CHAPTER 4

CANONICAL FORMS OF THE EQUATION OF TRANSFER

4.0 Introduction

In this chapter we begin a systematic construction of the main laws of radiative transfer theory 'by means of the principles of Chapter 3, with the particular goal in mind of deriving certain special types of transfer equations for the main radiometric concepts. These equations have been found most useful in the applications of the theory to the study of light in both the sea and the atmosphere. This task will occupy our attention during this and the following four chapters. In the present chapter our purpose is to obtain the canonical equations of transfer for radiance,

The sense in which we use the word "canonical" is twofold, First of all,, "canonical" is to denote a fundamental well-established form of .the equation of transfer--a form which has evolved and eventually gained universal acceptance over a two hundred year period of development. This is not to say that the canonical form of the equation of transfer is given first priority in every mathematical investigation of the transfer of radiant energy in optical media; rather, it is simply intended to signify the fact that the canonical form of the equation of transfer has been applied and independently rediscovered with sufficient frequency in various fundamental investigations in different sub fie lds o f radiative transfer over the years, that it has eventually taken on the role of an enduring useful landmark in the general theory. The second sense of the word "canonical" as used here is of a more technical nature; it is to denote the fact that the equations are written in a form of great simplicity without decreasing generality, and in a way that is independent of any particular coordinate system. Of the two senses, the first by far is to be considered the dominant sense in what follows.

The earliest recorded appearance of the canonical form of the equation of transfer was in the work o f Bouguer, in whose classical treatise [28] appears a special but unmistakable form of the equation. This equation was unearthed and dusted off by Middleton in his studies of Bouguer's work. Specifically, Middleton observes [281 that: "Bouguer integrated the contributions of many elementary layers (dx) by a geometrical construction t and showed that [in modern notation] the apparent brightness of an object at distance x is

BW $_{ae}$ -ax + b(I-e-axe r„ ~I] The salient features of this equation, those that make it "canonical" in the technical sense, can be described in terms of the concepts developed in Chapter 3. First of all we observe that (1) has the Gestalt of (5) of Sec. 3.13, where the term ae-ax corresponds to NYO in equation (5) of ec.3.13, the term b 1- e' axe, corresponds to N_r , and the term B (x) to N_r. Thus B (x) is interpretable as the apparent radiance of an object (Sec. 3.13) as seen over a path of length x, where the path radiance of the path is btI-e¯ax) and the inherent radiance of O e object is a. The particular manner in which a, b, and e occur in the algebraic form of (1) characterize (1) as canonical. Equation (1) is substantially the algebraic form of B(x) deduced by Bouguer from empirical observations. According to Middleton, however, Bouguer ostensibly missed the full physical significance of the terms a and b. Hindsight and a fully developed theory now let us view a and b in quite simple terms. Thus a in (1) is the inherent radiance of the object which is transmitted over the path with beam transmittance e-ax. Hence a must be the attenuation coefficient of the path (our a of Sec. 3.11). The term b is a simple instance of the general concept of equilibrium radiance which will be introduced and studied in detail in this chapter. Physically, b is the radiance of a very long uniformly lighted homogeneous path. Mathematically, b i s simply the limit o f B (x) as x-low. The radiance b is independent of location along the uniformly lighted homogeneous path, and in real life is closely approximated by the horizon radiance under suitable atmospheric conditions. The horizon radiance remains ostensibly constant,-for example, on a transcontinental jet flight at 10,000 m altitude over large segments of the flight path. The observed horizon radiance seen by the jet pilot is the real counterpart to the equilibrium radiance b in .(I),. Of course similar interpretations of a,b and corresponding interpretations of (1) apply to horizontal lines of light in the sea, under suitable conditions.

In the present chapter we shall develop a hierarchy of canonical equations of transfer for radiance starting with the simplest of applied situations and concluding with what appears to be the most comprehensive canonical equation of transfer for physically meaningful contexts, Equation (1) will fall somewhere in the lower middle of this hierarchy, that is, somewhere in the neighborhood of the Koschmieder equation of Sec. 4.3. Throughout this chapter, unless specifically noted otherwise, all optical media will be considered emission-free, in the steady state, and of constant index of refraction. This condition does not constitute any significant loss of generality in terrestrial settings while permitting a simple exposition of the main idea of the canonical equation.