3,11 Derivation of the Volume Attenuation Function

The volume attenuation function is a measure of how much radiance a light beam loses per unit length of" travel under the joint action of scattering and absorption processes. In this section we shall develop the concept of the volume attenuation function with the beam transmittance function as a starting point.

Let Fr(x,~) be a natural path in an optical medium x with associated beam transmittance T_r(x,~). If a parcel of radiant flux of unit initial radiance traverses er(x,E), then on the one hand Tr(x,O is the amount of radiance transmitted over (D_r (x , O , and on the other hand

1 - Tr(xp&)

is the amount of radiance lost over ~Pr(x,o . Hence if we write:

for

r

then we can say that ar(x,0 is the average amount of radiance lost per unit length for a beam of initial unit radiance traversing Rr (x , &) .

We are almost at our goal. It remains to write:

The function a which assigns to each x in X and E in the non negative value a(x,Q given in (1) is called the volume attenuation function. The dimensions of a(x,Q are Lr" (inverse radial length--see note (h) to Table III of Sec. 2.12). That a(x,&) is a non negative number follows from the contraction property of Tr (x, E) , ((8) of Sec. 3.10).

Once again we have assumed the existence of a limit without appropriate mathematical preamble. As in the case of (2), (3) and (5) of Sec, 3.10, we are concerned here only with the formal conceptual content of the interaction principle. There should not be any concern at present about-the existence of the limit (1) above and the limits in (2), (3), and.(S) of Sec. 3.10. These limits can always be made to exist in an acceptable and workable setting by postulating physically reasonable regularity properties of the underlying radiative process. However, what is of greatest importance here is the fact that there now exists a formal deductive chain of arguments connecting the volume attenuation function with the interaction principle. In this way we have shown that the volume attenuation function is an inherent optical property of an optical medium and a property whose conceptual roots are logically linked to the same principle which yields the reflectance and transmittance operators for surfaces and general subsets in the medium.

A useful connection between a and T_r is the exponential representation of T_r using a. This connection is derived by using the multiplicative property of T_r ((6) of Sec. 3.10) to write:

s s

$T_r =$ $(^{-a}s)^{T}r$

The definition of a_s was used to obtain the second equality. Letting s-o-0, we obtain;

 For given x and ~ this is an elementary differential equation for T_r, with known function a, whose solution is:
r
Tr = exp. - a dr' , n
or in more explicit notation: r
T_r (x , ~) = exp - a (x (r') , C (r')) dr' (3) d
We have used the identity property for T_r ((7) of Sec. 3.10) to find the integration constant for the particular solution (3) of equation (2). Here x (r') and E (r') are the location and direction of a variable point within e r (x , 0 , a t distance r' from x along the path. I f the index of refraction were constant, then x(r') = x + r'C; C(r') = E for every r',
U ~sr' _-r;r ; and (3) would become r
T_r (x , ~) = exp - a (x + r' E, E) dr' , (4) and (2) would take the form:
dT_r (x , 0
= - a (x + rg,&)T_r(xf0 (5)
dr
For a discussion of (5) in the case of variable index of refraction, see Sec. 17 of Ref. [251]. In that section there is also an alternative derivation of the function a using empirical radiances and empirical attenuating volumes. An experimental procedure for determining a is given in Sec. 13.4 of this work.