

Finally, we note that if we have a spherically symmetric field about a submerged light source, the estimation of $a(s)$ at radial distance s from the source is given by (44) of Sec. 9.2.

13.9 Operational Procedures for Apparent Optical Properties

The operational definitions of the apparent optical properties of stratified natural hydrosols are given in detail in Sec. 9.2, so that our present discussion may be limited to a brief summary of their definitions with particular attention to various features of the general depth behavior of the properties observed in natural waters. These features should be helpful in devising experimental procedures for the measurement of the apparent optical properties.

The principal apparent optical properties for stratified plane-parallel media are given in the following list.

$$\left\{ \begin{array}{l} D(z, \pm) = \frac{h(z, \pm)}{H(z, \pm)} \end{array} \right. \quad (1)$$

$$\left\{ \begin{array}{l} R(z, \pm) = \frac{H(z, \mp)}{H(z, \pm)} \end{array} \right. \quad (2)$$

$$\left\{ \begin{array}{l} K(z, \pm) = - \frac{1}{H(z, \pm)} \frac{dH(z, \pm)}{dz} \end{array} \right. \quad (3)$$

$$\left\{ \begin{array}{l} k(z, \pm) = - \frac{1}{h(z, \pm)} \frac{dh(z, \pm)}{dz} \end{array} \right. \quad (4)$$

$$\left\{ \begin{array}{l} k(z) = - \frac{1}{h(z)} \frac{dh(z)}{dz} \end{array} \right. \quad (5)$$

The preceding group of properties falls into two divisions: the first consists of the distribution function pair (1). The second division is the main group (2)-(5) of apparent optical properties and consists of the seven concepts shown. Closely associated with these concepts and lying halfway between them and the inherent optical properties α, σ are the hybrid optical properties:

$$\left\{ \begin{array}{l} \alpha(z, \pm) = \alpha(z)D(z, \pm) \end{array} \right. \quad (6)$$

$$\left\{ \begin{array}{l} s(z, \pm) = s(z)D(z, \pm) \end{array} \right. \quad (7)$$

$$\left\{ \begin{array}{l} a(z, \pm) = a(z)D(z, \pm) \end{array} \right. \quad (8)$$

along with:

$$f(z, \pm) \quad \text{and} \quad b(z, \pm) \quad , \quad (9)$$

as given in Table 4 of Sec. 9.6, or (7) and (8) of Sec. 8.3.

The Fundamental Irradiance Quartet

The principal family of apparent optical properties of stratified natural waters consists at present of a set of seven quantities (2)-(5) whose numerical values depend on the angular structure of the light field as well as on the physical composition of the water.

The first observation we can make which is pertinent to the determination of the seven apparent optical properties is that they can be obtained from four basic irradiance measurements. These measurements take the form of two pairs of irradiance quantities: one pair consists of ordinary irradiances, the other of scalar irradiances. In each of these pairs, one member is assigned to upwelling flux, the other to downwelling flux in the medium. The reason that there are precisely four such quantities stems from our conceptual decomposition of the flow of radiant energy in any natural hydrosol (stratified or not) into two streams: an upward flowing stream and a downward flowing stream across each horizontal plane in the medium (cf., the two-flow theory of irradiance in Chapter 8).

The four basic irradiances are:

$$\begin{aligned} H(z,+) \\ H(z,-) \\ h(z,+) \\ h(z,-) \end{aligned} \quad (10)$$

$H(z,+)$ and $H(z,-)$ are the upwelling (+) and downwelling (-) irradiances, respectively (Sec. 2.4). They are induced by the upwelling and downwelling flux streams at depth z . These quantities may be obtained from field radiance measurements, or they may be measured by flat plate collectors exposed to the appropriate hemispheres. See Fig. 13.14. In like manner, $h(z,+)$ and $h(z,-)$ are the upwelling (+) and downwelling (-) scalar irradiances, and refer to upwelling and downwelling flux, respectively, at depth z (Sec. 2.7). They may be obtained by simple numerical procedures from field radiance measurements. Alternatively, appropriately shielded spherical collectors may be used to measure these quantities. A possible experimental arrangement is shown in Fig. 13.15. See also Fig. 2.18. Observe that the collectors are complete spheres in each case. The sphere that measures $h(z,-)$, for example, should be shielded from the upwelling flux by some device which at the same time impedes as little as possible the interchange of flux across the horizontal plane at depth z . In analogy to our discussions of the relation between h and $h_{4\pi}$, we can show that the *downwelling spherical irradiance* $h_{4\pi}(z,-)$, measured by the shielded sphere shown schematically in Fig. 13.15(a), is related to $h(z,-)$ by:

