12.10 Instantaneous Radiance Field over a Dynamic Air-Water Surface

We go on now to the final optical problem of the present chapter: We shall show how a general solution may be found to the problem of the time-averaged radiance distribution of wind-blown, air-water surfaces. This will be done by suitably combining the interaction principle of Chapter 3 with parts of the hydrodynamic and: -harmonic analyses above. The problem will be solved in four main stages. The first stage will be accomplished in the present section wherein we shall in imagination stop the complex randomly moving. surface and take an instantaneous mathematical snapshot, so to speak, of the light field as it plays over the hills and in the hollows of the frozen surface. This information of the instantaneous light field will be made sufficiently general so that it will hold over the entire surface and for any instant in a continuum of instants. Then, in Sec. 12.11, we shall perform certain averaging operations on the instantaneous radiance fields over a sufficiently long time interval to obtain the requisite time-average radiance fields over the surface. In Sec. 12.12 the theory of the time-averaged light field within the hydrosol will be developed, and finally, in Sec. 12.13 the results will be synthesized into a complete mathematical solution of the problem of time-averaged light fields in natural hydrosols. Applications and concluding observations are made in Sec. 12.14.

Because of the necessity to work explicitly in both aerosols and hydrosols in the next few sections, it is also necessary to take into account the change in radiance when going from one medium to another with differing indices of refraction. In order to avoid cumbersome appearances of the square n^2 o.f the index of refraction in the various formulations, we shall make the. standing convention for Secs. 12.1012.14 inclusive: n^2-convention: all radiances N appearing in the equations of those sections will be understood to be divided by n , where n is the index of refraction of the medium (air or water) at the point at which the radiance is considered. The basis for this convention rests in the radiance invariance law (4) of 'Sec. 2.6,*

*It may be well to repeat once again the convention used throughout this work, concerning unpolarized flux (cf. Sec. 1.1). The significance of this convention for the discussions in Sec. 12.10-12.14 inclusive is that the equations are materially simplified without loss of essential generality. To develop the corresponding theory for polarized flux, one merely replaces N by its observable vector counterpart N (cf. Sec. 2.10) and the Fresnel reflectance by its matrix counterpart (cf. [292]) and the volume scattering function by its own corresponding counterpart (cf. Sec. 13.6). In this way, products of the form Nr,Na are replaced throughout by the vectors Nr,No. Since the theory is linear,, it follows that this is the only change needed to elevate the scalar theory to its vector (polarized) level.

The Geometrical Setting

Figure 12.55 depicts a portion of a dynamic air-water surface -S at time t. The surface S is of arbitrary geometric structure, such as is found in natural wind-blown settings, and not necessarily of a simple sinusoidal structure, nor even a superposition of sinusoidal surfaces. Only relatively recently have realistic analytical-representations of the dynamic air-water surface, along the lines of (71) of Sec. 12.4, been pressed into use in the study of hydrodynamic problems. An example of a parametric representation of the Lagrangian formulation ((I) of Sec. 12'.3)$^{-}$ of hydrosol motions may be found in [240]. Therefore with the knowledge that various realistic analytical models of the random water surface S exist for possible computation use, we can proceed with the main task of describing the interreflection process of the light field in the vicinity of S.

We establish a local reference frame at each point x of S, as shown in Fig. 12.55. To S at x we assign a unit outward normal n(x), and this in turn induces a partition of the unit sphere of directions into W+(x,t) and ._(x,t) respectively, the outward and inward hemispheres of directions to S at x at time t. =.$_{+}$(x,t) consists of all directions in ? making an angle less than 90° with n(x). The set _ (x,t) consists of all other directions in -. The main reference frame for the hydrosol is established relative to a mean horizontal

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FIG. 12.55 Direction conventions at an arbitrary point x on the dynamic air-water surface at a given instant t.

plane s kxrf ace S, shown in Fig, 12-55. The uni outward normal to S is .k and distances are measured from h positive downward ..,(in the direction of -k) as usual.

The Integral Equation for the Instantaneous Surface Radiance N_{+} (,S)

The general equations governing the radiance distributions at the points of surface S have been derived via the interaction principle in Example 4 of Sec. 3.5. In particular, the requisite equations are given in (14) - (17) of Sec. 3.5. That set of four equations may now :be used to obtain the two basic surface radiance distributions N+ and N_ in the present problem. Before proceeding, however, it may be helpful to the reader, in order to achieve complete understanding of what follows, to study Example 4 of Sec. 3.5,
and an appropriate amount of the material prerequisite-to it,

The particular forms of (14)-(17) of Sec.. 3.5 most appropriate^{-}for the present setting may be found with the help of Fig.- 12.56. This figure shows schematically that, despite the awesome complexity of the air--water surface, its "outside" never sees its "inside," as, for example, is the case in^{-}(e) or (f) of Fig. .3.16. This fact allows a simplification in the structure of the set (14)-(17) of Sec. 3.5. For it is "quite clear that the auxiliary functional equations (16) and (17) of Sec. 3.5 now become^{-}:

FIG. 12.56 The four radiances associated with an arbitrary. point x on the dynamic air-water surface at a given instant t.-

$N_-(S) = N_+(S) \cdot$
$x_+(S)\, t_m(S)$ (1)

$N_+(S) = N_+(S)\, .$
$y_-(S)\, t_m(S)$ (2)

