8.1 Invariant Imbedding Relation for Irradiance Fields

Our point of departure for the present discussion is the set of principles of invariance (7), (8) of Section 3.7. We recall that these statements were deduced from an application of the interaction principle to an arbitrary subslab X(x,z) of a plane parallel medium of the type X(a,b), schematically depicted in Fig. 8.1. The results may be written:

I. H(y,,+) = H(z,+) T(z,y) + H(y, r) R(y, z) (1)

II. H(y)-) = H(x,-) T(x,y) + H(y,+) R(y,x) (2)

Y

FIG. 8.1 The setting for the principles of invariance governing irradiance fields on plane-parallel media.

where a < x < y < z < b. The four numbers T (x,y), R(y,x) , T(z,y), R(y, z) are the various transmittances (T) and reflectances (R) o_f the pieces X(x,y) and X(y, z) of the partitioned slab X(x,z). The present goal is to derive the invariant imbedding relation for x(a,b) in the irradiance context. Our activity will parallel very closely that in examples 4 and 5 .of Section 3.9, thereby casting light on those earlier computations and in turn adding evidence to the belief that the manner of approach to the invariant imbedding relation, at least on the algebraic level, is independent of the geometry of the medium and the radiometric concepts used in the approach.

Most of the work toward attaining the invariant imbedding relation is already contained in the results (9), and (10) of Section 3.17; for if we now apply those equations to the subslab x(x, z) of the present setting (by letting z =x, b = z) and write:

"6 (x,y, z) " for T (x,y) R(y,z) / 1-R(Y.,x)R(y. z) (3)

lloftx ~y, z) tf for T (x,) / P--X1--y (4)

"k(z,y,x)" for T(?.,y)R(ax) / 1-R Y,x ypz) (5)

".Y(z,Y,x)'~ for T(z,Y) / 1-R y,x R y, z) '

then those equations can be written:

H(Y)+)= H(z,+)_J (zI,Y, x)+ H(x.-)q(x,Yaz) (7)

+ H(z,+)R(z,Yax)

Equations (7) and (8) present an excellent opportunity for the reader to become acquainted with the invariant imbedding relation in a relatively simple setting. It was for this reason that the irradiance example was presented first in Chapter 3. In this chapter we shall have more opportunity to explore the irradiance context of the invariant imbedding concepts.

The operators 6~ and" defined in (3) through (6) above are the complete reflectance and complete transmittance factors, respectively. Observe that the following special cases of &. and 7" hold:

&'t (x, x, Y) R(x,y)(9)

(x a Y, Y) T (x, Y)(10)

~. (x ty"Y) - 0 (11)

7(x, x, Y) 1 (12)

These statements may be obtained directly from (3) and (4) upon suitable substitutions, and by appeal to (13) and (14) of Sec. 3.7. A complementary set of four equations can be obtained from (5) and (6). It follows that the invariant imbedding equations (7) and (8) contain the principles of invariance (1) and (2) as special cases. The equations (7) and (8) can be cast into matrix form by writing:

~r(z~YI x) ~ (z P Y~x)

~~y(x ~Y ~ z)'} for (13)

6~ (x'9Y 9 Z) Y (x , Y . , z)

so that we have:

(H (Y;+) ~H(Y.,-)) = (H (z,x) .H(x,-))M(x,y, z)(14)

This is the required invariant imbedding relation for the irradiance context. It should be observed that we are adopting in the irradiance context, without essential change, the notation for standard and complete operators used earlier in Chapter 3 for the radiance (operator) context. This affords a great economy of terminology, retains a useful and suggestive notation, and serves to strengthen the conceptual unity of radiative transfer theory attained by means of the invariant imbedding techniques. There shall be no confusion arising from this practice, for it very rarely happens that the

irradiance field and the radiance field are simultaneously under study in a given one-parameter medium, since these are two very distinct levels of description of given radiative transfer phenomena: the irradiance description presently under study is a simple numerical description of a light field while the radiance description is a more detailed functional description of the light field.

As it stands, the invariant imbedding relation (14) is the general form of the solution to the radiative transfer problem in x(a,b) for the irradiance field: knowledge of M(x,y,z) for every three successive levels x,y,z in a subslab X(x,z) of x(z,b) allows one to compute the irradiance field (H(y,+) ,H(y, -)) at every level y in X(x, z) knowing the incident radiance field (H.(z ,+) ,H (x, -)) on x (x, z) . The complete reflectance and transmittance operators OF and in fY_(x,y, z) depend in turn on the standard operators R and T as shown, in (3) through (6). Therefore the complete solution of the irradiance transfer problem in y(a,b) devolves on knowledge of the standard R and T factors, in exact analogy to the radiance transfer problem studied in detail in Chapter 7. Consequently, knowing the R and T , or better still the 4 and -) factors, for a medium x(a,b), we can write down by sight the answer to every question about H(y,±) for every level y in X(a,b). We shall in the course of this chapter obtain methods for the determination of$_d$ the standard R and T factors and the complete factors and D'. For the present we go on to formulate further equations governing the irradiance field.