$$h_{4\pi}(z,-) = \frac{1}{4} h(z,-) \quad (11)$$

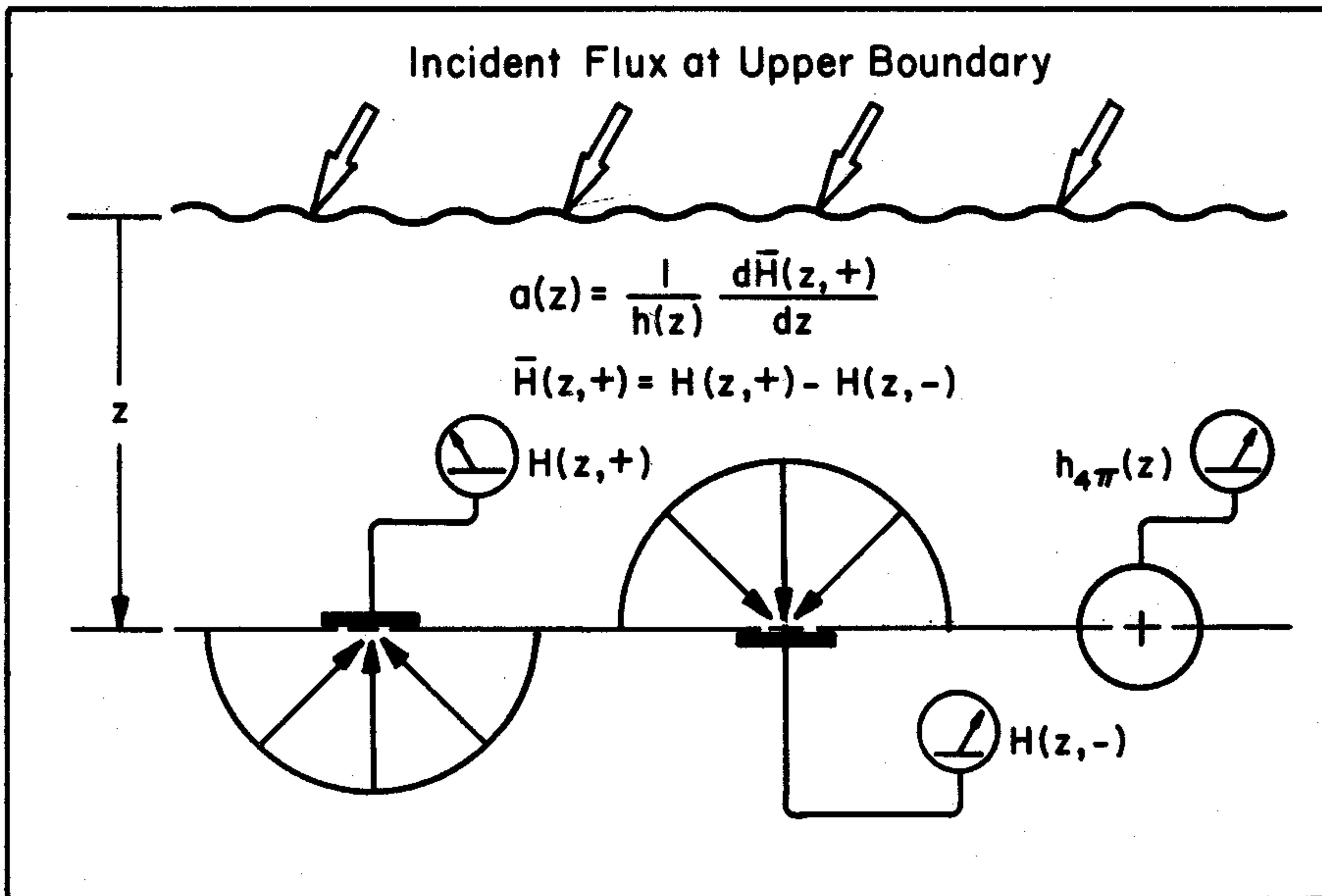


FIG. 13.14 Schematic arrangements for measuring up (+) and downward (-) irradiance and scalar irradiance at depth z . These measurements lead to exact calculations of the volume absorption function values $a(z)$.

