12.13 Synthesis of _Time-Averaged Radiance Fields

The complete description of the time-averaged light field in a natural hydrosol with a wind-blown. air-water surface will now be attained by gathering together the various pieces of the description fashioned in the preceding three sections.

The synthesis to be given is facilitated by casting equations (18) and (44) of Sec. 12.11 into operator form. Toward this end, and with (18) of Sec. 12.11 in mind, let us write:

~ ~ R 0 (S) it

for S (0 [ ] Q_+ (~') T_+ (~') dQ (E') (2) f .

where we^have dropped reference to the point x in the mean surface S. For example, "S (~)" is therefore a contracted name for S (x, ~) , defined in (18) of Sec. 12.11. Furthermore, writing

R

rwe can then write (18) of Sec. 12.11 as:

N_+ (S) = $N^O R\_$ (S) + N+ $(^X)^T$+ (S) + N_+ (S) R_ (S)

where "N_+ (S)" denotes the function N governed by ^(18) of Sec. 12.11, with the subscript "+" as a reminder that N is the averaged response of S over '$_+$(x,t). The surface radiance superscript i_%-now dropped as being understood. N° is the time-averaged sky radiance distribution (defined ovel S_3 and A_+ (x) is the time-averaged response of thq body X of the averaged hydrosol whose boundary is S. Thus N_+(x) is the time-averaged radiance function N° over W_+(x,t) occurring in (12) of Sec. 12.10 and in (40) of Sec. 12.11. (Recall the n^2-convention stated at the outset of Sec. 12.10.)

In a similar manner (44) of Sec. 12.11 can be cast into the form (again all radiances being s u r f a c e t y p e radiances, the superscript "+" may now be dropped):

"T_ (S) tv

for (7)

f or(8)

The functions R_+, T_+ are defined in (23), (26), (45), and (4b) of Sec. 12:11. By virtue of the conclusions following (11) of Sec. 12.12 we have:

N_+ (X)= N_ (S) R_ (X)

where. R- (X) „is the time-averaged reflectance operator for the medium X. This operator equation is obtained by applying the steady state theory of radiative transfer in plane-parallel media, with appropriate modifications, to the time-averaged equation of transfer (11) of Sec. 12.12, defined in the medium with

upper boundary S. The salient modification relative to the static case is that R_(R) is an integral operator with a representation of the form:
with E in ~. In other words the integration is over all of rather than just E7_ (as is the case for R(a,b) in the static theory of Chapters 3 and 7). Furthermore, the response of X can be along-every ~ in ~. The visualization of this new situation is quite easy, after the derivations of Secs. 12.10-12.12 have been thoroughly assimilated. Otherwise, the theory of R_(X) (and the other three time-averaged operators R_+ (X) , T_+ (X)) is exactly analogous to the static plane-parallel case and is essentially as developed in Chapters 3 and 7. The further exploration of this aspect of the time-averaged theory. will be left to future students of the discipline of radiative transfer.
We continue the present synthesis by showing that the three equations (4), (5), and (9) may be f2rmally solved to yield the three radiance fields 9+(S) and N_+ (X) . By (4) we have

N+(5) N__Q_ (S) + N+(X)a0"'(5)
(10)
where we have written:

(12)
Furthermore, (5) and (9~ may be combined to yield:
N_- (S) = N_°T_° C5) + N_+ (S) T:. (S) + N_ (S) R_ (X) R+ (S)
which in turn yields t4e`following formal-solution for in terms of N_+ (S) and N_-^0:
n
N_ (S)

I N_ (S) = No 7^0 (S A X) + N_+ (S) Y (S , X) (13)

where we have written

.. ..
' ' °
(14)
7 (S , X) "
for T (S) [I
(S tt ~'
- . (1 5)
3, X) for T_ (S) [I R_ (X) R+ (S)

the„operator (15)^acts like a transmittance operator for N_+ (S) because N+ (S) is the averaged surface radiance of S leaving S and descending down onto S to be transmitted through S and hence to contribute to N+-(S). With the help of (9), we can w1ite (1Q) so that, like (13) ., it involves only the inputs N_ and N_ (S) to S .

N+ (S) _- Nato (5) + N_ (S)Q_ (S v X)
(16)
where we have written
t ._(S . X) ft
for R_(X)-'+tS)
We have reached the penultimate step in the formal solution of the time surface radiances &+(§), &-(§). Equations (13) and (16) are a system of operator equations in the requisite unknown§ N±_(S) with given 6l and Y operators and given input radiance N° associated with the sky radiance distribution. These equations can be solved formally on the operator level in the manner illustrated repeatedly throughout Chapters 3 and 7. The -results . a.re:

(18)
(19)

where we have written
A
ir6-II for 0 (S) + ~` (S_AX) (21) Equations (18) and (19)completely solve, in principle, the problem of the time-averaged radiance distribution of the dynamic air-water surface irradiated by skylight 92 . (Recall the n„-convention stated at the outset of Sec. 12.10.) The operator,)'- is a general time-averaged complete transmittance of 9, and 6f- is a general time-averaged complete reflectance of 9, where the inner structure of ~q_ and ~'_ is completely determinable by retracing the thread of reasoning beginning with (20) and (21) and working back to Sec. 12.10.

Comparison with the Static Case

In closing, it is of interest to compare (18) and (19) with the representation of the radiance of the static air-water case given in (13) and (14) of Sec. 12.2. (At this point the reader should recall the n -convention for radiances stated at the outset of Sec. 12.10.) The operator Lr(- I, 0, z_j) in (16) of Sec. 12:2 is a special case of ~5r_ occurring in (18). Furthermore, the operator ~R(-1, 0, z ,) in (17) of Sec. 12.2 is a special case of V occurring in (19). This operator, when applied to N°,. yields. N+ (S) . The .similarity in the structure of these,- operators with their _ static counterparts is quite striking and : is traceable, of course, to the interaction principle underlying all algebraic descriptions of radiative transfer phenomena. The reader will find it instructive to show that D"I and 6,_ reduce exactly to (16) and (17), respectively, of Sec. 12.2 as the dynamic air-water surface S continuously approaches 9. This may be done for example by adopting the Neumann spectrum model for S (Sec. 12.8), letting U_a +0, (i.e., letting the equilibrium wind speed go to zero) and using the gaussian representations of the weighting functions Q, Q°, Q+, Q_. Or it may be done ,by simple intuitive considerations on the necessary properties the Q-functions must have for any reasonable model of the air-water surface, as the air-water surface continuously approaches the static plane form.