3,17 Further_ Examples of the Interaction Method

We conclude the illustrations of the interaction method in this chapter with a brief listing of some further important radiative transfer phenomena which can be methodically subsumed under the interaction principle. We begin with two concepts which we have already studied:, the path function and the path radiance (cf.. Sec. 3,12), Now we approach these familiar concepts in perhaps the most interesting way of all,

Example 1: The Path Function Operator

The equation connecting a radiance distribution N(x,•) at a point in an optical medium X and the associated path function distribution N*(x,•) at the same point in X was finally attained in Sec, 3.14 after a relatively laborious struggle which first had to bring into the light of day the concept of volume scattering function. We now connect N*(x,•) and N(x,•) in an alternate and less arduous way, However, what we gain in elegance and mathematical insight by taking the present approach, we lose in physical meaning. The earlier route taken, however long and detailed, has the virtue that it suggests operational means of measuring a in situ, i.e., within an optical medium. The present approach has the virtue of showing the logical structure of the relation between N*(X,'), N(x,•), and a(x;.;•), and does so with unprecedented clarity.

Let X be an optical medium and let the present subset S of X be a singleton {x), i.e., a one-point subset of X, Hence we will be using the special point-level interpretation of the interaction principle (re: Sec. 3,2), Let the set A1 of incident radiometric functions on {x} be radiance distributions like N(x,•), Let the set BI of response functions be the path functions like N_* (x, •) and defined using (3) of Sec. 3,12, Then m = n = I in the interaction principle of Sec. 3.2., and there exists an interaction operator R such that

N_*(x, e) = N(x, •)R 10

In the terminology developed in the closing paragraph of Sec, 3.16, in particular with reference to (15) of Sec. 3.16, b is now N_*(x,•), a .is now N(x,•), and s is now R. The sets C and D are each now the unit sphere '=, and -v is solid angle measure P on ~. R gives rise for each fixed 9 in to a positive linear functional ,so that for a particular fixed in D (_ ~) we obtain by Theorem A of Sec, 3,16, an interaction measure u(x; •;~) such that

N*(x,~) = f N(x,E') dv(x;~' ;~) 49 (2)

w

u (X; • ; &):clearly has the AC property with respect to 0, Hence by Theorem B of Sec. 3.16 there is an interaction kernel a(x; •;~) such that for every subset D' of

D'

Z(x;E' ;E) dtl(E') (3)

Equation (2) corresponds to (17) of Sec. 3.16; Equation (3) corresponds to (18) of Sec. 3.16, in which y is now-E: In the present instance the interaction kernel K for {x} is the volume scattering function a. The present specific instance of (2v)-of Sec. 3.16 is obtained by means of Theorem C of Sec. 3.16:

N*(x,~) = N(x,~)a(x;9' ;9) dil(V) f
V

and which is to be compared with (8) of Sec. 3.14. Thus we have

We call R the path function operator.

Example 2: The Path Radiance Operator

* The equation which represents the path radiance $N_r(z,\&)$ over a path $R_r(x,\&)$ in an optical medium in terms of the path function N* (• , &) def fined over'r (x, E) was obtained in (15) of Sec. 3.12 after some rather delicate analysis but in .which each step was completely meaningful physically. We now establish (15) of Sec. 3.12 using the interaction principle in a radically different way; one that exhibits the logical interrelation of these concepts with a minimum of direct appeal to physical meaning.

We begin by choosing a path ^{62}r(x,0 in an optical medium K (see Fig. 3.33). This path is a one-dimensional subset of K, and so we will be using the line-level 'interpretation of the interaction principle (Sec. 3.2). We let Al be the set of all incident radiometric functions on Rr (x, ~) , in this case all path functions like N*(•,~). We let B, be the set of all path radiances like Nr(z,g), where z = x + &r (see Fig. 3.33). Then the interaction principle yields an interaction operator T such that:

N^* (z f) = r N* (' , E) T (6)

In the terminology of Sec. 3.16, in particular (15) of Sec. 3.16, b is now Nr(z,~) , a is now N*(•,~), and s is now T. The set C is Pr(x,0 , and the set B(D) is the set of all path radiance values (non negative real numbers) N*(z,~) on the set D = {(z,E)). The measure v is now the length measure 1 along epr (x , E) .