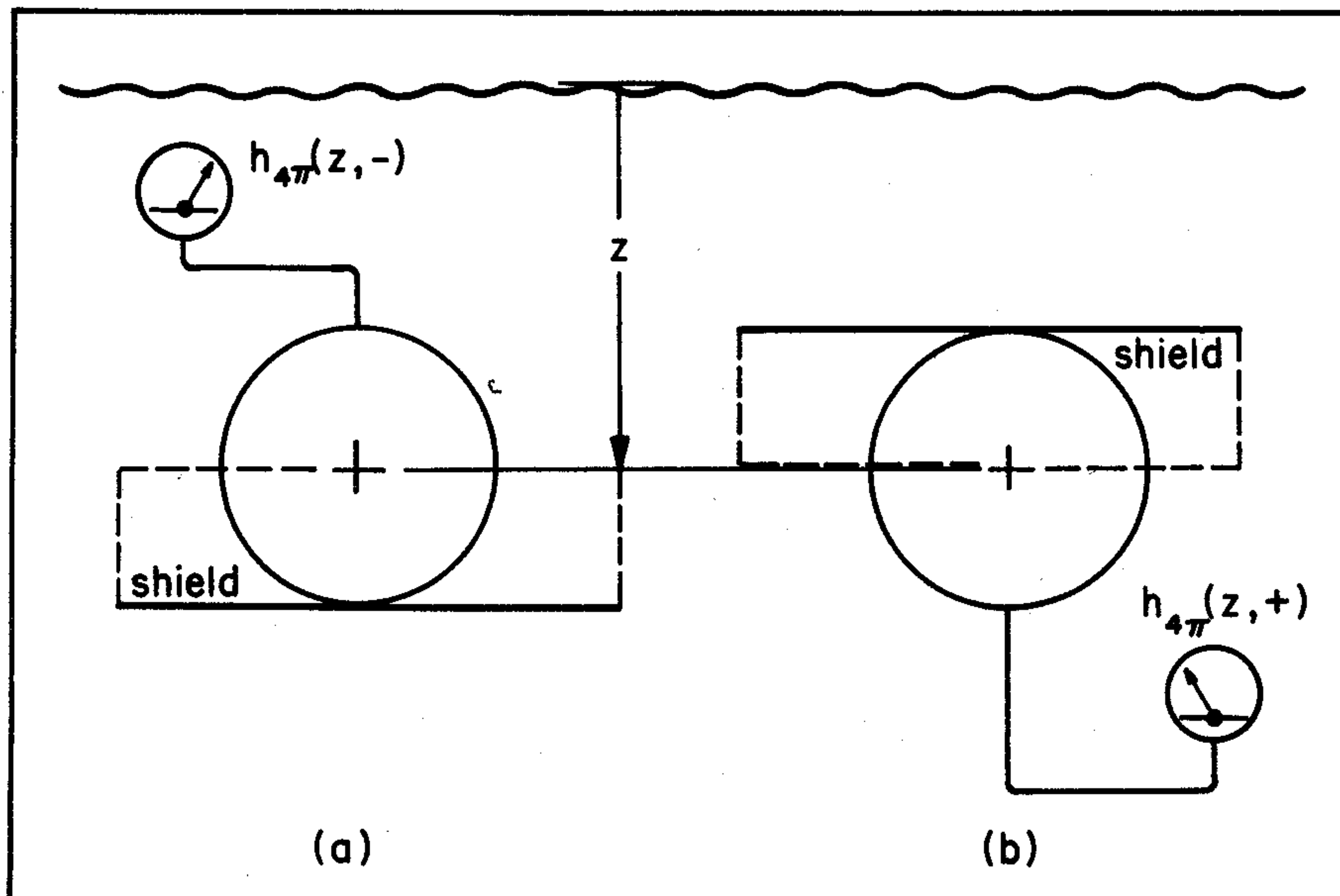


FIG. 13.15 Schematic arrangements for measuring up (+) and downward (-) spherical irradiances at depth z .

