2.7 Scalar Irradiance Radiant Energy, and Related Come is

The radiometric concepts studied in this section are those of scalar Irradiance, radiant energy, and related radiometric concepts. The first of these concepts is designed to quantitatively describe the volume density of radiant energy in a way which is amenable to operational methods of determination. In addition to the notion of scalar Irradiance, we shall develop in this section several closely related notions which together with scalar Irradiance comprise a set of useful measures of the volume density of radiant energy. The first of these is radiant density.

Radiant Density

The notion of radiant density is one of several concepts designed to give a measure of the radiant energy per unit volume at a point. Consider a steady beam of radiant flux normally incident on surface S at .point x at time t, as shown in Fig. 2.16. Let the field radiance of the beam at this instant be N, its cross sectional area be A, and its solid angle be 0. The amount of radiant flux incident on S at time t is then NASA. An instant t later, the flux of the beam will have moved-on a distance r = vt, and the flux will have swept out a cylindrical volume of magnitude V = Avt. During this time the beam has been steadily pouring an amount of radiant energy into the volume at the rate of NAn watts. Hence the radiant energy content of the beam is NAUt, and its average content per unit volume is NADt/Avt = NQ/v,

Suppose that point x were simultaneously irradiated at time t by an arbitrary finite number of narrow beams of radiance N_i, i= 1,..., n, and corresponding solid angles n_i. Then the radiant energy u(x,t) per unit volume at x is given at time t, by means of the D-additivity of 0 (equation (7), Sec. 2.3)

area A

FIG. 2.16 Setting up the connection between radiance and radiant density

$$u(x, t) -- \quad | \quad . \quad N. Q \quad \sum_{i=l}^{n}$$

The transition to the continuous case is immediate. Toward this end, let us continue to write "u(x,t)" for the radiant density, i.e., we shall also write:

$$fl_u (x_{10} t) \text{ if}$$

for

$$\frac{1}{v(x,t)}$$

fN(x_t~$_v$t) dil(~) .   (2)

The units of u(x,t) are aoutes/m'. We may use either the field or surface interpretation of radiance in this definition.

Scalar Irradiance

Let us go on to write:

"h (x, t)"   for   N(x,   t)   dQ(t) (3) V

h(x,t) is the scalar irradiance at x and time t. The field radiance interpretation of N is most often used in (3), and this interpretation will be in force unless

specifically noted otherwise. The reason for singling out h(x,t) for special consideration will be made clear in a moment. For the present it suffices to note that in general:

u(x,t)v(x,t) = h(x,t)

By virtue of (3) it follows that in this equation the field interpretation of u(x,t) is to be understood , and that while the units of u (x, t)   are joules/ma, those of h (x, t) are watts/m2 Hence, the term "irradiance" in the name "scalar irradiance" is appropriate. The reason for the modifier "scalar" will also become clear subsequently after vector irradiance has been defined in (2) of Sec. 2.8. A generalization of (3) is obtained by replacing E⁞ by a subset D of E. In that case we would write:

"h(x,D,t)" for fN(x,~,t) dn(E)

D

The radiant density associated with h(x,D,t) is u(x,D,t) and (4) holds for these two quantities.

Spherical Irradiance

We shall now show why scalar irradiance is singled out as an alternate (and an actually preferred) description of the radiant density at a point in a radiant flux field. Consider the light field at a point x in a natural optical medium at time t. Let N(x,•) be the radiance distribution at x. Now imagine a small spherical collecting surface S of radius r in the field so that its center is at x. We then ask: what ia the average amount of radiant flux incident per unit area over S?