T is a positive linear functional, so that we obtain directly from Theorem A of Sec. 3.16 an interaction measure u 0 pr (x , 0 ,P ` , z) such that:

N* r Cz ~ = N* (x' * 9) du (R_r(x q 4) ,x' .q z) • (7)

(Pr (x, ~) , • , z) clearly (i.e., on physical grounds) has the AC property with respect to l(•,z) the measure which assigns to - x' in $6P_r$ (x, E) the distance 1 (x' , z) = r' between x and x' . Hence by Theorem B of Sec. 3.16 there is an interaction kernel KC&"r(x,0,x,z) such that

u &52$_r$ (x,) E, z) = f x ~$_{r\ (x,)}$!x' , z) dl (x' , z) (8)
E

for every subset E of R_r(x,g). In the present instance, the interaction kernel is none other than the beam transmittance function such that:

K(Pr(x,Q~x',z)

so that in particular:

Being able to write down (9) at this time is of course the result of hindsight. Were we. Martians developing radiative transfer theory for the-first time, having been given only the interaction principle and none of the developments of Sec. 3.10, we would retain and work only with "K (6~'r (x , 0 , x' , z) " and perhaps eventually deduce in our own way the Multiplicative, identity and contraction properties of the beam transmittance function-Sec. 3.10), the differential governing it, and finally the volume attenuation function.

Finally, by means of Theorem C of Sec. 3.16 we come to:

. $_L9_r$ (xV.P

N* (x' ,) K (,p_r (x, ~) x' , z) dl (x' , z)

r

N* (x' ,) TL-Y. (x' , E) dr' , C 11)

by virtue of (9). Hence

r

r

f [ ]T r-r, (x', t) dr' (12)

We call T the path radiance operator associated with the path &Or (X14) .

R

R Radiometrically relates two directions in a given point

T Radiometrically relates two points in a given direction

FIG. 3.34

Before closing this example we wish to point out an interesting geometrical duality between the path radiance operator T and the path function operator R. This duality is best described in ideographic form in Fig. 3.34. In other words, if we interchange the words "direction" and "point" in the description of R, we obtain that of T, and conversely:

Example 3: The Volume Transpectral Scattering Operator

We now formulate the definition of an important extension of the volume scattering function--the volume transpectral scattering function. As its name implies this new scattering function relates incident radiance of frequency vt at a point x to resultant scattered radiance at x of frequency v. In short we shall now consider scattering of flux not only from one direction to another, but also from one frequency to another.

The use of the interaction method has been illustrated often enough by now so that it will suffice in this and the remaining examples to be somewhat less detailed in the explanations.

Stage one: Construct a function Ns(x,•,v) (the transpectral path function) at point x in medium X such that its value at E is the radiance of frequency v generated by inelastic (transpectral) scattering at x, of an incident radiance distribution N(x,•,v') of frequency v'. This stage corresponds to the definition of the path function

using (3) of Sec. 3,12.
Stage two. Proceeds exactly as in Example 1 above. Instead of (1), we now have from the interaction principle:
N_s (x, • ,v) = N (x, • , V') R
The resultant integral representation is:
Ia(x;E' fE;v' ,v) dQ(E') (13)
The interaction kernel in (13) is called the volume transpectral scattering function: This function is a proper generalization of the monochromatic volume scattering function as can be seen by setting v' = v.
Alternate Stage two: Proceeds analogously to Example 1 but now the incident radiance distributions have two free variables C' and v', so that the principle yields the operator equation:
A
Ns(x,•,V) = N(x,•,•)R

The resultant integral representation is:
R
f .. 4.0
a(x~ ' s ;vt ~v) dl(v')dS~['~ (14)
A
where A is the spectrum. The operator k in (14) is called the standard transpectral scattering operator.
„The undecomposed transpectral scattering operator combines R of (14) and R of (5)
f-f E (Q (x W 's ;v)6(- V) + Q(xi ! ~~IV^1,v)] d1(v^1)d11(E^1) (15) A
where d is the Dirac delta function. The dimensional distinction between a and the two a's should be noted. We shall also call Q' the volume transpectral scattering function. Operator (13) is useful when only a finite number of discrete frequency transitions are considered, Operator (14) is a natural choice when continuous frequency transitions are considered.
Miscellaneous Examples
We leave the applications of the interaction principle open-ended at this stage and merely list some further possibilities for consideration by interested students of the subject
(i) Interaction Operators for Internal Sources (cf. (37) of Sec. 3.9).

iii) Path Function Operator for Polarized Radiance (and hence the genesis of the volume scattering matrix --see Sec. 112 of Ref. [251])-.
(iii) The Path Radiance Operator for Polarized Radiance (and hence the genesis of the beam transmittance matrix--see Sec. 112 of Ref. [251]),,
(iv) Time Dependent Operators--the time dependent versions of all the kinds of operators considered so far. (See (4) of Sec.,3.15 and Sec. 127 of Ref. [251]
(v) The Photometric Operators Y [j9A , 2 (a,M). (See (13) of Sec. 2.12 and (1) of Sec. 2.13.)
(vi) The Operator C (x) .(See Sec. 2.1.1.)

(vii) The Operators of the Mueller Phenomenological Algebra (Refs. [192], [193] , [194]$_0$ and Sec: 137 of Ref. [251]),.