4,3 Koschmieder's Equation for Radiance

A classical problem of radiative transfer theory in either the atmosphere or in the sea i s to determine the a parent radiance o f an object as seen along a path o f sight ,(x,0 which lies in a homogeneous and uniformly lighted region of an optical medium. Specifically, the problem is to determine the apparent radiance N_r (z , ~) given a and a along fix,] , and N_o (x,) at the initial end point x of the path ~ along with the fact that each point of x &) is irradiated by the same radiance distribution (which may, however, depend arbitrarily on '~. This situation (or some reasonable approximation of it) arises often in the atmosphere and the sea, notably along horizontal paths of sight,

and the reader should be able to cite many personally observed instances of it. Koschmieder studied this classical setting in detail, and in 1924 published in (141] his analytic expression for N_r(z, 0 which was derived after lengthy preliminaries and under the radiometric conditions stipulated above. We turn now to a modern derivation of the expression for N_r(z,)•

Returning to (1) of Sec. 3.15 we assume a and a are independent of z along Or (x, O . Then

Tr(x,~) e-ar

where "a" denotes the assumed fixed value of the volume attenuation function along Ga_r (x,) , Furthermore, since the radiance distribution N(z,.) is independent of z along the path then N* (z,E) is also independent of z along the path and we shall abbreviate this fixed value by "N*". Equation (1) of Sec. 3.15 then reduces to:

N_r(zgE) a N_0 (x, E)e-ar + N*

$_e$_a(r_r^f)d_rI

and with the abbreviations for N_r (z ,) and "N_o" for N_o (x,&) , this simplifies immediately to

where we have written:

f1

for N*/a

Equation (1) is Koschmieder's equation which relates apparent radiance N_r to N_o on a path 0 in an optical medium along which a and a are constant value and along which the value N* of the path function is constant. The radiance N is called the equilibrium radiance for O_r. The significance & of N is seen by letting r}w in (1) , or alternately by contemplating the integrodifferential equation for N_r associated with or as given in (3) of Sec. 3.15

dN

Under our present assumptions, (3) is a relatively innocuous first order differential equation in which a and N_* are constants and N_r is the unknown function. Using (2) we can rewrite (3) as

 from which we can immediately read the physical significance of N_q: If $N_r < N_q$ at a point on the path, then $dN_r/dr > o$, i.e., N_r is increasing at that point. In general, N always tends toward the fixed radiance N , and $^4dN_r/ dr = o$ if and only if $N_r = N$. Therefore N tales on the aspect of an equilibrium value (in an ever' day sense) toward which the values N_r unceasingly tend. The equilibrium radiance N_q is often observable over long horizontal uniformly lighted paths through a homogeneous natural aerosol or hydrosol.

It should be observed that the derivation of (1) places no conditions on the orientation or the location of the path (?r in an optical, medium. The essential point to observe in the derivation is that (1) follows from (1) of Sec. 3.15 upon assuming only that a, a and N_* are constant long fi_r. This leaves hr free to be vertical, inclined, or horizontal, as the case may be. An interesting example of (1) for inclined paths of sight in the atmosphere may be obtained from the results in [71].