

### 11.5 Bibliographic Notes for Chapter 11

The concept of a universal radiative transport equation was introduced in [240]. The mathematical vehicle of the universal radiative transport equation is that of a Riccati differential equation in factored form:

$$\frac{df(x)}{dx} + p(x)f(x) - f(x)^2 = q(x)$$

\*It will be futile to employ more [principles] when it is possible to employ fewer.

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and with each radiometric quantity  $f$  (or  $K$ -function, or reflectance function, etc.) the equation associates two auxiliary functions:  $f_1$  and  $f_2$  which, as was seen in the text, play the roles of attenuation and equilibrium functions, respectively. For an elementary discussion of the nondegenerate Riccati equation, see, e.g., [116]. For modern developments of the theory of nondegenerate Riccati equations pertinent to possible radiative transfer applications, see the work of Redheffer [254], [255], [256], [257], and also Reid [261] and [262]. These mathematical studies are also of potential applicability to the operator equations for the  $R$  and  $T$  operators in Chapter 7, as noted in Sec. 7.1S. In view of the work of this chapter and that of Chapter 7, along with the results of the work of Redheffer and Reid, it is clear that the Riccati differential equation enters a productive new phase in mathematical physics.