

11.3 Universal Radiative Transport Equation and the Equilibrium Principle

For the purposes of this section, let us refer to the thirteen quantities studied so far as the standard concepts (namely $N(z, \delta, 0)$, $H(z, \pm)$, J^{\pm} , $h(z, \pm)$, $h(z)$,

$K(z, \delta, \sim)$, $K(z, \pm)$,

$k(z, \pm)$, $k(z)$, and $R(z, -)$). A directed standard concept is

any of the preceding, standard concepts except $h(z)$ and $K(z)$.

The evidence gathered in the preceding discussions may now be assembled in the form of:

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PRINCIPLE. Let X be an arbitrarily stratified source-free plane-parallel medium with arbitrary incident lighting conditions. Let r & Z

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denote any one of the standard concepts. Then associated with $r(z)$ are two

functions $e_a(z)$ and $\delta_r(z)$, the attenuation and equilibrium functions for $r(z)$,

respectively. The standard concept $F(z)$ together

with $f_a(z)$ and $(2)_q(z)$ satisfy the functional relation

$$de(z) = u(Z) \delta_r(z) - (z) | EC(Z) - e(Z)$$

z a a

where $P(z)$ and δ are known parameters depending on $C_r(z)$. The relation

(1) is the universal radiative transport equation. (It is degenerate if $\delta =$

0; and normalized if U

if $O(z)$ is a directed standard concept and $\rho(z) > 0$ then:

$$(2) \text{ whenever } C_r(z) > C_q(z)$$

(2)

and if $e(z)$ is any standard concept, and X is eventually

homogeneous, then:

$$a_{\sim} = \lim_{n \rightarrow \infty} C_0 e_a(Z)$$

exists,

$$= \lim_{n \rightarrow \infty} C_a(z) \text{ exists,}$$

eq (00)

and.

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$$\lim_{z \rightarrow \infty} We(z) = C'(00)$$

The proof of the statements (1), (2), (3), and (4)

have essentially been covered in the preceding discussions either directly (as in the

case of (1)), or indirectly by references to the appropriate portions of the present work

(as in the case of (2)–(4)). Table 1 below gives the explicit forms of $\rho(z)$ and S for the

thirteen standard-concepts: An examination of Table 1 shows that if $R(z, -)$ is removed

from the list of standard concepts, a considerable simplification is effected in the form of

(1). However, in the interests of completeness we have included $R(z, -)$ within the

purview of (1), and we note that by a change of z -scale, the equation is

normalizable.

SEC. 11.4 UNIVERSAL TRANSPORT EQUATION 281 TABLE 1
Standard Cases of the Universal Radiative Transport Equation