To answer this question it is useful to conceptually decompose the great number of radiant flux streams at x into a discrete set of flows. Two such flows are shown in Fig. 2.17. The lines of flux of one of these flows along the direction E~-. have been fitted with little direction cones of solid angle magnitude Ski. Suppose the radiance at x in the direction &i is Ni. Then the irradiance at x on a plane normal to ~i is Nini. If the sphere is small, say the size of a ping pong ball, then for most natural light fields in the air and sea, Ni will not vary in the region of space taken up by the volume of the sphere. From this we see that we can treat the radiance function N as a constant with respect to location in the vicinity of the sphere and of value Ni for the direction t-. It follows that the amount of radiant flux incident on the sphere contributed by the stream of flux in the direction ~i is $(n_i Q_i) n r^z$. This estimate is based on the

FIG. 2.17 Computing the radiant flux intercepted by a spherical collector in a general light field

assumption that the amount of flux of a narrow beam intercepted by a curved hemispherical surface is the same as the amount intercepted by the great-circle area associated with the hemisphere. The assumption is rigorously defensible for transparent media using the concepts of vector analysis and Stokes Theorem. For the present the reader's intuition will readily allow this assumption to stand

even for the case of turbid media as long as r is kept very small; The "line of flux" interpretation will help the intuition considerably in this matter.

The main task in answering the above question has now been dispatched. It remains only to add up all the contributions by the various beams of flux, using as justification Equation (7) of Sec. 2.3. The result is:

πr^2

$$N_i \Omega_i$$

The average radiant flux per unit area of the sphere S is then obtained by dividing this quantity by $4\pi r^2$. Let us designate this average by writing:

$$\text{"h (x,t)" for } \frac{1}{4\pi} \sum_{i=1}^{n} N_i \Omega_i \quad (5)$$

and agree to call it spherical irradiance. We shall retain this terminology and notation for the continuous formulation. That is, we shall write:

$$\text{"h (x,t)"} \quad \text{for} \quad \frac{1}{4\pi} h(x,t) \quad (6)$$

Definition (6) is the basis for an operational determination of scalar irradiance using a spherical collecting surface S. For the average radiant flux per unit area on S is readily measurable and this amount differs multiplicatively from u(x,t) by a fixed numerical factor. Hence, by only slight changes in optical design, the same photoelectric devices used to determine H and N can be directed to obtain scalar irradiance h.

Therefore it is spherical irradiance or scalar irradiance which is directly measurable by photoelectric devices.   The concept of radiant density u(x, t) is by way of contrast a theoretical concept related to the empirically based concept h (x, t) by means of (4) .

Hemispherical Irradiance

One of the most useful mathematical models of light fields in-natural waters is the exact two-flow model to be considered in detail in Chapter 8. A radiometric concept which arises in that theory, and one which also has been found of intrinsic interest to experimenters, is the concept of hemispherical scalar irradiance. We now discuss this concept.

Figure 2.18 (a) depicts a small spherical collecting surface S with center x which is exposed to flux from only one hemisphere of '. Let N(x,•) be the radiance distribution

at x. Let us say that light is incident on the sphere in the direction of [ ). We ask: what is the average amount of radiant flux incident per unit area over S? Clearly every point of S is in principle exposed to the light field over 5[ ). Fig. 2.18 (b) shows how an obliquely incident beam with a direction in [E] can come close to illuminating the "north pole" of the little spherical surface. If we divide up '(E) into pieces analogously to the manner used in deriving the expressions above for spherical and scalar irradiance, then it becomes clear that the integral of N(x,•) over '(C) yields the appropriate scalar or spherical irradiance component,

FIG. 2.18 Details for a shielded spherical radiant flux collector

Thus, using field radiance let us write:

"h (x,~,t)" for $N(x,E',t)$ dg(El) (7) V

and analogously, we write:

t'h4w(x,~,t)" for

$$\Xi(\xi)$$

We call h4 r (x , ~, t)the hemispherical irradiance at x , over the hemisphere W(&), at time t. Further, h(x,&,t) is the associated hemispherical scalar irradiance. It is clearly a special case of h (x , D, t) defined after (3) above. Methods of measuring hemispherical irradiances will be discussed _in Chapter 13. It follows immediately from (3) and (7) that

h(x,t) = h(x, ~, t) + h(x, -E,t) (9)

