3.2 The Interaction Principle

With the preliminary example complete, we turn now to the statement of the central principle of radiative transfer theory:

The Interaction Principle : For every X, 5, A, B, m and n, i f X is an optical medium and S is a subset o f X, and A ;, Am)) is a class of sets Ai consisting of incident radiometric functions on S, and B (_ (B I, . . . , Bn)) is a class o f sets Bj consisting o f response radiometric functions on 5, and m and n are positive integers, then there exists a unique set (s_{ij} : i=1,...,m, jnI,..., n) of linear (interaction)

operators sij with domain Ai and range Bj with the property that for every element (al, • • . ,a_m) o f A there exists an s te ment (b Y, . + , , bn) o f B such that:

b_j

m

Z ^{a}i^{s}iJ i'1

or in matrix form

where we have written:

"$_a$ll for

Co$_b$" for

II$_S$fI for

b = as

$(a_1, \ldots, a_m)$

(bi,•..,bn)

$s_{1=}$ s 21

r .

r

^{s}ml ^{s}m3 Smn
R
s_{12} ... s_{1n}
s_{22} ... s_{2n}

Discussion of the Interaction Principle

We shall discuss in some detail the meanings of the various terms in the interaction principle. First of all, the meaning of the term "optical medium" as used in the statement is quite broad and ,for example, is intended to have as real designata such parts of the world as lakes, oceans and various portions of the atmosphere. From the mathematical point of view, "optical medium" may be interpreted simply as part of Euclidean three-dimensional-space such as the region between two infinite parallel planes or the interior of a sphere, etc., in which we assume that the principles of geometric: optics hold, in particular, Fermat's principle. There will eventually evolve, as the studies progress and the basic constructs assume their final form, a relatively technical, version of what we mean by the term "optical medium" in the fully developed theory (re: Def. 5 of Sec. 9.1). However, for the present the term may have either of the simple meanings suggested above.

The meanings of the terms A, B, and sij in the principle can be illustrated using the preliminary example of Sec. 3.1. Let us return to the setting summarized by Eqs. (4) and (5) of Sec, 3.1. In that setting the optical medium was some (physically) vacuous region x of Euclidean three-space containing two plane surfaces S_3 and S_x. We concentrate attention on Si. Then 5, is an instance of $S_.$ in the principle. Consider the set of all incident radiances like NO on Si.

This set of incident radiances becomes the set A_i in the principle. Consider the set of all incident radiances like N_{ti} on Si. This becomes the set At in the principle. Together, (AI,AI) constitute the incident class A in the principle, so that m a 2. It should be noted that Ai and At are each closed under the operations of forming sums and products by nonnegative numbers (linear closure). Thus if N, and Nt are in A_i, then so,is cNi + dNt where c and d are nonnegative numbers, This feature of Ai and At comes automatically with the requisite linearity of the sib. The class B of response functions Si consists of one set Bi, with Niz as a typical element. Therefore in the case of Si we have m = 2, and n = 1, with E?i and Eiji as the present instances of ail and sti, respectively . Hence one invocation of the interaction principle for the case of Si yields (4). Another and distinct invocation in the case of St yields (5).

The alternate example summarized in (10) of Sec. 3.1 provides a further illustration of the principle's linear algebraic statement. In (10) of Sec. 3.1, x is the same space as above. Now, however, Si and Sz are considered parts of one and the same subset, say S of X. Consider the set of all ordered pairs of incident radiance on S like (N,, , N?) . This becomes Ai in the principle. Consider the set of all ordered pairs of response radiances of S like (Ni 0,Nz11) . This becomes Bi in the principle. Therefore in the present case of S, we have m o n_o= 1, and sii is 0^0 . As we select any new incident pair (N1,N1), there correspofs the associated response pair
(Ni 8,N2y) given by (10). Clearly (10) is the present instance of the matricial form of the principle's algebraic statement.

As we progress along the line of examples of the interaction principle we shall be gradually less explicit in pointing out the particular parts of the current form of the interaction principle, leaving the details of correlation more to the reader as he becomes familiar with the principle. In all the subsequent uses of the principle, we shall look upon it as a convenient working principle, i.e., a rule of action for the formulation of subordinate principles, the various laws, and everyday problems of radiative transfer theory. The practical uses of the Principle are directed to determining the light field in natural Optical media by finding the interaction operator sij, supplied by the basic principle, for a given medium. The determination of the structure of the operators sij and the various functional equations they satisfy constitutes one of the more interesting and challenging problems of modern radiative transfer theory. We shall begin the investigation of these operators in the present chapter and continue it in Chapter 7.

