of Volume Absorption Function

The volume absorption function a(x), by its very nature, must initially enter the domain of radiative transfer theory in an indirect manner, that is, by means of the defintion which characterizes a(x) as the difference between $\alpha(x)$ and s(x), as was done in (4) of Sec. 4.2. In this section we shall deduce from this definition alternate operational procedures which lead to determinations of a(x) by means of direct measurements of radiometric quantities.

The formula which serves as the basis for the present operational definitions is (15) of Sec. 8.8. Thus, solving for a(x), we have:

$$\boxed{a(x) = -\frac{1}{h(x)}\nabla\cdot\mathbf{H}(x)} \quad . \tag{1}$$

This operational definition of a(x) inverts the usual way of looking at the divergence relation (15) of Sec. 8.8. In (1) we imagine the relation to have its roots in the operations of the real radiometric world. By performing the indicated operations on the right side in (1), a(x) is thereby determined virtually independently of $\alpha(x)$ and s(x).

Fig. 13.13 depicts an operational realization of equation (1). Suppose a janus plate (i.e., an instrument which measures net irradiance $\overline{H}(x,\xi)$, as depicted in Fig. 2.21) is oriented so that it measures $\overline{H}(x,\mathbf{i})$, $\overline{H}(x,\mathbf{j})$, $\overline{H}(x,\mathbf{k})$ at a point x in an optical medium. Suppose further that the janus plate is moved back and forth along the i-axis a distance r about each side of x, as shown in Fig. 13.13. Then:

$$\frac{\overline{H}(x+r\mathbf{i}) - \overline{H}(x-r\mathbf{i})}{2r} ,$$

approximates to:

$$\frac{\partial\overline{H}(x,\mathbf{i})}{\partial r} ,$$

for suitably chosen r. The remaining two derivatives occurring in the representation of the divergence $\nabla\cdot\mathbf{H}$ can be approximated similarly. If the scalar irradiance h(x) at x is determined, then (1) leads to a(x). Equation (1) also suggests several alternate modes of measurement of a(x) in which spherically or cylindrically symmetric light fields, artificially induced, can lead to appropriate forms of $\nabla\cdot\mathbf{H}$ using spherical and cylindrical coordinate systems. These special instances of (1) are best left for adaptation, by interested researchers, to individual cases (cf., e.g., (43) of Sec. 9.2).

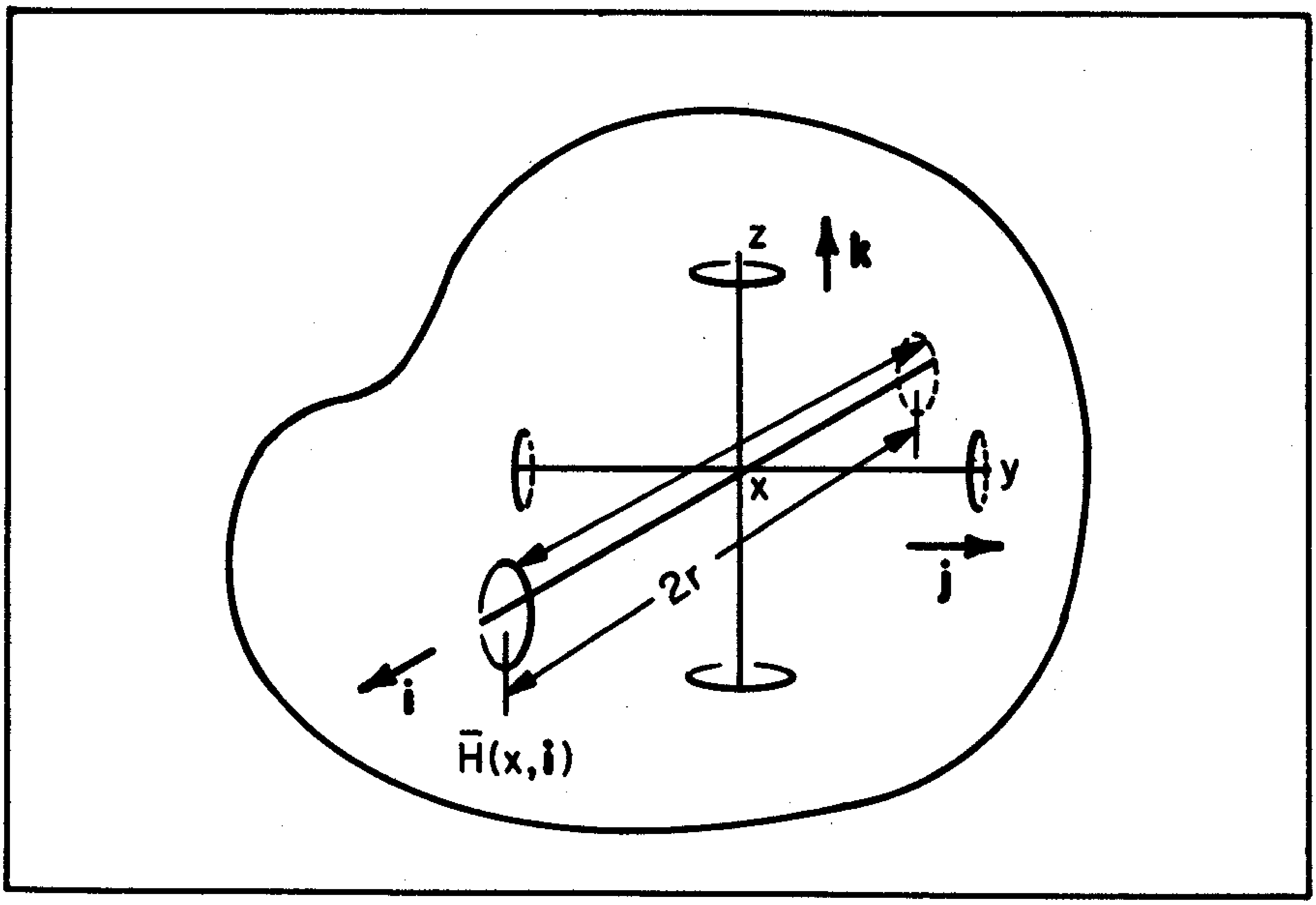

FIG. 13.13 Required motions of a janus-plate probe about a point x which can lead to an estimate of the divergence of the irradiance vector.

