2.10 Polarized Radiance

In this section we shall develop an operational definition of polarized radiance. The development shall take as a point of departure the notion of empirical radiance introduced in Sec. 2.5. The details of the development shall be kept to a minimum, as we will not in this work make extensive use of the concept of polarized radiance. For a somewhat more detailed theoretical discussion of polarized radiance suitable for geophysical applications, the reader is referred to Chapter XII of Ref. [251),

Before going into the technical details of how to measure polarized radiance, a few comments may be made on the reason for wanting to measure polarized radiance in natural optical media. The first and most important reason is that the systematic documentation of the state of polarization of submarine [and atmospheric] light fields increases our store of basic optical knowledge of the world in which we live W For those of a more practical turn of mind, it may suffice to add that knowledge of the kind and amount of polarization extant in a natural light field could yield efficient means _of increasing visibility in-both the atmosphere and the sea. For, the contrast of objects seen against a sky or underwater background is occasionally increased when viewed through a material which can transmit polarized light in various amounts depending on how the material is held and oriented. If we possess systematic tabulations of polarized light fields and some workable theoretical models of such fields, these empirical observations can be more deeply explored and applied. Finally, there is the question, still not fully resolved-especially for the hydrologic optics branch of geophysical optics--of whether and to what extent polarized light is used by creatures in navigating, in foraging, and in their biological growth cycles.

In order to help resolve such questions and in order to add to our knowledge of the light fields in natural hydrosols in a systematic manner, we must develop a precise but workable means of measuring and theorizing about polarized light fields. A small but definite beginning in this direction will be attempted in this section and subsequently in Chapter 4 wherein the equation of transfer for the polarized radiance vector is used to derive a theoretical model of polarized light fields in natural optical media.

One final comment is in order. It will be recalled that our approach to hydrologic optics is through the tenets of radiative transfer theory and, as a consequence, we are committed to-study the natural light fields on a phenomenological level. In particular, as pointed out in Sec, 2.0, we have agreed to adopt those instruments of investigation which make quantitatively precise, all of the optical phenomena visible to the human eye. One may then--in view of this observation--argue that in extending the capabilities of our instruments to detect and measure polarized light fields we are transcending the bounds originally set down by us when we embarked on the development of the concepts of radiometry. It may be observed, however, that whenever it is deemed necessary to extend the radiative transfer phenomena of concern to hydrologic optics in particular, or geophysical optics in general, the extension will be made solely on its merits to add to the descriptive power of these branches of radiative transfer theory.* In the present discussion, the extension of the radiometric concept of radiance to the polarized level not only fulfills this general criterion, but interestingly enough, still keeps the collection of radiometric concepts within that small, select circle of concepts which are directly observable b the unaided eye. For indeed, the polarization of the light of the sky or a submarine light field is directly observable to the unaided (but practiced) human eye. The physiological basis for this capability of direct observation is the dichroic nature of either the material comprising the yellow spot of the retina or perhaps that of certain of the optic nerve fibers themselves. (Dichroic materials are also found in natural deposits, e.g., in the form of tourmaline crystals, and were already used in the early devices for detecting polarized light.) It is the small but adequate amount of dichroic material in the retina which thus permits the unaided eye to detect and the brain to record the presence of linearly polarized light in a natural light field. This innate ability of the eye to detect polarized light was reported by Haidinger in 1846, and the elusive but yet visually observable pattern seen by the eye is known as Haidinger's brush. An informative description of how to facilitate the detection of Haidinger's brush in skylight is given by Minnaert in Ref. [182]*,

'The case for an extension of the classical scalar theory to the polarized level ultimately involves no less than the consistency of the classical scalar theory in the context of polarized light fields. See (I?) of Sec. 13.11.

FIG. 2.25 Schematic details of a radiance meter fitted with-a polarizer P and a variable wave plate W, for measuring polarized radiance distribution.

Operational Definition of Polarized Radiance

The operational definition of polarized radiance we shall adopt has been chosen for its inherent simplicity and its amenability to be linked with the classical Stokes vector for polarized light. Thus the polarized radiance vector will, on the one hand, be tied directly to observable qualities of natural light fields and, on the other, be rigorously representable by means of concepts extant in the electromagnetic picture of light.

We begin with a radiance meter, as described in Sec. 2.5, and adjoin to the meter, at the base of the tube, a polarizer P and a variable wave plate W. The order in which the entering light encounters these devices is important and is depicted in Fig. 2.25: the light is to encounter the variable wave plate first, and the polarizes second;, then it. Passes on through the filter to the Photo element below. This relative placement of W and P is the essential point to observe here; where the filter is relative to W and P is, however, immaterial as far as ideal detectability of polarized flux is concerned.