Similarly, the *upwelling spherical irradiance* $h_{4\pi}(z,+)$, measured by the other shielded sphere shown schematically in Fig. 13.15(b), is related to $h(z,+)$ by:

$$h_{4\pi}(z,+) = \frac{1}{4} h(z,+) \quad . \quad (12)$$

These relations are simply definitional identities based on the discussion of Sec. 2.7. The demonstration of the connection between $h_{4\pi}$ and the spherical irradiances defined above, assuming ideal shielding is straightforward. The result is:

$$h_{4\pi}(z) = h_{4\pi}(z,-) + h_{4\pi}(z,+) \quad . \quad (13)$$

Furthermore, as in (9) of Sec. 2.7:

$$h(z) = h(z,-) + h(z,+) \quad . \quad (14)$$

The practical connection between $h_{4\pi}(z)$ and $h(z)$ is discussed in Sec. 13.1; in particular, see (17) of Sec. 13.1. This connection holds also between $h_{4\pi}(z,\pm)$ and $h(z,\pm)$. Observe that the connection factor C_r in (17) of Sec. 13.1 also holds for the hemispherical irradiance connections.

Discussion of the Reflectance Functions

The physical interpretation of $R(z,-)$ in (2) is straightforward: It represents the ratio of upwelling irradiance at depth z to the downwelling irradiance at depth z , so that $R(z,-)$ may be thought of as the reflectance, with respect to the downwelling flux, of a hypothetical plane surface at depth z in the medium. For completeness, we have included in (2) the reflectance $R(z,+)$ for the upwelling stream. However, this is simply the numerical reciprocal of $R(z,-)$. In actuality, $R(z,-)$ depends on the scattering properties of the entire medium above and below level z . It will also depend in part on the reflectance properties of the upper and lower boundaries of the medium if these are within sight of the flux collectors. $R(z,-)$ is not an inherent property of the medium (as defined in Sec. 9.3) for experiments and theory show in general that for a given medium and a given depth in that medium, the value $R(z,-)$ changes with the external lighting conditions. Some theoretical relations helpful in studying these variations are given in Sec. 9.4.

Relations (2) are completely general: They apply to any medium, be it deep or shallow, irradiated by the sun in a clear sky or by any type of overcast. Because of this generality, very little can be said about exactly how the values of $R(z,-)$ should depend on depth. No simple statements beyond those made in Chapters 9, 10, and 11 can be made which assert that $R(z,-)$ should always increase with depth, or that it should always decrease with depth, or that it should go through maxima or minima at certain depths, and so on.

Despite this unwillingness of $R(z,-)$ to have its characteristics typed generally and in very fine detail, there are

certain gross characteristics as we have seen in Chapter 9, 10, and 11, which make it an indispensable tool in engineering calculations: in optically deep homogeneous hydrosols, it is an observational fact that $R(z,-)$ varies very little with depth. Near the surface of these media, it shows relatively high variability with depth which depends on the state of the surface and incident lighting patterns, but soon settles down and approaches a constant value independent of depth. $R(z,-)$ thereby takes on the status of an apparent optical property of the medium. Furthermore, in media that have no self-luminous organisms, $R(z,-)$ behaves as any respectable reflectance should: It is never greater than 1. In fact, in most clear natural hydrosols the values of $R(z,-)$ are usually found to be somewhere in the neighborhood of 0.02, give or take 0.01, for midspectrum wavelengths (around 550 $m\mu$). For turbid, nearshore or inland waters, $R(z,-)$ can rise to 0.08 or more. In media containing self-luminous organisms distributed throughout some layer it is quite possible, however, for the values of $R(z,-)$ to approach 1 as this layer is approached, and even become greater than 1 just before it enters the layer. Some examples of $R(z,-)$ are given in Table 1.

