12.14 Observations on the Theory of Time-Averaged Radiance Fields or Dynamic Air-Water Surfaces

The theory of time--averaged radiance fields developed in the preceding four sections contains a great variety of special cases of practical interest in applied hydrologic optics. It is the purpose of this section to classify and discuss the main set of these special cases and to indicate their use in practice.

A Hierarchy of Approximate Theories

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Starting with the. basic equations (12) of Sec. 12.10 and (40) of Sec. 12. I1., . we may construct a sequence' of approximate theories of the time-averaged light fields for the dynamic air-water surface of natural hydrosols. As one proceeds along this sequence, the descriptive powers of the models decrease and the numerical tractability increases. The main sequence of approximations is as follows:

(i) The Exact Time-Averaged Theory. Staring with (12) of Sec. 12.10 and (40) of Sec. 12.11 with N^o given along with the prescribed motion of the air-water surface S, the radiance N (x, ~, t) at point x on S in the direction E for every t in some interval (0,T) of time may be computed. The resultant set of radiances may then be averaged to obtain a numerical estimate of A(X,~) which is, in principle, exact in the sense that no special assumptions are used to obtain the averages.

(ii) The Statistical Z Time--Averaged Theory. The theory summarized in (18) and (44) of Sec. 12.11 and in the equations of Sec. 12.13 is based on certain observed or theoretical statistical regularity properties of the dynamic air-water surface such as the stationarity condition (5) of Sec. 12.11, the independence condition (10) of Sec. 12.11, and the ergodic hypothesis (31) of Sec. 12.11 (which allows space averages to be interchanged with time averages). This theory contains the effects-of the radiometric interaction of S with itself and with the body of the hydrosol in addition to the hiding effects of finite wave slopes and finite wave heights from the sky radiance distribution.

(iii) The Wave-Slope, Wave-Height Time-Averaged Theory. The statistical time-averaged theory in (ii) is made somewhat more tractable but less descriptive by dropping the self-interaction term in (18) of Sec. 12.11. The result is:

N (x, E) = S (x, ~) A_+ (x s o (

for all ~ in E', and which is a straightforward integral representation (and no longer an integral equation) of N (x, ~) which may be evaluated once 1v, Q , Q_+, R_, T_+ are prescribed. It is clear that the effects of finite wave heights and finite wave slopes are still included in this approximation. From (19) of Sec. 12.13, the upward time-averaged radiance of S is given in the statistical theory by

In the present wave-slope, wave-height mode, OR_ takes the form:

-. R^C (S)

(S) [I - R_ (X) R+ (S) I R_ (y) T+ (S) (

That is, the effect of dropping the self-interaction term ---i-s to require

R_- (S) =9

T_- (S) = o

whence, in particular

Lro (S) TO ~S) ~5 J_ (S ~X) _ 0

[iv] The wave-Slope Time-Averaged Theory. The wave slope, wave-height time--averaged theory of [iii] is further simplified by neglecting effects of the wave heights of the dynamic air-water surface. Thus the immediate effect is to require q_ (x, ~) = o , whence by (35a) of Sec. 12. 11 Q (x, 0 = 0,V and so that by (27) of Sec. 12.11 we now have

R°(xso + R+(x,o = 1

Furthermore, by;t4e definition of S (x, ~) in (18) of Sec. 12.11, and (1). N(x,~) = o for ~ in E_. Otherwise, the general form of A_+ (x, E) is the same. The resultant theory is of great importance to, optical studies of the dynamic air-water surface since the distribution of wave slopes rather than wave- heights is more often the dominant physical determinant of the time-averaged radiance of air-water surfaces. For example, as a fresh wind blows over an otherwise calm water surface on a clear day, a fine dark patina quickly overspreads the windblown surface in which the higher parts of the dark sky are mirrored. On the other hand, when the observer--sun geometry permits, a bright glitter pattern springs into existence as the wind produces a great range of wave slopes of capillary waves. In such transient settings the overriding importance of the wave slope state of the sea is clear.

(v) Partial Time-Averaged Theories. Occasionally it is of interest to study the reflected or transmitted radiance from a dynamic air-water surface S which is due only to the light of the sun or only to the light of the sky or only to the radiance incident on the underside of S from the hydrosol proper and in which self-interaction, the wave-height and wave-slope effects may or may not be included. Equations (5) and (9) of Sec. 12.13 summarize these -component radiances in clear intuitive form. For example, &-(§) in (5) of Sec. 12.13 describes the inherent radiance of the lower side of the time-averaged surface S as being composed of transmitted time-averaged sky light N"-, self-interaction light N_+ (S) ,

and reflected time-averaged hydrosol light, N_+ (X) , and is therefore, of the same general form as the static radiance of curved surfaces considered in Example 1 of Sec. 3.5t Similarly, N_+(S) in (Io) „of Sec. 12.13, namely the upward time-averaged radiance of S, is seen to be composed of reflected sky

light and transmitted hydrosol light. Any one of the five terms on the right of (5) or (10) of Sec. 12.13 may be isolated for the study of + (S-] in a partial time-averaged .theory.

