4.6 Canonical Representation of Polarized Radiance

In this section we shall extend the notion of the canonical representation of apparent radiance to the polarized context. One consequence will be a representation of polarized radiance distributions in stratified natural hydrosols comparable in simplicity and utility to the scalar equation (2) of Sec. 4.4. The resultant polarized canonical form also suggests some interesting experimental programs that may be performed for polarized light fields in natural hydrosols. These will be briefly outlined at the conclusion of the section.

In order to establish the polarized version of (15) of Sec. 4.5, it seems natural to try to repeat the constructions between (1) and (15) of Sec. 4.5, now for each of the four components iN of the polarized observable radiance vector N (Sec. 2.10). Thus let us write

"Xi" for ¯DiN/iN (1)

for each component iN of N, i = 1, 2 , 3 ,4 , and let us write (7) of Sec. 3.15 as

E•PN- -aN+N* where we have written:

"N*" for N p dSl V

where p is the standard observable volume scattering matrix. All that we need know about the standard observable volume scattering matrix p in the present derivation is that it is a 4 by 4 matrix with entry pij in the ith row and j th column.

In particular the problem of how the pij are obtained in principle or in practice is immaterial for the present derivation, since we are concerned only with the mathematical process of constructing the vector counterpart to (15) of Sec. 4.5. The matrix p is defined and discussed in detail in Sec. 112 of Ref. [251].

The canonical equation of transfer in the scalar context now becomes four coupled scalar equations in the polarized context as follows. We first write:

11_P i

for (pli' P2i' P3i' poi)

Next we

read off the ith component of (2), i = o_iN Z - a iN + iN*

1,2¯,3,4:

where we have written

1f N t1 Z

for

V

It follows from (3) and (6) that:

($_1$N*, $_2$N*, $_3$N*, $_4$N*) Using (1) in (5) and solving the result for iN:

This is the canonical equation of transfer for polarized radiance, which holds for each i = 1,2,3,4. Continuing as in Sec. 4.5, we deduce for i = 1,2,3,4:

r

$_iN_T/_iN_0$ = exp {

- J E ·iC_i are } 0
which is the vector component Applying the notation $^{11}T_r$ [f I 11 text, (9) may be written:
counterpart to (2) of Sec. 4.5. of Sec. 4.5 to the present con
$_iN_r$ = iNoTr[_E.xi
and we observe that
o _
N o T $_r$1¯A1
i^N r i
It now follows readily that for every i
10
(10)

(12)
which is the desired canonical representation of polarized radiance,
The set of four equations (12) are coupled by means of the terms
iN_*. For example, the representation for 1Nruses
1N * where
1^{N*}
f *.W
1Np 11 + $_2$Np $_{Z\,1}$ + 3Np 31 + 4NP $_{41}$ dQ
A Simple Model for Polarized Light Fields
We now give some attention to the construction of a simple model for polarized light fields in stratified natural hydrosols, the constructions being guided by the successful scalar prototype in Sec. 4.4. In the scalar case, the effective step was to assume that there was a non-negative number K, less than a, such that:
-K (z-z) 0 (13)
This suggests that we take each iN_*, i = 1,2,3,4, which by (6) has the form:
iN* ` 1^Npli + 2^Np2i 3^Np3i + 4Np4i dO (14) [(j
and agree to write:
" . . N_*I• for .Np_J . . 1 dO , {15~
M w
so that jN_* will have the representation
i^{N*} = ii ^{N*} + 2i ^{N*} + 31 ^{N*} + 4i N* (16)
Then, still being guided by the prototype (13) we agree to make the following assumption: the four non-negative real numbers Ki,- as defined in (1) , are each less than a, and such that

w
(17)

for every i, j = 1,2,3,4, where K• now is the z-component of X---the only nonzero component of K• by virtue of our current assumption about the stratification of the light field in natural waters, Under these assumptions, (12) reduces to

N$_r$(z f~) = $_i$N (Z E)e-ar + + i $_{1-e}$- (a + K$_i$ cos e) r
0 0 a cos e
(18)

for i = 1,2,3,4, and where:
i N$_*$(z,E) _ i•N*(zo,E)e-KI(z-zo) I
2i
•N$_*$ (z$_o$.* ~) e$^-$K2 (z$^-$zo)
3i N*(zo.%E)e-K3(z-ZO)
+ 4i N* (zot O $_e$ $^-$K4 (z-z$_o$)
or more compactly:
_ 4 -K'(z-Zo)
$_i$N$_*$ (z .4) - j j iN* (zo 0 ~) e j=1
(19)