While the problem of the find detail of the depth dependence of $R(z,-)$ is mainly of academic interest, we note that there is no dearth of theoretical approaches to this interesting problem. One model of the light field which is particularly useful in the study of this problem is the two-D theory of Chapter 8. This model is relatively simple to use and is still sufficiently detailed to supply a multitude of

TABLE 1

Examples of the values of $D(z,\pm)$, $K(z,\pm)$, $a(z)$, $R(z,\pm)$

m (meters)	$D(z,-)$	$D(z,+)$	$K(z,-)$	$K(z,+)$	$a(z)$	$R(z,-)$
4.24	1.247	2.704	-	-		0.0215
7.33	-	-	0.129	0.126		-
10.42	1.288	2.727	0.153	0.150	0.115	0.0184
13.50	-	-	0.178	0.174	-	-
16.58	1.291	2.778	0.174	0.172	0.118	0.0204
22.77	-	-	0.171	0.170	-	-
28.96	1.313	2.781	0.169	0.169	0.117	0.0227
35.13	-	-	0.167	0.167	-	-
41.30	1.315	2.757	0.165	0.165	0.117	0.0235
47.50	-	-	0.162	0.163	-	-
53.71	1.307	2.763	0.158	0.158	0.112	0.0234
59.90	-	-	0.154	0.154	-	-

Explanation of Table 1: Depths and units are in terms of meters. Data are associated with a wavelength of 480 $m\mu$ and were derived from radiance information summarized in Ref. [298]. The optical medium (Lake Pend Oreille, Idaho) was found to be essentially homogeneous; the volume attenuation coefficient being $\alpha = 0.402$ per meter. The sky was clear and sunny with the sun at about 40° from the zenith. The values $a(z)$ were obtained by means of (2) of Sec. 13.8.

examples of the depth dependence of $R(z,-)$: It supplies cases in which $R(z,-)$ can increase or decrease over pre-selected depth ranges. (See Sec. 10.4.) In all cases, however, the model states that there is some value R_∞ which $R(z,-)$ approaches asymptotically with depth in optically infinitely deep media. This asymptotic value depends in a calculable way on both the inherent optical properties of the medium and on the limiting lighting conditions. Further discussion on the behavior of $R(z,-)$ at great depths are made in Chapter 10.

Discussion of the Distribution Functions

A particularly simple means of characterizing the depth dependence of the shape of radiance distributions, without resorting to an actual measurement of the radiance over all directions at each depth, is given by the distribution functions:

$$D(z,-) = \frac{h(z,-)}{H(z,-)}$$

$$D(z,+) = \frac{h(z,+)}{H(z,+)} .$$

It is readily seen from the definitions of h and H that if the *shape* of the radiance distribution changes with depth, then $D(z,-)$ and $D(z,+)$ will change with depth; and conversely, if the values of the distribution functions vary with depth, the radiance distributions must be changing shape with depth (cf. Sec. 8.5). It is clear from the definitions that $D(z,-)$ gives an index of the shape of the radiance distribution in the upper hemisphere (i.e., for the downwelling flux), and $D(z,+)$ does a similar job of characterizing the shape of the radiance distribution in the lower hemisphere (i.e., for the upwelling flux).

Numerical analysis of detailed experimental studies of the light field by Tyler in Lake Pend Oreille show that both $D(z,+)$ and $D(z,-)$ exhibit relatively little change with depth (cf. Ref. [306]). Furthermore, this independence of depth is found whether the external lighting conditions are sunny or overcast. Under either of these conditions, and for blue-green light, the values $D(z,-)$ hovered very closely in the neighborhood of 1.3, while the values $D(z,+)$ clustered around 2.7. Examples of $D(z,-)$ and $D(z,+)$ are given in Table 1. It appears at present that these values should be typical of the values that one may find in many natural hydrosols and for the blue-green portion of the spectrum.

Of course, as in the case of $R(z,-)$, the quantities $D(z,-)$ and $D(z,+)$ will obstinately refuse to have any sweeping generalizations made about the fine structure of their depth dependence. However, as in the case of $R(z,-)$, simple theoretical tools exist which can be directed toward such problems if the need ever arises to discuss depth dependence in detail. These are given in Chapters 9 and 10. Furthermore, the ultimate depth dependence of $D(z,-)$ and $D(z,+)$ in

deep media is quite regular and predictable, as was shown in Chapter 10, as a consequence of the proof of the asymptotic radiance hypothesis.

The observed constancy of the distribution functions with depth has important practical consequences. In homogeneous media exhibiting this type of behavior a few well-selected measurements of the inherent optical properties together with radiance distributions near the surface would suffice as the basis for an estimate of the quantity and quality of the light field for *all* depths in the medium. Such estimates could be made by means of the two-D model of Chapter 8 or the simple radiance model built around (2) of Sec. 4.4.