The polarizer P, which is made from a dichroic crystal or a sheet of Polaroid , is mounted so that it is rotatable about the axis of the cylindrical tube of the radiance meter. The orientation of the optic axis of the polarizes is important in what follows; therefore it is essential that some

means be provided for the clear marking of the position of the optic axis relative to the radiance meter's tube ,.or some other fixed part of the radiance meter. Further, if absolute radiance measurements are desired, the transmittance of P over the spectrum is required. The ideal transmittance of P is 1/2 for unpolarized light.

The wave plate can be made of some negatively doubly refracting material such as calcite, and is assembled at least for the introductory discussion below) so that a wide range of optical path lengths is available at the twist of a knob. For example, a Babinet compensator type of arrangement may be employed. Later on, when the radiance meter is readied for field use, the wave plate may be replaced by an attachment fashioned from a single sheet of some good grade of circularly polarizing material of known transmittance over the spectrum. The ideal transmittance of W is 1, and that of circularly polarizing material, 1/2. The fact that a circular polarizing material can be used in lieu of a variable wave plate will become clear after the observable radiance vector has been defined.

The next step in the present operational definition of polarized radiance is to take the radiance peter, set for frequency v, to a point x in the environment, and direct it so that flux enters the tube along the direction E at time t. Then one systematically varies the angle * of the polarizer's optic axis, starting from the vertical plane, with a given fixed retardation e -Z 0 of the wave plate. (See Fig. 2.26)

In fact one varies * from 0 radians (so that the optic axis is in the vertical plane) and increases # clockwise (when seen looking into the tube from the front of the tube, i.e., looking along the direction of travel of the photons) to radians (so that the optic axis is again in the vertical plane). As this is done, one should note how the recorded flux varies with and that the variation is of period w. In particular, for e 0 and a general light field, the variation of radiance turns out to be representable in the form:

$$\frac{1}{2}\left[I + Q\cos 2\psi + U\sin 2\psi\right]$$

where we have written:

it ¡#t	for	29
t#Qtt	for	2AN cos 2f 0
#tuft	for	2AN sin 2* 0

where, in turn, we have written:
t#N"
for $(N_{max} + Nmin)12$
"AN"
for $(N_{max}$! %in)l2 0

FIG. 2.26 How to measure and record the standard observable radiance vector, and where "Nmax" and "Nmin" denote, respectively, the maximum and minimum radiance readings when * is varied from = Q to
V _ W, *o is the angle of occurrence of the maximum reading Nmax

Further experimentation, with now a general e-setting on W and with 0 varying over the interval [o,n], shows the full form of the radiance variation to be:

2

11 + Q cos 2* + (U cos e-V sin e) sin 2*

where V is determinable by a simple trigonometric analysis of the recorded curves obtained by fixing * ~ D and varying e. (See, e.g., Ref. (193]). If this reading is obtained at point x, for the direction C at time t, for frequency v, and with a P-setting *, and a W-setting e, then we will agree to denote

it by "N(x,C,t,v,*,E)", or, if x,E,t, and v are understood, we will denote it simply by for short. The development of the empirical basis of this quantity (using the 'IS", "D", "F" notation for radiant flux) is parallel to the unpolarized radiance case of Sec. 2.5, and therefore need not be repeated here. Expression [1] constitutes the desired: operational definition of the polarized radiance N(x,E,t,v,*,e)

The Standard Stokes and Standard Observable Vectors

The operational definition (1) for polarized radiance and the experimental considerations leading to it draw out the remarkable fact that the most general polarized radiance field can be characterized by four functions, I,Q,U and V whose values are determined once a selection of *,E along with x,&,t and v are given. In view of the potentially infinite variety of specific forms that polarized radiance fields can assume, this is indeed a remarkably simple characterization and representation of the entire class of possible fields. This theoretical characterization of the polarized light field by I;Q,U, and V was first systematically studied by Stokes in 1852. We shall write:

for G ,Q,U,V)

and call S the standard Stokes vector. S is a function which assigns to each choice of *,E, along with x,&,t, and v,. an ordered quadruple of radiance numbers obtained as described above,

There is an alternate method of quantitatively documenting a polarized light field. Instead of obtaining I,Q,U, and V as described above, one may obtain four direct readings

N(*,E) for the following four special pairs of settings * and e. We write:

it I Ntt	for	N (0 , 0)
11 2N"	for	N(n/2,0)
it sN tt	for	N(n/4, 0)
it 4N 11	for	N(w/4,n/2)

We then go on to form an ordered quadruple from these numbers; we write:

"N"

for (jN,2N, $3N_s$4N) ,

and call N the standard observable vector. N is a function which assigns to each choice of x,&,t, and v the four numbers shown. Observe how N requires use of W only for the setting 4N. Readers familiar with-the concepts of polarized light will see that each N(A/4, w/2) can be obtained by means of a single reading using a piece of circularly polarizing material.