Experimental Questions

The derivation o f the canonical representation (18) for polarized radiance incorporated several assumptions which, even though suggested by the well-established scalar case of Sec. 4.4, require some critical examination before they are fully accepted. These assumptions in turn raise certain specific questions concerning the nature of polarized light fields in natural hydrosols in general and the nature of the K-functions in particular, We shall conclude the present section with a brief statement and discussion of these questions.
•N$_*$ (z I
First of all, the definition of each Ki as given in (1), is a constructive definition and hence presents no difficulty in being translated into operational terms, so that actual experimental determinations of the Ki are possible in principle. These determinations should parallel very closely those already developed for the function K in (20) of Sec. 4.5, because Ki, as K, is a logarithmic derivative of a radiance function. The main difference between Ki and K is simply that each Ki is associated with the component of a vector valued function while K is associated with a scalar valued function. Thus with the extra attachments of wave plates and polarizers on the radiance meter required to measure the polarized radiance, one performs essentially those

operations with the radiance meter that one performs to find K, but now four times over for each Ki, i = 1,2,3,4.
With the matter of the measurability of the Ki settled, at least in principle, we now ask the first question that comes to mind concerning the Ki : I s there some observable

regular pattern in the individual depth-behavior and in the relative magnitudes of _the four , functions K_1, K2, K3, K4? This is actually two questions in one, and it may be simpler to phrase them separately. The first question may be phrased: Is there some observable regularity in the depth behavior o f each Ki ? The second question may then be rendered as: Is there some observable regularity in the relative magnitudes of the Ki? As far as the first question is concerned, it is expected on simple physical grounds that the individual depth behavior of each Ki should follow very closely that of the scalar K defined in [20] of Sec. 4.5. In particular the depth behavior of the Ki at relatively great depths in homogeneous media should be quite regular and should follow the patterns discussed in Sec, 7.10 and Sec. 10.6 dealing with the asymptotic radiance theorem. Some attention to this question has been given by Lenoble [157]. The second question is more difficult to answer and, in view o f the present state of development of the theory of polarized light fields in natural optical media, it appears likely that a definitive answer will be forthcoming first from experimental investigations. Nevertheless, it is interesting to speculate on the possible interrelations among the Ki . Thus, suppose that the Ki are all equal to a common value, then the set of equations in (18) assumes a particularly simple form. It follows that any differences between" iN_r and j N_r will depend solely on the state of affairs between them at the surface of the medium. On the other hand, i f there are two Ki f s which differ at all depths then the radiance component associated with the larger Ki will decay with depth more quickly than the other. As a result, those components of N with the smallest Ki's will persist down to greater depths than the others with larger K-'s. By contemplating these possibilities and by taking into account the known properties of the unpolarized light field, the general state of affairs for the functions Ki will most likely turn out as follows: Near the surface the K-'s will differ, and there will be some permanent characteristic pattern of relative sizes discernible among them which is related to the state of the sea surface, and to the polarized state of the sky; however, the transmitted sky-polarization and under-surface reflection-induced pattern will eventually disappear with increasing depth in such a manner that in the limit, all the values K} tend to a common value independent of the state of the sky s polarization, with an attendant asymptotic value of the polarization of the light field. This common value of the K~'s will be that of the depth decay rate of scalar irradiance (z), which should be determined only by the inherent optical properties o f the medium ,just as in the scalar case. It remains to be seen how this conjecture is borne out by experimental studies. Our review of the experimental work of Ivanoff and Waterman in 1.2 shows some encouraging agreement in this direction.

While attention is directed toward the possible structure of the functions Ki at great depths and while conjectures

about the Ki are being made, it might be in place to add some further conjectures about the light field itself in addition to its depth-rates of decay Ki. When one imagines the natural light field at great depths one is led to picture a predominantly downward feeble flow of light, the radiance pattern being graphically depicted by an ellipsoid-like surface with vertical axis. If this light field is conceptually analyzed for polarization features, it seems--on intuitive grounds--that the radiance vector for vertical downward or upward flux should have the form (1/2) (N,N,N,N) , i.e., vertical downward or upward-radiance should be unpolarized. Furthermore, it seems that the horizontal radiance should be horizontally linearly polarized, i.e., have the form (1/2) x (0,2N,N,N). This follows from the fact that the flow is predominantly vertical and beamlike (and of course very feeble) at great depths. Since natural light fields change continuously - rather than abruptly in most macroscopic settings, we would expect the radiance vector components to vary continuously between these two extremes as the angle of the radiance direction varies from e = 0 (vertical upward), or n (vertical downward) to 6 = w/2 (horizontal). A simple model for this radiance N(e) which comes readily to mind and which satisfies these conditions is:

$N(e) = {}^{1} (N \cos^{t} 9, N(1 + \sin^{g} e), N, N)$

where e is measured from the zenith-and N is the fixed references radiance for a = o at each depth. All these assertions are at this stage of our knowledge of course conjectural, being based on a modicum of physical experience with polarized radiance fields in natural waters, and are intended primarily to perform a heuristic service. It will be left to interested researchers to carry this matter to amore satisfactory state of affairs, both theoretically and experimentally. A possible theoretical approach can be based on the polarized version of (21) of Sec. 10.7, or on (29), (31) of Sec. 7.10, These approaches may show that the preceding conjecture must be modified to take into account the structure of the volume scattering matrix (cf. (24) of Sec. 13.6) of the medium.