Illustrations of Some Classical Partial Theories

Some of the features, of the theory of ,time-averaged light fields as developed above can be illustrated by selecting for examination each of the four terms in (18) and (19)

of Sec. 12.13. By doing so we shall also deduce some of the classical instances of the theory of time-averaged reflect-ed and transmitted radiance at the surface of natural hydrosols.

We ~ begin with the component of the time-averaged upward radiance N-+ (S) of S contributed by reflected sky and sunlight. Thus from (19) of Sec. 12.13 we consider N-^{0}61-M. For simplicity of exposition„we shall omit the self-interaction effect of S so that 6_ (S) = R_ (S) , which follows from (5) . Hence from (1) of Sec. 12.13 and the present selection of effects:

go(E')Q°(~')R_(~';~)d2(~')

with A in W*, and where we have dropped reference to the point x in 9, the uniformity of optical properties over all of S being assumed.

The f actor Q^o (~') in the integrand o f (6), as defined in (11) of Sec. 12.11, takes into account the shielding effect of the wave heights and wave slopes against the light of the sky and sun. Q° (~') may be evaluated using, e.g., .(27), (29), and (35) of Sec. 12.11. „Even when wave heights are considered negligible (i.e., q_ (x, E) = 0), so that Q (Q = 4, we see from ° (29b) , and (35b) of Sec. 12.11 and (b) that wave slope shielding maybe i effect.. By means of (23) of Sec. 12.11, we may represent () Q in greater detail as. follows

where r (~' , t) is the value of the Fresnel reflectance function for incident and reflected directions ~' ~ , respectively, in w..- and w+ . The function p (~' , ~) is the probability that the wave normal to S is such that the incident and reflected directions E' and ~ occur for Fresnel reflection. If the sea. surface is statistically gaussian, then. either (24) of Sec. 12.11 may be used or its anisotropic generalization (28) of Sec. 12.5, after an appropriate change of variables in (7), if desired, from direction space E to slope space E 2 (i.e., the two-dimensional Euclidean plane).

As a specific illustration of (7), suppose that the sun is of uniform radiance N° and positioned such that the radiance of its center is directed along $~_o$, and that its direction set is "o Then since Q(E) is quite small, we may approximate the contribution to NEE) by sunlight ,by:

$N(E) = N^{\circ}Q^{o}(_a)r(_0 94)p(~_0 3~0001^4 '_0)$ (8)

A plot of N(~) as a function of E will, therefore, yield a radiometric map of the sun's glitter pattern over +. If $p(~_o, ~)$ is given by (24) of Sec. 12.11 or (28) of Sec. 12.5, then the glitter pattern is seen to be a function of the root mean slope o, and, owing to the general presence of $Qo(~_o)$, to the root mean elevation m_{od} It is, in fact, quite possible to use (8) to form a systematic tabulation of the values

$R_j(E) = Q^{\circ}(_j)r(-~)P(jP)~(w_j)$ (9)

as a function of ,-, and the statistical sea surface parameters a and m_{oo}. Then, when the radiance distribution of the sky hemisphere ._ is known over a partition {-.J, =2, 600_P W $_n$1 of .~- with central direction ~j, j = 1, . . . , n, for each j19 we can compute which is a discrete version of (7). A different kind of approximate formula for computing reflected sunlight from a roughened water surface is given by Cox and Munk in [56]. Equation (10) also may be used to estimate the time-averaged radiance of an air-water surface which has an extraordinary ground swell under the usual statistically stationary jumbled surface. This would be accomplished by literally tilting the coordinate axes used throughout the preceding derivations so that the previously horizontal reference frame now lies in the rising or descending part of the swell. Computations then proceed as usual in this new frame of reference. With some tare, the additional shielding effect due to the swell's geometry can be included to supplement the QO factor shielding effect,

As another example of a classical representation .of N+ (S) we consider the upward transmitted component N_+ (x)

.T? (S) in (10) of Sec. 12.13. Once again, for simplicity, the self-interaction effect is omitted so that Z7'+O(S) = T_+ (S) and we have for the case at hand:

N() = N0 ('

J;; Q_+ (E I) T_+ (E'~E)d2(E') ..+ n

2

where ~ is in w and where N°(') is the radiance of the hydrosol incident on the lower side of 9 in the direction fit.

FIG. 12.65 Setting for the classical apparent contrast reduction formula by time-averaged refraction at the air-water surface.

We have momentarily set aside the n^2-convention, stated at the outset in Sec. 12.14, so as to avoid an accidental misapplication of (11) by readers using only this discussion as a reference source (n for air is taken as n = 1).