Of the two equations (14) and (15) of Sec. 3.5, the one we shall single out for illustration is that describing $N_+(S)$, that is, the radiance distribution of the air-water surface as seen from above the water.. The solution procedure for $N+(S)$ can be inferred from that for $N+(S)$, which will be given in detail in the course of the present and following sections. The representation of $N_+(S)$ is the more immediately useful of the two and fortunately can be discussed
for the most part independently of specific knowledge of the radiance distributions in the hydrosol and aerosol-bounding . the surface. However, in the final analysis---as in the statlc case of Sec. 12.2--all three media: aerosol, boundary, -and hydrosol, actively participate in the solution procedure and
both $N+(S)$ and $N_{+}\_(S)$ must be determined. Accordingly, the time--averaged form for N^{+}-(S) is summarized in (44) of Sec. 12.11.
In accordance with the preceding remarks, we consider (14) of Sec. 3.5 in which $N+(S)$ have been replaced by their representations given in (1) and (2):
$N+(S) = A_+(S) + [N+(S) \cdot x+(S)\, t_m(S)]\, r-(S)$
$+ [N\pm(S) \cdot X-(S)\, t_m(S)\, l\, t_+(S)$
where we have written:
"$A_+(S)$ ft for $N^O(S) r^\circ(S) + N^O(S) t^o(S)$ (4)
A further simplification may now be made in the structure of (3) which in no way will incur any loss of generality of the results. We shall assume that the aerosol above S is perfectly transparent and that the hydrosol below S is perfectly opaque. In other words. we shall white convexity S from above and black convexity S from below (cf. Sec. 3.8 and Example b of Sec. 3.9) and thereby isolate it for study pending the description of its subsequent interactions with the media above and below it via the generalized principles of invariance. This will be done in Sec. 12.12 after suitably averaging all the equations involved. With these convexifications invoked, (3) becomes:
$N+(S) = A_+(S) + [N+(S) \cdot x+(S)\, t_m(S)\;\; r_-(S)$ (5)
Equation (5) clearly determines $N+(S)$ under the present conditions since it may be formally solved to yield:

$N_+(S) = A_+(S)\, I\, I - (x_+(S)\, t_m{}'\;\; r_-(S)\, -'$ (6)
in terms of the incident external radiances $N^\circ(S)$ from the sky and $N+(S)$ from the water body, and the reflectances and transmittances of the air-water surface. For the present, then, $N^\circ(S)$ will be assumed given.
We can convert (5) from operator form to its specific functional form (as illustrated in Example 4 of Sec. .3.5) preparatory to the averaging process... The result is (dropping p subscripts , for brevity) ,
$f \;\; N^O(x \;', t)\, t_+(x; \;' \;)\, dQ(\;')$ ~+ (x 'V t)

N(x-rm '   'st)r_ (x   ' ;)ds~(')
D(S,x,t)

where ~ is in "+ (x, t) , and where D (S, x, t) is the time-dependent version of D (S, x) defined in Example 4 of Sec. 3.5. D°(x,t) is the subset of -_ (x,t) over which radiant flux may arrive at x at time t from the sky (see Fig. 12.55). Therefore the first integral on the right in (7) gives the contribution to N(x,E,t) by reflected skylight. The second term gives the contribution to N(x,~,t) by transmitted hydrosol light. The final term is the interreflection term where, because of the local concavities of S, S may feed light to itself. It is clear that the natural movement of the water surface is so slow relative to the speed of light that the interreflection process over each instantaneous configuration of the water surface virtually attains its steady state before the configuration can sensibly change: thousands of terms in the natural solution can be formed over a gravity wave of 1 meter wavelength before that gravity wave can change its linear proportions by 1 percent. Therefore (7) gives an instantaneous--or as is sometimes described, .a quasi-steady state--description of the light field over S as it is lit by the sky, the sea, and itself. The exact interconnection among the domains of integration of the integrals in (7) will be needed subsequently, and is as follows. First, by Fig. 12.55 it is clear that for every x on S and every time t,

The inward hemisphere '-(x,t) in turn has the decomposition D^0 (x,t) UD(S,x,t)
Combining (8) and (9), we have:

+(x,t)UD°(x,t)UD(S,x,t) = W (10)

Next we may introduce the characteristic functions X of a set E is such that X(x,E) = 1 whenever x is in E, and X(x,E) = 0 whenever x is not in E. It follows from this definition and (10) that for every direction E in E, we have:
x(E.9W+(x~t)) + x(aD°(x~t)) t X(sD(sjxst))

The introduction of the characteristic function serves the purpose of allowing the domains of integration in (7) to be uniformly replaced by ' and the integrands to be prepared for the ensuing averaging process:
S E W_+ (x, t) '.x c S
(I2)

This is the required integral equation governing the surface radiance of the air-water surface S, at an arbitrary point x of S, at time t ad for all directions ~ in ~+ (x,t) . The incident radiance N^o consists of two parts: the part Nv(~) from the sky and the part N (x, ~) from the body of the hydrosol below. When these two radiances are given or assumed known, (12) completely determines N fix, ~, t3 . We note in passing a certain asymmetry in the decomposition of EE above.

Thus we have partitioned 'E_ into D° and D as in (9), but have not made the analogous partition of ~.E_+. This is a result of our decision to treat the aerosol above S and the hydrosol below S in two distinct ways using the technique of convexifying the surface S. The consequences of this decision are summarized in the discussion of (2fl) of Sec. 12.12, below.