2.5 Radiance

We now define the radiometric concept of radiance, discuss some of its various forms, and study some of its basic geometrical and physical properties of particular use in hydrologic optics.

For those who are studying the concept of radiance for the first time, we may introduce it by saying that radiance is designed to yield a simple mathematical representation of the percept of brightness experienced by the human eye as the eye is directed along various paths of sight. Some introspection will show that when one directs visual attention to a point in his environment, such as a point on a desk or a wall, the brightness sensations of neighboring points of the point under scrutiny can be willfully suppressed. The result is a possible conscious comparison of "brightness" of successive neighboring points in one's environment. Now when one attempts to simulate this sensation of brightness by means of a radiant flux meter, one must introduce a mechanical means of directing the 'attention' of the collecting surface along a narrow bundle of directions. The collecting surface by itself is obviously incapable of the complex and partly automatic process that takes place in the eye-brain circuits within a human head when visual attention is directed along a narrow bundle of directions. Some sort of "blinder", usually in the form of a long narrow circular cylinder, must be fitted around a circular collecting surface so that its axis is normal to the plane of the collecting surface. The result is a radiant flux meter with a relatively narrow conical set D of directions along which radiant flux may be incident on a plane circular collecting surface S. Such an assembly is depicted in Fig. 2.5, and is called a radiance meter.

The operational definition of the radiometric concept of radiance can be given in terms of a radiance meter as follows. The radiance meter is taken to a point x in a natural optical medium such as the atmosphere or a natural hydrosol. The center of the collecting surface is placed so as to be at point x. The axis of the cylindrical collecting tube of the meter is directed along a direction & so that radiant flux from the field of view is funneled along the set D in the general direction of The sensor component of the radiance meter records an incident radiant flux P(S,D) on the collecting surface. The area A(S) of S and the solid angle ~2(D) of D are known instrumental constants. The quotient:

$$P (S ,D) /A(S) Q(D)$$

is called the (empirical) radiance at x along . Radiance, therefore, is a nonnegative number which is paired with the

dimensions of pourer per area per solid angle (per unit frequency in and with convenient units such as watts per square meter per steradian (per unit frequency interval). We will write:

"N (S,D)" for P(S,D)/A(S)Q(D) (1)

collecting surface S
photoelectric element
collecting directions D
(f i xed)

dial R

FIG. 2.8 Schematic details of a radiance meter

or in more complete notation:

"N (S,Dlt,V)" for P(S,D9t,v)/A(S)D(D)

Since A(S) and a(D) are fixed numbers for a given radiance meter, the radiant flux reading can be calibrated directly in terms of N(S,D). Experimentation with variously proportioned radiance meters indicates that those meters with solid angle magnitudes Q(D) on the order of 1/30 steradians, and with collecting areas A(S) on the order of that for a circular surface of a centimeter in diameter, are adequate for radiometry in most natural optical media. Of course, the smaller A(S) and Q(D), the sharper are the radiance maps obtainable (while still remaining above the level where diffraction and general interference effects set in).

Recall the definition of empirical irradiance H(S,D) in (1) of Sec. 2.4. It follows from (1) above that:

N(S,D) = H(S,,D) /a(D) .

Corresponding to (4) of Sec. 2.4 we shall write:

"N (x,D)" for H(x,D)/D(D) , (3) "N(S,~)" for 1im H(S,D)/Q(D)

and

where the central direction of D is normal to the plane of S at x. We are then led to write:

"N (x, g) " for 1 im N (x, D) D-o-{g?

We will occasionally use the more complete notations "N(S,D,t,F)rt, 'IN(x,C,t,F)f=, "N(x,C,t,v)II.. etc., for radiance when time and frequency parameters are explicitly required. The time symbol "t" and the frequency symbol "v" may be included or omitted as needed. In the case of the first of those listed above we agree to write:

"N (S, D, t, F) " for O (S, D, t, F) /A(S) Q (D) .

The quantity N(x,Q is the (theoretical) radiance at x in the direction E. It exists as a mathematical entity by virtue of the D-additivity and D-continuity properties of 0 cited in (7) and (8) of Sec. 2.3.