We shall see that S and N are equivalent descriptions of polarized light fields in the sense that knowledge of either description allows the deduction of the other. Since N is operationally the simpler of the two radiance vectors, we shall henceforth work with N. But before going on to the exclusive use of N, we shall establish some important and useful connections between S and N. Then, since S is computable directly from electromagnetic theory, we will have available a direct tie between N and electromagnetic concepts, to be used when needed.

As an illustration of how N characterizes the commonly occurring polarized radiances, consider the following list of special cases where "N" denotes the relative magnitudes of the components $_i$N of N.

Verbal Description	Observable Radiance Vector
vertically linearly polarized radiance	1(2N,n,N,N) 2
horizontally linearly polarized radiance	1(U,2N,N,N) 2
linearly polarized radiance at +45°	1(N,N,2N,N) 2
linearly polarized radiance at -45°	1 $_2$(N, N, U v N)
right circularly polarized radiance	I (N,N,N,o) 2
left circularly polarized radiance	1(N;N,N 2N) 2 '
unpolarized radiance	1(N,N,N,N) 2

To tie in these conventions with electromagnetic conventions, recall first that the optic axes of P and W lie initially in the standard preferred orientation, i.e., they lie in a vertical plane. Now the E-vector in vertically polarized light by convention lies in a vertical plane as it crosses planes perpendicular to the direction of travel. (Recall that the E-field is a transverse field.) In +45° linearly polarized light the E-vector lies in a plane tilted +45° from the vertical plane containing the direction of travel of the ray associated with the electric field E. The + direction of. the +45° is measured clockwise as one looks along the direction of travel of the ray. Finally, right circularly polarized light is by convention that light associated with an E-vector whose tip describes a clockwise circular motion on a stationary plane perpendicular to its direction of travel as seen on the incident side of the plane. The most general light field can be resolved into its linear and elliptical components. This is the Polarization Composition theorem of Stokes.

Analytic Link Between S and N

The connection between the two vectors S and N is easily established by means of (1). On the basis of (1) and the definition of iN, i s 1,2,3,4, we have:
N = 1 [I 2
N = 1 [I 2
N 1 [I 2
N _1 [I

From this set of equations we may construct the matrix which transforms S into N. Thus let us write:
D
Then:
where
" $\wp$ "
for

	1	1	1	1
(2)	1	- 1	0	0
	0	0	1	0
	0	0	0	- 1

N a S f '   (2)

S = N P   (3)

1	1	-1	1
1	- 1	-1	1
0	0	2	0
0	0	0	- 2

As an example of the use of (3), we obtain the following representation of vertically polarized radiance in terms of Stokes vectors:

$1 \; _2(2N,O,N,N)$

1 1 - 1 1

1 - 1 - 1 1

(N , N , 0 , 0)

0 0 2 0

0 0 0 - 2

$\mathcal{P}^{-1}$

The reader will find it instructive to use (3) to obtain a list of Stokes vector representations of the seven special observable vectors given above,

Standard and Local Reference Frames

Up to this point in the exposition of the polarized radiance vector all operational activity has implicitly taken place in a terrestrial coordinate system. In particular the -setting of the polarized radiance meter was such that if

0, then the optic axis of the polarizer P of the meter is in a vertical plane. (See Fig. 2.26 (a).) Such a frame of reference for polarized radiance measurements we call a standard reference ,game, We now introduce a second reference frame--the local reference frame--whose main virtue and reason for being is that at each point in an optical medium it permits a simple means of experimental determination of the polarized volume scattering function. Furthermore, the introduction of the local reference frame considerably facilitates the formulation and handling of the various forms of the transfer equation in the polarized context.

To establish a local reference frame at A point x in an optical medium, two directions E' and ~ must be given (in, say, a terrestrial coordinate system) such that C' and

uniquely determine a plane. In other words, the only requirement on ' and E is that they not be collinear. We shall call V and , respectively, the incident and scattered directions, and the plane they determine together with a point x, the

plane of scattering. See Fig. 2.27.

Once a plane of scattering is determined by x,V and C, the incident polarized radiance is measured as follows: place the radiance meter at x so as to allow the flux in the direction ~' to enter the meter's collecting tube. With the setting at 0 with respect to the meter's tube, rotate the entire radiance meter around V so that the optic axis of the polarizer P lies in that plane A' through t' and perpendicular to the scattering plane. With the radiance meter so oriented, perform the four operations leading to -N, i = 1,2,3,4. Designate the local observable vector by "N~,"where "~" denotes the angle through which the vertical plane through t' must be rotated clockwise around E'--when looking along the direction of V--so as to become coincident with the plane A'. Thus 0' varies from 0 to n; similarly with ~ for the radiance determination in the scattered direction E. In general it can be shown (Sec. 111, Ref. 0251)) that the standard observable vector N is related to a local observable vector No by means of the equation:

N_0 = Ndf(0)   (4)

where we have written:

FIG. 2.27 The plane of scattering, and associated angle conventions.

$$" \mathcal{L}(\phi) "$$

for

cos2~ -1 sin 2# sin'# +1 sin 2# sin'# -1 sin 2$ 2   2   2

sine# -1 sin 2# cos^2# +1 sin 20 sin'# +1 sin 2~ 2   2   2

sin 2~   -sin 2#

cos 2#

0   0

It is useful to observe that ~(#) has the properties:

$$\mathcal{L}(\phi_1+\phi_2) = \mathcal{L}(\phi_1)\ \mathcal{L}(\phi_2) \qquad (5)$$

X ⁻'(#) M o$^{\%}$((⁻#)   (6)

Thus, in particular, the inverse X"(~) of v°(~) is obtained s i m p l y by replacing 0 by -0 in ;1~(O).

The following example will illustrate the use of (4). Fig. 2.26 (b) depicts a beam of +45° linearly polarized radiant flux proceeding along direction & at point x. Hence

N a 1/2 (N,N, 2N,N) .   The reference frame is now switched from the standard reference frame at x to a local reference frame at x defined by a rotation 0 of magnitude +45° around E.   In order to find the components of the given beam of polarized f l u x in this new frame, we first note that:

v	1	0	0
0	1	1	0
1	-1	0	0
0	0	0	1

Hence*

fl 1 0 v   0 1 1 0

N

45* =   Z   (N , N ,   2N,N)

o   0 0 1

" 1(2N,Q,N,N) 2

that is, NJ* is vertically polarized radiance in the new frame of reference, as was to be expected.

The primary advantage of introducing local reference frames and their corresponding local observable radiance vectors lies in the fact that the volume scattering matrix obtained in most natural optical media can be given a simple standard form whenever such frames are used. The general relation (4) permits the resultant scattering matrix to be fitted into discussions using the standard observable radiances.

Radiant Flux Content of Polarized Radiance

Having extended the concept of radiance to the polarized context, the question now arises as to the necessary connection that exists between the readings of a radiance meter with and without polarization attachments. Specifically, let N be (jN,2N,3N,4N), the observable radiance vector (in either standard or local form) at a point and a given direction at that point. Further, let N be the simultaneous radiance reading of the meter at the same point and same direction with the polarizer P and wave plate W removed. What is the connection between N and N? This question, interestingly, cannot be answered within the theoretical framework of radiative transfer per se; of course it can be answered on the empirical level quite easily. However, to establish the desired theoretical connection one must appeal to some relatively finer grained picture of light phenomena, such as electromagnetic theory. On such a more fundamental level both N and the components of N are representable in terms of the principal construct on that level: the electromagnetic wave. The-desired connection can be established by suitably relating these electromagnetic representations of N and the iN. On that level the desired connection is readily forthcoming (see, e.g., Ref. [43])' and is of the form:

$${}_1N + {}_2N \quad .$$

This relation is interpreted as described above and under the assumption that P has ideal transmittance 1/Z for unpolarized flux and W has ideal transmittance 1.

Of course these ideals are not attained in practice. However, with (7) as a starting point, the associated practical version is readily established. The customary operational
definition of the transmittance of the polarizer assembly is as follows. We write:
for max$_i$ (N (V , 0) /N)(18)

Further, N(*,O) is, as the notation implies, the radiance reading with the W setting c equal to o (or W removed entirely) and with P in place and with optic axis rotated an amount
in the usual way. For example, it follows from (1), in the case of linearly polarized radiance that N(*,O) varies

ideally as cos^i*, Hence N(*,D) reaches its ideal maximum of
N(O,U) N at 0. In practice, however, N(0,0)< N, and
so T <1,, It is now easy to establish that the practical counterpart to (7) is:

$N = ({}_1N + {}_2N)/T$

where ,N and $_z$N are measured with the same polarizer P as that used to obtain T in (8) and with the identical disposition of W as that used for (8), i.e., either W is in the tube and c = 0, or W is removed entirely.

To summarize, if a radiance tube is fitted with attachments to allow the determination of the observable radiance vector (,N, 2N, sN, 4N) , then the associated reading N of the meter without these accouterments is generally related to the vector components by means of (9), with T defined as in (8). In this way the radiant flux content N of N is established.

On the basis of relation (9) or its suitable generalizations, it is possible to use tabulated polarized radiance data to compute all the usual unpolarized radiance, scalar irradiance and vector irradiance quantities, etc. formulated in the preceding sections, simply by replacing "N" everywhere in those formulas by "(1N+2N)/T". In this sense then we understand polarized radiance data to be more general than unpolarized radiance data, for it includes the latter as a special case.