Procedures for Stratified Light Fields

In natural light fields, which are usually found to be stratified, (1) reduces to:

$$a(z) = \frac{1}{h(z)} \frac{d\overline{H}(z,\mathbf{k})}{dz} \qquad (2)$$

which shows how to find a(z) at depth z in terms of the vertical derivative of net irradiance $\overline{H}(z,\mathbf{k})$.

When a natural stratified light field decays at a regular (the asymptotic) rate k with depth z in an homogenous medium, then (2) suggests the following integrated form:

$$a = \frac{k[\overline{H}(z,\mathbf{k}) - \overline{H}(y,\mathbf{k})]}{h(y)\,[1 - e^{-k(z-y)}]} \qquad (3)$$

for any two depths y,z $y \le z$ in the medium. By measuring $\overline{H}(z,\mathbf{k})$ and $\overline{H}(y,\mathbf{k})$, along with h(y) and k, it follows that a(=a(z) for all depths z) is determinable using (3).

Procedures for Deep Media

If the optical medium is known to be optically infinitely deep, then, in (3) we let $z = \infty$, so that, since $H(\infty,\mathbf{k}) = 0$, we have:

$$\overline{H}(y,-\mathbf{k}) = \frac{ah(y)}{k} \quad , \tag{4}$$

which is the basis for the formula:

$$a = k\,\frac{\overline{H}(y,-\mathbf{k})}{h(y)} = \frac{k}{h(y)}\,H(y,-\mathbf{k})\,[1 - R(y,-)] \tag{5}$$

for every depth y; and was considered earlier in Sec. 10.8, along with alternate useful estimates of a.

General Global Method

Formulas (3) and (5) are based on special cases of the general formula:

$$\boxed{a = \overline{P}(S,-)/vU(X)} \tag{6}$$

which holds for any homogeneous subset X of an optical medium in which the speed of light is v and whose radiant energy content is steady, with magnitude U(X). Thus by measuring U(X) and the net inward flux $\overline{P}(S,-)$ across the boundary S of X, a is determinable (cf., (33) of Sec. 8.8 for a derivation and discussion. See also (8) of Sec. 10.8). Equation (6) should suggest to experimenters several powerful means of determining the volume absorption coefficient a in both laboratory and natural optical media, (Compare (5) of Sec. 5.13 for an earlier application of the principle behind (6).)

Further Procedures for General Media

We close this discussion of the methods of experimental determination of a(x) by observing that in stratified light fields, (25) of Sec. 9.2 may be solved for a(z) so that: we obtain the following exact formula for a(z):

$$\boxed{a(z) = \frac{K(z,-) - R(z,-)K(z,+)}{D(z,-) + R(z,-)D(z,+)}} \quad . \tag{7}$$

Thus a(z) may be obtained at any depth knowing the D, R, and K functions for irradiance. Equation (7) is quite general, so that the medium may be arbitrarily stratified, of arbitrary depth, and in which the angular dependence of $N(z,\cdot)$ is arbitrary at each a.

Finally, we note that if we have a spherically symmetric field about a submerged light source, the estimation of a(s) at radial distance s from the source is given by (44) of Sec. 9.2.

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13.9 Operational Procedures for Apparent Optical Properties

The operational definitions of the apparent optical properties of stratified natural hydrosols are given in detail in Sec. 9.2, so that our present discussion may be limited to a brief summary of their definitions with particular attention to various features of the general depth behavior of the properties observed in natural waters. These features should be helpful in devising experimental procedures for the measurement of the apparent optical properties.

The principal apparent optical properties for stratified plane-parallel media are given in the following list.

$$D(z,\pm) = \frac{h(z,\pm)}{H(z,\pm)} \tag{1}$$

$$R(z,\pm) = \frac{H(z,\mp)}{H(z,\pm)} \tag{2}$$

$$K(z,\pm) = -\frac{1}{H(z,\pm)}\frac{dH(z,\pm)}{dz} \tag{3}$$

$$k(z,\pm) = -\frac{1}{h(z,\pm)}\frac{dh(z,\pm)}{dz} \tag{4}$$

$$k(z) = -\frac{1}{h(z)}\frac{dh(z)}{dz} \quad . \tag{5}$$

The preceding group of properties falls into two divisions: the first consists of the distribution function pair (1). The second division is the main group (2)-(5) of apparent optical properties and consists of the seven concepts shown. Closely associated with these concepts and lying halfway between them and the inherent optical properties α,σ are the hybrid optical properties:

$$\alpha(z,\pm) = \alpha(z)D(z,\pm) \tag{6}$$

$$s(z,\pm) = s(z)D(z,\pm) \tag{7}$$

$$a(z,\pm) = a(z)D(z,\pm) \tag{8}$$

along with:

$$f(z,\pm) \quad \text{and} \quad b(z,\pm) \quad , \tag{9}$$

as given in Table 4 of Sec. 9.6, or (7) and (8) of Sec. 8.3.