It is instructive to disassemble the definition of theoretical radiance layer by layer until the primitive concept of radiant flux 0 is recovered. Thus, beginning with (4) and using (3):

N(x, t) = lim H(x, D) /D(D) D+{0

Then by means of (4) and (1) of Sec. 2.4:

N(x,~) = lim lim P(S,D)/A(S)Q(D) S+{x) D+{~)

Finally, by means of (3) of Sec. 2.3 we have (in full notation)

N(x,~,t,v) - lim lim 1im 0(S,D,t F)

S+{x} 1+{V} F->0} A(S) Q(D)1(F)

1im 1im 1im N(SjD,tj,) S+{x} D-),- {t? (. F+{v} I (F)

This is the basis for the fact that, in the last analysis, all radiometric concepts are reducible to the primitive physical concept of radiant flux embodied by 0 and the appropriate geometrical and analytical notions of limit and measure. Hence all equations of pure and applied radiative transfer are resolvable into expressions containing only one primitive physical notion, namely O(S,D,t,F), and auxiliary geometrical and analytical concepts.

[Those who desire radiant energy as the most primitive physical notion, may then start with U(S,D,T,F) where T is a finite time interval, so that O(S,D,t,F)=1imT,P-{t)U(S,D,T,F)/1(T).

In Vol. I, U was taken as a primitive concept; in this and subsequent volumes, U will be derived from 0 as in (17) of Sec. 2.7, e.g.]

Now that the definition of radiance has been established it is an easy matter to return to the definition of the phase density n (x , t , t , v) of photons in (1) of Sec. 2.2 and by energy and dimensional arguments conclude that

N(x, C, t,v) = hvvn(x, ~, t,v)
(5a)

which together with (5) connects N(x,&,t,v) with some of the more basic constructs of radiometry (0, and photons). Statement (5a) can be cast into terms of wavelength X by using the transformation (32) of Sec. 2.12.

To gain some insight into the magnitudes of radiances found in nature, we append Tables 1,2, which are constructed from the graphs in parts III, IV of [26] and which form part of a four-part series of compilations of sky (field) radiance distributions. The skies in the present tabulation were morning (0800 hours) skies at sea level, covered 40% with scattered clouds. Two regions of the spectrum are considered: Table I gives orders of magnitude of field radiance in the wavelength interval [400, 500] mu, and Table 2 is for the interval [580,700] mu. The main purpose of the tables is to complete the statement: "daylight skies (away from the sun) have radiances on the order of lon watts/ ($_M$2 X steradian), where n = ?" It is clear that the answer is around na -1,0,1. By way of contrast, recall that the radiance over the sun's disk is on the order of 2×10^7 watts/ (m2 X steradian) as seen just outside the atmosphere, and for a wavelength interval [0,°°] mu (cf., (3a) of Sec: 1.2). Hence N in the vicinity of the sun runs from 10° to 10' units of radiance. The data were taken June 21, 1958 in balloon flights over central Minnesota. For angle conventions, see part (a) of Fig. 2.3.

Table 1 Sample Radiances Morning Skies 400-500 mu, Sea level, sun zenith angle a 70° watts/(M^2 x steradian)

Azimuthal Polar 8	sun's azimuth $_0$0	= 800	180^0
zenith			

e=45°			+
e 90[10]	7.0. .	2.2	

Table 2

S a m p l e Radiances, Morning Skies n rw~ rrrrr rrr rr+-~~i~ nr 580-700 mu, Sea level, sun zenith angle = 70° watts/ (ml x steradian)

zimuthal Polar e	sun's azimuth = 0°	= 80°	=
e = 0° zenith	.45	.45	.4
0 = 45°	3.0	.62	.4
e = 90°			

Radiance Distributions

We have seen how the operational definition of radiance leads to the theoretical radiance displayed in (4). This in turn leads to the construction of a function N which assigns to each point x in an optical medium and direction at that point, a radiance N (x, &) of the natural 'light field. N(x, ~) is the number of watts of radiant flux incident per unit solid angle, in the direction ~ normal to a unit area at x. Implicit in the notation is the time t of the measurement and the frequency v of the energy passed by the filter of the meter. The totality of all values . N (x, ~) paired to (x, E) as x ranges over all points of a selected optical medium X and as & ranges over the unit sphere w is called the radiance function on Xx .= and is denoted by "N". If attention is restricted to an arbitrary fixed point x and the totality of values N(x,~) are considered for all C in H, then that totality of values is called the radiance distribution at x and is denoted by "N (x, •) ". The radiance function is the most important radiometric function in geophysical optics and in particular, in hydrologic optics. For an exhaustive empirical study of radiance distributions in a natural hydrosol, see the classic work of Tyler [298]. The importance of the radiance function rests in the fact that from knowledge of the radiance function alone, all other radiometric quantities are relatively easily calculable. This fact will become increasingly apparent as the discussion of this work proceeds, and we begin below with a f ir'st example of this fact. (See also Figs. 1.231.25)

