3.10 Derivation of the Beam Transmittance Function

In this and the following four sections the interaction principle is used to derive the basic inherent optical properties and the integral equation of transfer for radiance in general optical media. In the present section the beam transmittance function is derived which in turn will yield the first of the inherent optical properties in Sec. 3.11, namely the volume attenuation function.

Let X be a general source-free optical medium. Consider a point x in X and a direction ~ at x. These together determine a natural path in X--a path for a light ray through x with direction . By specifying a length r, a path segment ,,^{40}r(x,~) is determined within x. For simplicity of exposition we shall introduce the beam transmittance concept in a medium in which the index of refraction is constant. Then P r (x , U is a sensed, straight line segment in X with initial point x, terminal point x+rt and length r. We shall write ^{11}z" for x+Er. The requisite steps for the definition in a completely general medium with variable index of refraction will be clear from the following derivation.

Our goal is to define the beam transmittance function-the well-known function of radiative transfer theory which assigns to an initial radiance N_o(x,E) at point x of r (x , 0 the residual radiance N?. (z , E) at z after traversing do_r (x , 0 . This residual radiance N O (z , ~) may be described as consisting of photons which have traveled the entire length of Pr(x,&) without having undergone scattering or absorption. Were we not in possession of the interaction principle, we would simply define the beam transmittance of ~'r (x, t) as the ratio N_r (x, t) /N_o (z, t) . Such a definition is quite acceptable and implicitly assumes the terms "scattering" and "absorption" as understood, using their meanings as grounded in other departments of physics such as electromagnetic theory, and thus requiring no further elucidation. However, we choose not to use the words "scattering" and "absorption" at this stage of the exposition. Instead we ask: can the basic idea of "beam transmittance" be communicated without presupposing the physical notions of scattering and absorption? If this is possible then the theory of radiative transfer, at least in this area, is kept self-contained and freed from unnecessarily using

FIG. 3.29 The general setting for beam transmittance, path function, and path radiance derivations. The basic path e9r[x,0 is imbedded in a cylindrical volume located in a general optical medium X (a portion of the atmosphere or hydrosphere),

undefined terms. But to do this requires essentially a geometrically-based definition of absorption and scattering. Scattering and absorption are not ostensibly geometric concepts as they occur in, say, classical electromagnetic theory. Hence there appears to be an impasse between us and our present goal along the path we wish to travel as long as we try to retain the strict electromagnetic meanings of these terms. However, if we pause and examine closely the meanings of the terms "absorption" and "scattering" as they are customarily used in radiative transfer contexts, we find that there is no essential reference to physical processes beyond the simplest conservation-type of activity.

Radiant flux is observed to enter and leave a medium and it is possible to take a census of the immigrating and emigrating photons. What happens inside the medium is not of immediate concern--only the phenomenological aspects o f the transfer o f radiant flux through the medium is of concern. Thus the terms "scattering" and "absorption" as they are used in radiative transfer theory proper are characterizable solely by means of a geometric measure of radiant flux. Such a view was taken in an earlier study (Sets. 11, 19, Ref. [251]) and it was found possible to give a complete geometric characterization of the concepts of residual radiance, scattering, absorption, attenuation, and hence beam transmittance as they are used in classical radiative transfer theory. We shall adopt the view of Ref. [251]

in the present discussion and with the concept of the 1d[P]operator at our disposal, we shall use it to define the beam transmittance function geometrically.

Let C be a right cylinder in X containing 'r(x,~) as its axis. This cylinder is depicted schematically in Fig. 3.29. The plane base of C at x is denoted by "a", that at z by "b". Let k(x) (_ -Q be the unit outward normal to C over base a. Then C is an instance of a one-parameter optical medium with distance along the direction -k(x) as the parameter of the space (cf. Example 2, Sec. 3.9 and part tea of Fig. 3.25).

Consider C -isolated. Since C is convex we shall not need to black-convexify it. Let N_(a) be an incident radiance distribution over a circular conical set D of directions with axis ~ and on base a. Let N_(a) be the only source of radiant flux in C. Since C is isolated, radiant flux leaving C is not considered to enter it again nor is the radiant flux ,in x considered to enter it. In short "isolated" means, as usual, that a subset is conceptually excised from its master set and placed in a dark vacuum for a controlled radiometric study.

We now direct attention to the radiance distribution

N_(a) X(a,y,b)

where y is an intermediate plane cross section of C, as in Fig. 3.29, and O"(a,y,b) is the complete transmittance operator for C (cf. (11) of Sec. 3.9). This' radiance distribution is thought of as that part of N_(a) transmitted from a to y. Hence:

(N w (a) %"(a fy j b)) T,..(y $_i$b,b)

is that part of N_(a) transmitted from a to b and emerging over base b. By (12) of Sec. 3.9 (with z = b) we have:

N$_-$ (a) J"(a.y0b) $_v$`r(y,b,b) = N_ (a) 0-(a, b, b) (1)

or, alternatively:

N_ (a) Jr(a,yab)T(y,b) ! N_ (a)T(a,b)

The crucial step in the definition of the beam transmittance can now be taken. We select the value of N_(a),'(a,b,b) at z for the direction & and let a-+4x}, which is tantamount to letting C} P_r(x,E). We then go on to write

"N^0 (z,)" for lim [N_ (a)T(a,b)] (z,) (2) C-* $4F_r$(x!)

where the notation indicates that N_(a)T(a,b) is evaluated at z in the direction E during the limit process. Let us also write

"N_0 (x, ~) " for

i.e., N_o(x,~) is the value of N_(a) at x and . Then we shall write evaluated for a unit radiance function N_(a) and at z in the direction ~. 1y (2) the definitional identity

N0(z~) = No(x,E)Tr(xf0

readily follows, where z = x + rE. For every x,E,r, the quantity T_r(x,0 is a dimensionless, non negative real number associated with the path 0r(x,&), and is called the beam transmittance of 4P (x,~). The radiance N

reet2 transmitted for residual or reduced or unattenuated) radiance.

