

## CHAPTER 11

### THE UNIVERSAL Radiative TRANSPORT EQUATION

these examples, which might be multiplied by the millions, are cases in which a long, laborious, conscious, detailed process of acquirement has been condensed into...one. Factors which formerly had to be considered one by one in succession are integrated into what seems a single simple factor. "

(From: "The Miracle of Condensed Recapitulation" in the Preface of Back to Methuselah Bernard Shaw)

#### 11.0 Introduction

The present chapter concludes the development of the basic theory of radiative transfer in the present work with a survey of the manifold transport equations for the radiometric concepts-introduced during Parts I, II, and the preceding chapters of Part III. The main purpose of the survey is to bring to light, especially for those readers interested in the theoretical aspects of radiative transfer, a recurrent symbolic theme which runs through every transport equation considered so far, and to go on to capture its essence in the form of a "universal radiative transport equation."

The universal radiative transport equation is an equation which, by suitable choice of its parameters, yields in turn such equations as the general equation of transfer for radiance, the general two--flow transport equations for irradiance, the transport equation for scalar irradiance, and the transport equations governing the apparent optical properties of an optical medium.

- The primary purpose of the universal radiative transport equation is to formulate in a single mathematical package all the important transport equations which have evolved during the past seventy years in the theoretical studies of the steady state transfer of radiance energy through scattering absorbing media of the stratified plane-parallel type. In this way a recapitulation of the evolutionary process of the transport equation's growth is achieved and a unification of all these important transport equations is attained. We shall illustrate the scope of the equation by selecting thirty--four

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types of transport equations discussed in this work or implied by the discussions of their principal functions, and showing how these various types may be uniformly subsumed under the regime of the universal transport equation.

A second purpose of the universal transport equation is to provide a new useful tool in the study of radiative transfer theory. For example, certain special forms of the universal transport equation have already been successfully used (Sets. 10. 5 and 10. 7) to obtain a solution to the long-standing practical problem of the existence of the asymptotic light field in deep stratified hydrosols, a mathematical task which appears to be simplified, and given interesting physical significance with-the introduction of the general type of functions \_associated with the universal transport equation. Further evidence of the usefulness of the universal transport equation as a tool which leads to new practical results will be illustrated below.

Before we go into the details of how the universal transport equation can achieve a semblance of unity in the classification of modern radiative transport equations, and of how it leads in some cases to new results which are beyond the immediate capabilities of the classical transport equations, it may be of help to the reader to

indicate the steps in the development of modern radiative transfer theory which have led to the idea of the universal transport equation. With such information in mind the reader can then easily follow the steps of the synthesis.

There are four well-defined steps in the development of modern radiative transfer theory which form the immediate background to the formulation of the universal transport equation-. These are, in chronological order: The adoption of the general equation of transfer for radiance and the development of the notion of equilibrium radiance [279], [111], and [43] ; the development of the unified two-flow irradiance equations  $F$  and the notion of equilibrium irradiance as recorded in Chapter 8; the development of the canonical equation of transfer and the notion of the radiance  $K$  -function as recorded in Chapter 4- the development of the theory of the asymptotic light field and the transport equation for the radiance function as recorded in Chapter 10.

In the following two sections we will illustrate these steps in detail and add still further illustrations which have been uncovered subsequent to the time of the fourth step.

In this way we will systematically build up evidence for the existence of a universal transport equation and for the equilibrium principle (described below) with which it is closely associated. After these concrete examples of the various transport-..equations have been assembled, the genotype of the universal transport equation is extracted from them and displayed ((1) of Sec. 11.3). The chapter closes with a brief survey of less common but equally important examples of transport equations which are also subsumed by the universal transport equation.