Irradiance from Radiance

As an illustration of the use of the concept of radiance, and to aid the reader to fix in mind its definition, we shall derive the relation between irradiances of the form
H(x,g) introduced in Sec. 2.4 and radiances N(x,~) just defined above. More detailed examples are reserved for Sec. 2.11.
We begin with the empirical connection between H(S,D) and N(S,D) established as a matter of course in equation (2). I f N (S , D) i s known,' we can compute H (S , D) using
H(S,D) == N(S,D)n(D)
It should be recalled that D is a narrow conical set of directions associated with the radiance meter, and that the central direction of the cone is normal to the surface S.
We now apply this general relation to the following, problem, which is formulated with the aid of Fig, 2.9. A surface S with inward normal is irradiated by n distinct sources of flux such that the i-th flux has radiance N (S i ,D-) and is incident on the points of S through a small conical set Di of directions centered on direction Ei. The sets D are pairwise disjoint (i.e., no two overlap) and all lie on the same side of S. What is the resultant irradiance H(S,D) produced by this given set of incident irradiances?

FIG. 2.9 Setting up the connection on going from radiance to irradiance

The first step in relating H (S, D) to the n radiances is to observe that by successive applications of the D-additivity property of radiant flux. (equation (?) of Sec. 2.3) we can write:
$H(S_O D)$ = H(SvD1) + H(S9,D2) +...+ H(S,Dn)
where "H(S,Di)" denotes the irradiance on S produced by radiant flux incident within the set of directions Di. The second step consists in using the cosine law for irradiance (equation (15) of Sec. 2.4) to relate H(Si,Di) and H(S,Di), for i = 1,..., n. Thus
$H(S,D_i) = H(S_i,D_i)$ cos
where " ly-" denotes the angle between the unit inward normal ~ to S anA Ci. We have chosen S smal 1 enough so that the conditions of the derivation of the cosine law (15) or (16) of Sec. 2.4, are satisfied. Furthermore, we use (4) above to permit slight adjustments of the choice of the Si as may be required to meet the cosine law derivation conditions without noticeably changing the value of the radiance of the flux on Si through Di. Thus, by definition, for every i = 1,...,n:
H(SifDi) ~` N(Si~Di)Q(Di)
and this constitutes the third and final step. By assembling the results of these three steps we have the desired connection:

When n = 1, we have the intuitively useful special case of

H (S , D) = N (S' , D) Q (D) cos h ,
r
where we have written 'IS"' for Si, and now D = D, in Fig.2.9.

The connection (6) is a useful relation in practical situations where knowledge of radiance distributions is applied to find irradiances on arbitrarily oriented surfaces.

By using terrestrially based coordinate systems (Sec. 2.4) equation (6) can be translated into a workable standard computation procedure. This task is facilitated by first establishing the theoretical counterpart to (6). Thus, let S-~{x),
so that also Si-)-{x). Then H(S,D)-1o.H(x,D) and N(Si,Di)-1-N(x,Di), according to (3).
Equation (6) then becomes:

n
H(x,D) N(x,D_i) cos 11J i ^{0}(Di) (7)

We now apply (7) to the case where : is divided up into n disjoint pieces "i, and we then let the number n increase indefinitely so that each Ei goes to zero. In this way we arrive at the integral counterpart to (7):

Htx,) Nix, ' ` ' dt~ ')

Recall that the symbol "E.(Q " denotes that hemisphere of consisting of directions &' which make an angle less than ninety degrees with C. Further, is the unit inward normal to the collecting surface S at x. Recall also that ~•V, by
the discussion of 2.4, equals the cosine of the angle between C and 9'. Thus (8) may be rewritten in terms of -&P. Before this can be done with complete clarity, we must express dQ(&') in terms of polar and azimuth angles. This we shall do, taking the opportunity to explicate at the same time the notion of "solid angle".