The compatibility of the limit in (3) with classical theory can only be fully made after a sufficient amount of the theory has been developed. Those readers wishing an im mediate indication of the compatibility may consult statements Ia and Ib of Sec. 23 in Ref. [251]. Statement Ia shows at any rate that the limit of T(a,b) need not go to zero as a+{x} and D-*{~} for short paths. Furthermore statement Ib states that R(a,b) does go to zero under these conditions. The rigorous proof of the existence of the limit in (3) requires the specific postulation of very mild regularity properties of the underlying radiative process. We shall not digress§ here to establish such fine points. Interested readers are referred to Chapter III of Ref. [251] for a complete discussion of what regularity properties are needed in this matter,

Next, by means of (42) of Sec, 3.7 we see that the statement:.

follows from:

lim ~T(a,y,b) = T(a,,Y) (5a)

C+ P_r(x)

1im R(y,b) = 0 (5b)

C+ 6P_r(x,0

for every y, a s ys b. Statement (5b), in turn, follows from the S-continuity of the function S(x,-•

, , ,). The S-continu

ity of S(x;•,•,•,•) follows from statement Ib of Sec. 23 of Ref. [251]. From (1), (3) and the preceding observation (5a) we have for every x,~,s and t such that s + t = r:

T_r(x,E) = T_s(x,&)T_t(x + sEtO

or

$^T s+t \; ^T s ^T t$

for short when x and are understood. Furthermore, from the fact that

J"(a,a,a)

we have for every x and E

or simply
T
0

for short. In other words the beam transmittance of .a path of length zero, is unity. Finally, from (15),(17) of Sec. 3.8 it follows that for every x,~,r:

or
T 1 r
for short. Property (6) is called the multiplicative (or semi-group) property; property (7) is the identity property, and (8) is the contraction property of beam transmittance. These properties can also be based on properly presented physical arguments (see, e.g., Sec. 16 of Ref. [Z51]). However, it is of some interest to see that they can be made to follow formally from the interaction principle and (in the case of (8)) the law of conservation of radiant energy.

The intuitive meaning of T_r (x, ~) for a path R_r (x x in an optical medium can now be discerned. In everyday words, the radiance N_ (a) 0"(a,b,b) (= N_ (a) T (a,b)) transmitted through a finite cylinder C arrives . at base b after having possibly been scattered several times. However, as C ap
P - roaches~ (x,&)$_f$ so that base a approaches the set {x} consisting of [he single point x, there is progressively less "elbow" room for the flux in transmission to scatter about within C on its way to b. In the limit as C goes to 6 (x,C), any radiant flux that travels from x to z along er(x,J has the "straight and narrow path" to follow. If it happens that a photon in transit from point x to point z is scattered exactly forward or backward a certain number of times, this photon is considered to constitute part of the residual radiance at z, for it is impossible at present to operationally distinguish, in the time averaged residual radiance, between a photon which has travelled along epr (x,0 without scattering and
one which has travelled along P_r(x , E) and which has been scattered. Hence it will not be inconsistent to speak of N^O(z,E) as radiance consisting of those photons originally

constituting ^lN_o(x,Q which have not been absorbed or scattered from ^{6}'r(x,Q as they travelled from x to o+r~ = z. This is the phenomenological interpretation of N_r (x $_P$&), and we shall adopt it in the present work.

The generalization of the foregoing results to the case of non-constant index of refraction is effected by repeating all steps with (N_(a)/n z) instead of N_(a), and building a tube of natural paths around 60_r(x,0 using cross section a as a base from which the paths begin with normal incidence. the motivation for using the quotient N-(a)/n rests in the n -law for radiance in (4) of Sec. 2.6.

This discussion is concluded with the observation that the beam transmittance $T_r(x,\sim)$ associated with the path &[1]'r(x,0 in X is independent of the radiance distribution in
X. This may be seen by returning to (3) and recalling that the standard transmittance operator R(a,h) is an integral operator whose kernel function Sb is derived from an interaction operator obtained via the interaction principle. A re-examination of the conclusion of the interaction principle will show that an interaction operator is independent of the members of its domain sets Ai and range sets Bj, in other words interaction operators do not depend on the radiance distributions (i.e., the light fields) in X.

Inherent and Apparent Optical Properties

The foregoing observation is of considerable importance in establishing the basic optical properties of a natural optical medium X. An optical property P of an optical medium X R i n the form of a number, function, or operator) which is independent o f the light fields (in the form o f radiance distributions) in X will be cal led an inherent optical _ property~ o f X; otherwise, P is an apparent opticalro ert . [r] Hence, the beam transmittance function $_r$ which assigns to the path P_r(x,Q in X the beam transmittance T_r(x,Q, being independent of the radiance distribution in X, is an inherent optical property of X. We shall return to the systematic study of inherent and apparent optical properties in Chapter 9.