5.2 Equation of Transfer for n-ary Radiance Diffuse Radiance _, and path function

The equation of transfer for n-ary radiance will now be derived. The equation is an elementary consequence of relation (11) of Sec. 5.1. To see this, suppose we fix attention on an arbitrary path (?r(x,0. Then holding the initial point x and the direction 4 of the path fixed, and differentiating Nn along the path with respect to path length r, we have

dNn $_d$ - ( )
r
= ^{d} N^{n-1}(x\` ,4')a(x' ;4' ;~)dII(C') T _ ~ (x' ,4)dr'
car r r fo M
r
r N^{n-1}(x' t^f)o(x' fE)do(&I) $dT_{r-r,}$(x' ,&) dr' r

At this point we observe that, by (3) of Sec. 3.11: dTr_ rI(x' 90 }
Then using (6) and (11) o f Sec, 5,1 we arrive at n~dN^n= n+ n
DN $_{ar}$ _aN N_* (1) which is the requisite equation of transfer for n-ary radiance with n y 1. Observe that the equation of transfer for Nn is not an integrodifferential equation for Nn; rather it

is a first order linear differential equation for Nn with known n -ary path function NV, once Nn - i s known. This suggests a conceptually powerful natural mode of solution of the general equation of transfer for N, which we shall study throughout this chapter. In the following section we shall place (1) into its canonical form, thus rounding out the studies of the canonical equation given in Chapter 4. In Sec. 5.4, the complete natural solution will be obtained. Before concluding this discussion on n-ary radiance equations, we mention two more transfer equations for radiometric concepts which are closely related to the family of equations in (1). Note that (1) holds only for n > 1, the case n = 0 being excluded. This singular case n =~0 is readily stated using (4) of Sec. 3.10 and (2) of Sec. 3.11. The result is

C•QN^0 = ^{dN0} = -aN0

for source-free media. A generalization of (2) for media with sources is given in (2) of Sec: 5.8. The remaining transfer equation to be noted here is that for the diffuse radiance N* (or path radiance when a specific path of length r is given somewhere in the medium). Thus, using the concept of n-ary radiance, let us write:

Go 0
for E NJ (3) j=0
"$_N$"
for .1_1 W (4) 7
and

j=2

Then summing each side of (1) over all n from 1 to a*, we have

• QNj = E dN - - E aN' + E N* (6) j=1 j=1 j=1

which, on applying (4) and (5) becomes

• oN* = dN = -aN* + N* + N*

This is the equation of transfer for diffuse radiance N*. By assuming that N* obeys(1) of Sec. 4. 4, i. e. , N* decays exponentially with depth at the rate K, then (7) supplies a somewhat more powerful description of the light field than

that given by (2) of Sec. 4.4. It is clear from the discussions of Sec. 5.1 and (5) that:

N* = N*It (8) We shall return to these ideas in Sec. 5.4, especially to that o f N., as defined in (3), wherein we will show that N so defined is a solution of the equation of transfer, Finally, by applying the operator R to each side of the equation of transfer for radiance, we find:

~ • VN* _ - aN* + N**

which is the equation of transfer for the path function, and where we have written "N**" for N*R (Io)