Toward this end, let us consider a set D of directions on the unit sphere Fig. 2.10 depicts a typical set D occurring in practice, i.e., one which consists of a single

FIG. 2.10 The unit sphere of directions as the natural setting for solid angle measurements,

connected part of -=. It is quite natural to characterize the "amount of opening" of the set D by specifying the amount of area that D occupies on the unit sphere. Thus, we denote by ^{11}00)" the number representative of the area of D on ~ . This is the standard definition of the measure of a set of directions D, the one which we have been using informally up to this point. (For a further discussion of solid angle measure, see Note (h) to Table 3, Sec. 2.12.)

We can now go one-step further and characterize Q(D) in terms of the polar and azimuthal angles 9 and # (measured in radians). Clearly a small rectangular patch on ? about the point specified by (e,t) and of lateral extents da and db is very nearly of area da db. But since the sphere W has unit radius, db = de.and da ft sin e d~. Hence:
da(Q a sin 0 de dt t9]
where (e,~) is associated with the direction (see Sec. 2.4). From (9) we obtain

1 f s i n e d 8 d m
9(D) = f dQ(~) a
D D
(10)
It should be clear that the radius of W plays no essential role in determining Q(D). In general, if we write "Q(D)II for A(D)/r', where A(D) is the area determined by the set D on a sphere of radius r, then equation (10) results once again for Q(D). We leave the ranges of integration in (10) undetailed, as the mode of specification of D varies widely. The number n(D) is customarily dimensionless, However, when dimensions of D(D) are needed, the system in Table 3 of Sec. 2.12 may be adopted. ,The standard unit of a solid angle is the steradian. It is important for a thorough understanding of solid angle, to make the distinction between the set D of directions and its measure Q(D): D is a set of points on , Q(D) is a number describing the size of that
t-.
se
As an example of the use of (1^0), consider the spherical cap D on ' consisting of all directions ~ with polar angles less than or equal to 6, See Fig. 2.11. Then:
e 2n
n (D) - sin 6^1 den d 1 e'=0 0'=0
= 2n(1-cos e) 49 . (General 9) (11)'
This formula is frequently used. It is also the basis for the following well-known estimate of Q(D) for smatt 0. In (11) let 0 be small so that 0^2 is much smaller than

FIG, 2.11 Solid angle measures of some simple shapes
(negligible compared to) e. Then we can approximate cos 9 by 1-(8^2/2), by truncating the series expansion of cos e at its second term. Under this assumption, (11) becomes:
Q(D) = we^e (Small e) (12)
From (11) we also obtain the solid angle measures for various special parts of which can be made up from spherical caps. Thus:

Q(D) = 2R (e = w/2, D is a hemisphere) (13)

Q (D) = 4n (9 = x, D is O) (14)

Q(D) = 2w(cos ei-cos e2) ($015^0$2t D is a' spherical segment) (15)

This last formula is, incidentally ,,a generalization of (11), for (11) is recovered by setting ei = o. Equation (15) can be used to obtain the solid angle measure of a spherical rectangle bounded by two latitude circles and two longitude circles of the unit sphere. Thus, if ~1 and ~2 are the bounding meridians with ~1S 02, then the rectangle bounded by them takes up the fraction (~2⁻~1)/2n of the spherical segment area bounded by latitude circles at ei and e_2. Hence
from (15)
Q(D) (02-0k) (cos 9_1-cos e 2) ($e_l$ss e 2 , 0 , S 0 2 . v (1 6) D is a spherical rectangle)
Equation (16) is a further generalization of (11). The latter may be obtained by assuming 02 = 01 + 2w and 9_1 = o. Of course (16) can also be obtained by direct appeal to (l0).
We now return from the preceding digression on solid angles and conclude our discussion of the computation of irradiance H(x,E), given a radiance distribution N(x,g). It remains to cast (8) into e,0 notation. Using (8), and (13) of Sec. 2.4 we have:

(1 7)
where is simply another name we shall use for E(E), when (e,~) is explicitly associated with . Further, s-~is the angle between g. and &', where the he latter direction is as sociated with (e'.01). Now, cos can be represented by means of (6,0) and (6^1,0^1) as follows. Recall first of all from Sec. 2.4 that if & is a unit vector, then:
i cos i + i cos ⁻T X 2 + k cos ⁻a 3
where i, V2, and Z P $_3$ are the angles between t and the unit vectors i, i, and k, respectively. Once again, now for ~':
' = i cos -^{9}1 '+ j cos ⁻t' 2 ' + k cos 'zX3 '
Then by the observations in Sec. 2.4:
cos V = g • g'= cos ^{0}1 cos ^{4}'t 1 ' + cos $^{4\text{`}0}$2 cos 4 $^{'+}$cos 1/3 cos ⁻t^{f}3'
(1 8)
By means of Fig. 2.4, or Fig. 2.10, we see that:
cos -a 1 = sin 9 cos
cos /^{1}-^{P} 2 = sin e sin ~ } (1 9)
cos ~$_3$ = cos e

There are three precisely similar equations for the 'Z^{P}i' associated with C'. In this way (17) is cast into a well-defined analytical formula involving only e , ~ and e',~', in an integration over B(e,~). This completes the detailed unfolding of equation (8). The result is a formula often used in practice to compute H (x , e m , given N(x,•) at point x. We will return to illustrate equation (17), and other formulas, in Sec. 2.11. For the present we continue with the development of further properties of the concept of radiance.
Radiance from Irradiance
As a further illustration of the interconnections between the concepts of radiance and irradiance we now reverse the considerations of the preceding discussion and show that from a given irradiance distribution at a point x in an optical medium, one can compute the radiance distribution at that point. As a consequence of this fact and the results of the preceding discussion, we see that radiance and irradiance distributions share equal informational content. In addition to this theoretical consequence, there is

also one of experimental import: it is possible, at least in principle, to measure irradiance distributions in natural hydrosols and aerosols and from this data deduce complete information about radiance distributions. In other words, one can in principle completely document the light fields in natural optical media solely by means of irradiance distributions.

We begin the illustration with the simplest possible case: we are given that the irradiance distribution H(x,•) at point x is generated by a radiance distribution N(x,•) which is of uniform radiance N over a small conical set D' of directions of solid angle magnitude SI(D') with central direction V and with N(xX) zero for all other directions.

It is required to find N. Now from (8) we have very closely:

H(x,C) = Ng'~'Q(Dl)

whence:

N = H(x,&)1~•E'Q(D')

where g is some specifically chosen vector such that g•V~0.

Suppose next that the given irradiance distribution is generated by a radiance distribution which is of uniform magnitude Ni over a narrow set D1' of directions with central direction ~$_1$' and of uniform magnitude N2 over a narrow set Dz' of directions with central direction ~$_2$' and such that Di' and Dz' are disjoint and N(x,~') is zero for all other directions. From (8) we have now:

H(x,g) = Nl~'&I'Q(Dl')X(+ N29*C2 'Q(D2')X(~)&2')

(2C

where x is a function with the property that x(~,&') = 1 or according as t' is or is not in (~), respectively, and whe is any direction in .

Now we may choose C at will from a large collection of possibilities, and use the given values H(x,C) to try to determine the two radiances N1 and N2, Clearly we must generally choose two directions g, and C2 in order to determine N1 and N2. This is readily seen if we write:

"C_{ji}" for $\xi_i \cdot \xi_j{}' \Omega(D_j{}') \chi(\xi_i, \xi_j{}')$

for each i

1,2, and j a 1,2, and furthermore, if we write:

"Hi" for H (x , &i)

for each i = 1,2. Then equation [20) yields, for E_ti and t=~2 the two equations:

H1 ' NIC11 + N2C21

H2 " NIC12 + NIC22

If we write:

licit f or

then the preceding set of equations can be written:

(H, pH,) " (N 1 ON2) C . (21)

or, more succinctly, as:

$$H = NC \qquad . \qquad (22)$$

We have written:

'=H" for (HI ,H2.)
and
"N" for (N1 ,N2)