The Place of the Interaction Principle in
Radiative Transfer Theory

It is not intended that the interaction principle categorically replace all classical instances of itself such as the principles of invariance and the invariant imbedding re lation, or other classical instances that occur in the literature or that arise during the subsequent developments below. Rather, it is intended that the principle be viewed by its. users simply as a working principle of radiative transfer theory, and to be used (and perhaps refined) by those students of the subject who prefer to envision the theory as governed by and derivable from a single idea, The place of the interaction principle in radiative transfer theory and in the mainstream of physics may be summarized by the following diagram:

r MW mm..MWW-W .-mm .-mm __ 1~
(Mainland of Physics) 1 (Mathematical Measure Theory
I ~I I I
I ELECTROMAGNETIC I I I ABC IOMS OF RADIATIVE I
I THEORY I
TRANSFER I
I I I I

saw .~ 0..M~11

__ 0wir .war ~ ass* _ J _ _ 0r &r _ _ ~ ~ ~ _i ~ ~ ~ ~ rri ~

INTERACTION PRINCIPLE I

i

I (Radiative Transfer Theory)
I

I

I Equation Principles Invariant I

I

I of of Imbedding
'

I Transfer Invariance Re la t ion

L J

As the diagram indicates, radiative transfer theory may join the mainland of physics via electromagnetic theory (see, e.g., Chapter XIV, Ref. [251]] or the theory may be made completely autonomous using an axiomatic formulation made elsewhere (Chapter XV, Ref. [2511). Direct interconnections also exist between the three principal parts of the theory (indicated in the diagram below the interaction principle). In fact the internal ties on the level of the general equation of transfer, the general principles of invariance and the general invariant imbedding relation are so strong that these ties are effectively logical equivalences. The details of the pursuit of these connections are mainly mathematical and are beyond the scope of the present work. For further details on this matter, the reader is referred to the various chapters of Ref. [251] .

Levels of Interpretation of the Interaction Principle

The great practical range and depth of the interaction principle arises from the levels of interpretation on which it may be applied. There are generally four main levels of

interpretation of the principle: the point, tine, Surface, and space levels. Of these, the surface and space levels of interpretation are operationally the most meaningful. The point and line interpretations are special theoretical artifices which increase the range of the principle in specific settings. The preliminary example above is an instance of the surface level of interpretation. In general, the surface level interpretation of the interaction principle subsists when one interprets the subset S of a space X as a subset of one less dimension than X. For three-dimensional spaces X, S would have two dimensions. For two-dimensional spaces
(which arise in certain mathematical models) S would have one dimension, etc.

In general the space-level interpretation of the interaction principle subsists when one interprets the subset S of a space X as a subset of the same dimension as Plane-parallel slabs, spherical solids in Euclidean three space are settings for the space-level interpretation. For two-dimensional spaces X, the subset S would have two dimensions, etc.

Of the remaining two levels of interpretation of the principle, the point level interpretation is the more widely used. In fact the point-level interpretation covers so much ground that it is convenient to regard it from two separate aspects. The general point-level interpretation of the interaction principle subsists when X is a general space whose points are arbitrary. The general point-level interpretation is of most use in the development of general discrete-space theory (Ref., [251]), The special point-level interpretation of the interaction principle subsists when S is a point or an optically small three-dimensional subset of space (Le., e.g., a point source) in which single scattering processes are to be dominant relative to multiple scattering processes. This special interpretation is commonly used to establish in an intuitive fashion the concept of the volume scattering function, which plays a key role in the theory (see Sec. 13.4). An alternate establishment of the volume scattering function could take lace strictly and rigorously in the space-level interpretation (see Sec. 3.14). The special point-level interpretation is also a useful and defensible ploy in setting up radiative transfer theory and is thereby retained and given a special status. (See, e.g., Example 1, Sec. 3.17.)

The final level of interpretation to be discussed is the line-level interpretation of the interaction principle. The tine--level interpretation subsists when one interprets the subset S of a space X ,as a one-dimensional subset of X. The line-level interpretation is not operationally meaningful as are the surface, space and special point-level interpretations. However, it is retained because it favors useful mathematical artifacts, as does the special point-level interpretation. Furthermore, like the special point-level interpretation, the use of the line-level interpretation is rigorously defensible by means of limit arguments starting with the space-level interpretation; for that reason it is retained as a useful technical device, We -shall use it below in viewing the path radiance as the response of a path in real optical medium to the incident path function radiances along the path. (Example 2, Sec. 3,17.)

Unless specifically noted otherwise, we shall henceforth mean by "optical medium" any three-dimensional part X of Euclidean three-dimensional space: This then will automatically set the dimensionality of S in the various interpretations of the interaction principle. (A formal definition of optical media, as they are studied in radiative transfer theory, is given in Sec. 9.1.)