In addition to characterizing the depth dependence of the angular structure of radiance distributions, as explained in Sec. 8.5, $D(z,-)$ and $D(z,+)$ play indispensable roles in the equations of applied radiative transfer theory, particularly in those equations which link the inherent and apparent optical properties of a medium. These roles are illustrated as a matter of course in the discussions throughout Chapters 8, 9, and 10.

Discussion of the K-Functions

The reflectance function, as we have seen, gives a running account of the *relative* magnitudes of the irradiance of each stream of radiant flux. The quantities which characterize the *individual* depth dependence of the upwelling and downwelling irradiances and of the scalar irradiance of the streams at each depth are called the *K-functions*. The motivations for the operational definitions of these functions are supplied by both theoretical and experimental precedent extending back over at least fifty years of applied radiative transfer theory, and we shall now devote some discussion to the concepts centering around their definitions.

The theoretical motivation for the K-functions for irradiance and scalar irradiance stems from an attempt to increase the usefulness of the Schuster equations for the two-flow analysis of the light field. The detailed development of this approach and its practical applications have been thoroughly explored in Chapter 9 (see Sec. 9.2).

The experimental motivation for the K-functions rests in early empirical relations of the kind:

$$I_z = I_0 e^{-Kz} \quad , \quad (15)$$

which simultaneously were to characterize the depth dependence of I_z and define its logarithmic depth-rate of decay K . In the above relation, the quantity I_z took many forms: in some studies it was downwelling irradiance, in others it was a scalar irradiance-like quantity; in still others, its exact nature was not quite clear. In every case, however, it was intended to be some measurable "intensity" of radiation. Nevertheless, there was no universal agreement as to what "intensity" meant and what radiometric quantity it

should represent. As a result, there was no agreement as to what was really measured. A plot of I_z on semilog paper with depth as abscissa yielded $-K$ as the slope of a curve which often appeared visually as a straight line. K could thus be defined operationally, via the equation:

$$K = -\frac{1}{z} \ln \left(\frac{I_z}{I_0} \right) \quad (16)$$

It suffices to observe here that these early theoretical and experimental approaches to characterize a K -like optical property of natural hydrosols were inadequate to the subsequent needs for precision and completeness in modern hydrologic optics. In current basic research I_z is explicitly replaced by any of the three precisely defined irradiances $H(z,-)$, $H(z,+)$ and $h(z)$. Furthermore, it has become necessary to distinguish not only between the magnitudes $H(z,-)$, $H(z,+)$, and $h(z)$, but also their logarithmic rate of change with depth. Careful measurements (see, e.g., Table 1) show that their logarithmic rates of change are generally different, and the difference far exceeds the range of experimental error. In general, semilog plots of $H(z,-)$, $H(z,+)$, and $h(z)$ also exhibit noticeable departures from linearity, especially in near-surface regions. This fact, of course, is part of the folklore of the study of hydrologic optics which has been extant for many years, but this nonlinearity has been considered more of an annoyance than a source of enlightening information. In particular this nonlinearity made it impossible to rigorously define a single unambiguous fixed number K , of the kind appearing in (16) which otherwise could be used to help classify the optical properties of the medium.

The current views in hydrologic optics are such that the departures from linearity by semilog plots of $H(z,-)$, $H(z,+)$, and $h(z)$, and even $h(z,\pm)$ are a source of extremely useful insight into the intricate structure of real light fields in natural hydrosols. Far from being ignored, these departures from linearity should be welcomed as harbingers of new and deeper understanding. The logarithmic slopes of the $H(z,\pm)$, $h(z,\pm)$, and $h(z)$ plots are defined in general as in (3)-(5). Useful interrelations among these magnitudes which can help guide the reduction of empirical data and further understanding of radiative processes in natural hydrosols are developed throughout Chapter 9 and 10.

13.10 Theory of Measurement of Local and Global R and T Properties

In this section we invert the usual way of looking at the principles of invariance, and more generally the interaction principle, and show that, by so doing, we encounter new ways in which to measure the inherent and apparent properties of optical media. We have already used to advantage this point of view throughout all the preceding sections to find operational definitions of such local properties as α , σ , a , s , the K functions, $R(z,\pm)$ and $D(z,\pm)$. The results of Sec. 7.8 are also pertinent to the present discussion. Now we wish to show how these procedures can be extended to the complete