The solution of Equation (22) is that N for¯which

H_r C21

N1 -1

H_2 C22

C11 H1

N2 0_1

C12 H2

where we have written:
SEC. 2.5 RADiaNCE 4 3
tt_a,t
for
C i l C 2 2 ¯C 1 2 C 2 1
and where the four Cij are such that 0 may be put into the form:
0 .
This solution
N =
HC''
(24)
where
Cxz ¯Cr z
c_I , -i
`Cz I Ca t
(25)
The pattern is now clear as to the means of obtaining a radiance distribution from the generated irradiance distribution. For, generalizing the two simple cases just considered, we now suppose that a given irradiance distribution at point x is generated by a radiance distribution at x which is uniform and of magnitude Ni over each of n narrow sets Di' of directions such that Di' and Dj' are disjoint and with central direction ~-' for each Di', i= 1,...,n. Hence the set {DI', Dg',...,Dn't of subsets of is an arbitrary partition of into narrow bundles of directions. From (8) we have:

n
H(x, 9) ut 21 NiE • E_i' D (Di' Wc, ci')
(26)
where 9 is any direction and 7C(g,&i') has the same meaning as before for the case n = 2.
We now choose n directions Ci, i= 1,2,...,n and write: "Cji",and Hi exactly as before, but now with i and j ranging over the general finite set {1,2,..., nI of integers. With _this notational convention (26) becomes:

H_j =

(27)
Writing:
ttHtt

for (HI,,- . . ,

for (NI,...,

H $_n$)
N_n)

	Cal	C 1 2 •. .	Cin
for	Czl	C 2 2 . . .	Can
	.		
	. r C_n i	C_n2 . . 0 C_{nn} ~	

we can then cast (27) into matrix form:

$$H = NC$$
(28)
The solution of (28) is given by:

N a HC⁻'
(29)
where C^{-}I, the inverse matrix of C, generally exists upon suitable choice of the 9i. The proof of the existence of the general continuous counterpart to C^{-1} is given in Example 15 of Sec. 2.11.

There the full equivalence of H(x, •) and N(x, •) is established. The rigorous proof of the equivalence requires relatively advanced concepts and for that reason is deferred to Sec. 2.11. However, the present discussion has been designed so that the practical details involved in the determination of N by H require no tools beyond those of the elementary theory of algebraic equations.

As a result of the preceding discussion leading to (29), we can view in a new light the observation that "radiance is the most basic of radiometric concepts". The radiance concept is most basic in the sense that from it all other radiometric quantities can be most conveniently derived; it is not "most basic" in the sense that there is only a one-way computational path from it to every other radiometric quantity. This brings up the interesting question of: just which of the radiometric quantities discussed so far have informational content equivalent to radiance? and: just what, in the last analysis, characterizes a radiometric concept which has equivalent informational content to radiance? These questions will be briefly considered in Example 15 of Sec. 2.11.

Field Radiance vs Surface Radiance

There is a distinction that can be made in practice between two types of radiance, a distinction which is analogous to that made in Sec. 2.4 between irradiance and radiant emittance. This distinction is depicted with the help of Fig. 2.12 which shows radiant flux across an hypothetical surface S in the indicated direction and within a narrow conical set D of directions around a direction & normal to S.

Now, corresponding to the conceptual distinction established between W(S,D) and H(S,D) in (17) and (18) of Sec.

2.4, we can write:

$11_N+_{(S,D)}fz$ for W(S,D)/SI(D)

and

"N^- (S,D)" for H(SD)/Q(D)

FIG. 2.12 Conceptual distinction between field radiance and surface radiance N^+.

We call N^- (S, D) the (empirical) ,fie td radiance, and N^+ (S , D) the (empirical) surface radiance (or specific radiance, or specific intensity). It is quite clear that, in the general context of Fig. 2.12:

. w.. . _~ ..,.~...~. ,

N+(SvD) a N_(SOD)

(32)

Despite the numerical equality, the conceptual distinction between field and surface radiance is useful to maintain. Indeed, some need for a conceptual distinction inevitably forces itself on the-attention of careful students of applied radiative transfer theory where on the one hand emitting surfaces, real or hypothetical, are characterized most naturally by surface radiance, and where measurements obviously result in field radiances, The term "surface" in "surface radiance" is a vestige of the days when surface radiance was associated with the radiant emittance of real surfaces enclosing sources of radiant energy. The present interpretation of "surface", however, includes the possibility of hypothetical surfaces anywhere in an optical medium. The term "field" in "field radiance" denotes the sense of "field of view". In practice, whenever possible, one of these two interpretations of radiance is usually fixed and agreed upon throughout a given discussion. Thus, we can omit the "+" (or "•") superscript from "N" when the type of radiance is understood.