5.12 Global Approximations of General Radiance Fields

In this and the following section some of the theory of time-dependent n-ary radiant energy fields will be applied to two general problems of radiative transfer theory. In the present section attention will be directed to the problem of. finding relatively simple approximations of. time dependent and steady state radiance fields in optical media. In particular it will be shown how the n-ary radiant energy fields may be used to obtain approximations of the observable radiance field such, that the approximations are exact on a global level over the given medium..

The precise meaning of this phrase will become clear during the course of the constructions of the approximations, to which we now turn. Unless specifically stated otherwise, all constructions will take place on a general optical medium X with arbitrary source conditions,

We begin with' the observation that the operator formula $_N$n 1 n-1

based on the theory of Sec. 5,1, suggests the following simple approximation, where we write:

"nrt U^n 1

N_g for --$_r$ N (1) U

Here Un, N > 1, is the n-ary radiant energy in X, and Nl is the primary radiance function in X. Ng is called the global approximation of Nn for n > 1.

The reason for such a name and structure of Nn lies in the following observations. Note first that Nn has scatter inn order "dimensions" of n-ar K radiance. Nexi, observe that the global approximation for N Yields the estimate

1 Un t Ni (x,, ~ ,¯t) d Q( ) v x U (t)

M

V

for the radiant density function u in X. If we write "unit for this function, then we see that

un _ Un ul 2 g T

U

for n > 1. Finally

u

JX

u^n(x,t)dV(x) U n ^{t} u^l(x,t)dV(x) U (t)

U . Ul fit) U (t)

U^n (t)

This shows that the approximation Nn to Nn has the property:

r

Un (t)

vTXT

N^n(x,&,t) doM

In other words, Nn yields the same radiant energy content of X at each time t as does Nn, the actual n- ary radiance function on X. Thus NTg yields an exact prediction of,

approximation of Nn on an overall (or global) basis. The directional or local structure of Nn is approximated by that of N , a relatively easily computed function.
The global approximation of Nn may be used to obtain a global approximation of the directly observable radiance N by means of .the natural solution representation of N^*_g, where we have written
"$_N$" 9
for I NJ (4) j=1 g
SEC, 5.12 GLOBAL APPROXIMATIONS
For, by the definition of the Ni we have:
N
ca 00 NJ = UJ g ~ ~
j=1 g
j=1 U

The requisite global approximation of N is obtained by writ

g

"N for $N^\circ + N^*$

g g

It follows that:

U (t) 1 v x Ng(x,~,t)dQ(~) dv (x)

X
so that N_g indeed endows X with the same radiant energy content as N, the actual observable radiance function on X. The function N may then be used to assign to each x in X, and in at ti e t the radiance
N (x,,E,t) = N^o(x'&'t) + U*(t) NI(xo~~t) g U1 (t)

where, in case standard growth conditions are in force in X, U* (t) (alias U (t ; s))and UI (t) are given by (14) of Sec. 5.9 and (12) o f Sec. 5.10. In the steady state attained under standard growth conditions, (8) yields

N^o(xto + 1 N^1 (x,)
which is defined for 9 < p < 1.
Global Approximations of Higher Order
The global approximation N
R in (1) above is but the

lowest rung on an infinitely high ladder of global approximations of the radiance function in the medium X. We now formulate the global approximation to N of arbitrarily high order. Thus let us for every n ? 1, write
"N n,, 9
for

 Here we choose to use the same name "Nn,, for the approximating function, and we have now written, ad hoc:
and

"N (k) " for k
I

j=I

jj$_U$ (k) it for k
I

jwI

N)
UJ
N^n is the global approximation of the kth order of N^n. It is e 9sy to verify that N^n again i s globally exact in the general
sense of (3). Define iKg N_g as in (6) and N* as in (4) , now for the kth order context, by stopping th§ sums in (4) and (b) at j = k, it follows that:

o U* t $_N$ (k) (x t) U (t)
(10)
we call Njk)k n (10) the global approximation of the kth order o N. N IM M is exact in the sense of (7), i.e.,
using N k) in (7) globally ill yield U k) (t) . Observe that this approximation also has the virtue of converging to N as k + co. That is
lim N (k) k-,
This follows from (10) and the facts that:
lirnk-1'-W U (k) (t) = U* (t) (12)
and that:

N*
In this way we see that the global approximations to N have one additional property over the truncated solutions of Sec. 5.5, namely the global exactness property. The steady
state limit version of (10) attained under standard growth conditions is:

N (k) (x~) N^0(x,$)   +   [11]~.-.- N (k) (XV&)
g   1- $_p$k
(13)

and which is defined for k > 1, and 0 < p < 1. Under standard growth or decay conditions, one may use in (10) the

expressions for U* (t) and U^n (t), developed in Sec.   5.11,   to generate useful approximations to time-dependent radiance fields . First or second order global approximations should suffice for many practical settings.

We note in passing that preliminary and informal numerical studies seem to indicate that the shapes (the directional structure) of Nn appear to be spherical (or very nearly   so) when n i s larger than some integer p which depends on the medium X and p.   I f this conjecture can be proved in general, (probably by means of the set up in 10.5) then an enormous advance in the practical utility of (13) can be made. This conjecture of the limiting shape of Nn as n -+ w, bears a striking analog to the asymptotic radiance theorem studied elsewhere in this work (cf., e.g., Chapter 10). An important application would be to diffusion theory (see (78) of Sec. .6).