An analogous connection to that displayed in (9) also holds between h4.$_{ff}$ (x, ±&, t) and h4, (x , t) . The introduction of

h4,ff (x , ~, t) into the family of radiometric concepts is motivated exactly for the empirical reasons that motivated the introduction of its full spherical companion

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When we are working in stratified light fields (Sec. 2.4) then it is possible to drop without loss of generality the "x" and "y" coordinate symbols from the notation and re

tain only the depth coordinate symbol "z" in the notation. In such contexts we agree to write:

$_{or}$ Irh(z,0,0,t)+' for h(x,g,t) - (10)

In particular, if E is k or -k, which occurs in the important case of the two-flow theory (Sec. 8.3), then we agree further to write:

"h (z,±,t)" for h(z,±k,t) , (11)

where we read upper signs together and then lower signs together to obtain two separate definitions. As usual; when the light field does not appreciably change in time, or when time is understood, we shall drop ' I t " from the notation. Applications of these concepts are taken up in Sec. 13.9.

Radiant Energy over Space

The discussion of this section is now continued by officially noting two interpretations of the term "radiant energy". The first interpretation centers on the simple connection that exists between scalar irradiance and radiant energy. Suppose X is a subset of an optical medium over which at time t there is defined a scalar irradiance function h for a given frequency v. Let "U(X,t)" denote the radiant energy content of X at time t. That is, by the definition of u (x, t), we agree to write:

i s $_{U(X,t)}$l t for u(x, t) dV(x) (12)

and from (4)

U (X , t) = f (h (x , t) / v (x , t)) dV(x) (13)

X

where "V" is the volume measure of the optical medium. As a special case, if v(x, t) and u(x, t) are independent of x and t then (13) becomes:

U (X) _ (h/V) V (X) (14)

where, for this case, we have written:

"U (X)" for U(X,t) "h" for h(x,t) t'V" for v(x,t)

I t is clear from (12) that U (• , t) is v-additive and v-continuous. That is, for every two disjoint parts X} and X2 of an optical medium:

$$U(x_i,t) + U(X_2,t) = U(X_l \cup X_2,t) \quad 9 \quad (15)$$

and for every X and t:

if V(X) 0

then U(X,t) w 0   It   (16)

Radiant Energy over Time

There is still one more interpretation that can be made of the term "radiant energy". The preceding interpretation of (12) is associated with the energy content of a given region X at time t. There is a complementary interpretation of the total energy incident on or leaving a surface S over an interval T of time. For this interpretation we write ,e.g.:

"U⁻ (S,T)"   for   f   f H(x, W (&) , t)   dA(x) dt   (17)

T S

i.e., U"(S,T) is the radiant energy incident on S over the time interval T. The hemisphere of incident radiant flux at each x is E(Q , with C normal to S at x, in the inward sense. A complementary definition can be made for $U^+(X,T)$ using radiant emittance.

It is worthwhile isolating the important concept, occurring in (17), of radiant flux across a genera Z surface S rather than just a collecting surface of the kind encountered in the sections above. Thus we write:

T ! p - (Sot.) i t   for   H(x,H[ ) ~t)   dA(x) (18) f

S

where C is the unit inward normal to S at x. A similar definition of F+(S,t) can be phrased. As usual, the signs "+" and "-" can be dropped whenever no confusion results, and also the "t" can be omitted for brevity.

Scalar Radiant Emittance

we conclude this section with the definition of the notion of scalar radiant emittance. This concept is the surface-counterpart to scalar irradiance h defined in (3). Thus, let us write

, 1 w (x, t) 1 1 for   f   N+ (x o   g ' i   t)   d Q   (E ')   (1 9)

w(x,t) is the scalar radiant emittance at x at time t. This concept is useful in describing certain sources of radiant flux distributed continuously over some region of an optical medium. The emittance counterparts to hemispherical scalar

irradiance emittance can now, be defined for w(x, t) . These definitions would exactly parallel those in (5), (6), (7), (8), (10), (11), and therefore need not be given in detail at this time.