Equation (11) can be unfolded layer by layer for particular applications just as we did with (6). Thus., by (26) of Sec. 12.11, we can write;

0

(~') t dS2

N N

( ) = n 2 4_+(~)t(V ~~)p(V s) ( ) (12)

where C is in

As a particular illustration of (12) let us estimate the time-averaged radiance in the vertical direction k for the case where NO(El) is of the form:

°

N W) - ~

N_t if ' is in $\sim_t$

{ N_b is in $\sim_b$

if ~' is .

where $_t$ and are the sets of directions which partition +, shown in Fig. 12.65. The set of directions E_t is subtended by a circular disk D.of uniform apparent time-averaged radiance Nt and ~~ is the set of directions of its back round. As a ¯result ¯of this choice of direction k, and form of $0.₉ (12) becomes

N_t 2 (13)

Now the presence of the Fresnel transmittance factor in (13) is such as to limit the range of integration of E' in =+ to a solid angle - f of half angle 41.4° within which t (t' ,k) >0, and outside of which t(VA) = 4 owing to total internal reflection. Hence the range of integration may be limited to

.t n.=$_+$ and. n w , when working toward actual numerical estimates of N (k~ . -To see the general order of magnitude of N(k), let us set:

for all ~' over =-+,. so that (13) becomes

N

(14)

The integrals in this expression have very simple interpretations: the integral over Et, for example, is the probability that the normals to the wave slopes at point P in Fig. 12.65 are tilted at less than the angle ~t from k, the angle for which the refracted rays from the rim of the submerged disk, D emerge from the hydrosol along the direction k.

(See insert, Fig. 12.65.) Using the gaussian model of the sea surface (24) of Sec. 12.11, we have at once from (17) of Sec. 12.5:

$\tan^g t$

e^{2a2} (15)

and

fW ~

- $\tan^g t$ 2

p W , k) dQ (t') = e ^{2a} (16)

With these representations, (14) may take the form:

N (k) Nt - a n

This approximate representation of N(k) is of historic interest, being essentially the result of the first calculation of a time-averaged radiance viewed through a moving air-water surface (cf. [82]). Equation (17) may be rewritten to use the observed time-averaged radiances N_t,gb seen just above the surface S. For this purpose we use (18) of Sec. 12.1 but retain the present assumption that t (E' ,E) = 1, so that (17) becomes

One of the first calculations of contrast reduction by time varying refraction was based on (18). Thus, as in [82], let us define the time--averaged apparent contrast of the apparent radiance Nt of the center of the submerged disk with respect to the background radiance 9b by writing:

li$_c$it

N .(k) - Nb

This is an extension of the standard notion of contrast, defined in Sec. 9.5, to the time-averaged case. The apparent contrast C for the static case is, by definition of Sec. 9.5, simply

N_t N_b

Here we are implicitly assuming that the time-averaged radiances N_t and Nb are the same as would be measgred$_2$in.the „ 2 static case, i.e., we are assuming that $R_t = N_t/n$, N_b = Nb /n . In order words, we are hypothesizing for the purposes of the present discussion that the movements of the surface -S do not change the average values of the apparent radiances over the disk D and over its background. With this assumption in mind we can obtain a simple and useful connection between _C and C using (18)

(21)

From this representation of C, despite the multitude of assumptions on which it has been based, we gain valuable quantitative information about the effect of the movement of an air-water surface on the apparent contrast of submerged objects viewed through that surface. Equation (21) is particularly useful when the submerged target is angularly small and the water surface is freshly crinkled with capillary waves. Then we can readily see how C varies with ~t and a: tb'e larger Ot, for aa given a, the larger C is, meaning the less the diminution f contrast by wave action. Thus small objects tend to be readily blended into their background by the water surface disturbances. On the other hand for a given target size, C goes down as a increases. For example, if we use the relation

= Y ^{U}a

for some constant y [cf. (28) and (29) of Sec. 12.5 and (11) of Sec. 12.8), [21] can be written as

tan'~$_t$

which for small targets and fresh breezes over an otherwise calm surface can be reduced to the rule of thumb:

/tan^{2}o $_t$ 2yU) C =C I

I

(23)

This equation allows one to readily see the direct variation between $t and -C, and the inverse variation between, U_a and U.

As a final illustration of dome partial representations of the time-.averaged radiance N_+ (S) , we consider (19) of Sec. 12.13. Thus we are to consider (compare with (2))

^ o^

N_+ (5) _ N _

Once again let us omit self-interaction effects over S so that (4) holds. Furthermore we concentrate on the transmitted upward-flux term in the preceding relation, thus we consider the second term of

N $_+$(S) = N_R_ (S) + N_$^{\circ}$T°(S) II - R_(X)R0

(s)]-' R_ (X)T+(S)

(24)

The interpretation of the second term of (24) is straightforward: given the incident radiance distribution N- from sun and sky, operate on N2 with T2 (S) as given in (7).

The result is a time-averaged radiance distribution transmitted downward across 9. The latter distribution is-operated on by the interreflection operator:

- R_ (X) R+ (S)] r

which may be approximated arbitrarily closely by computing a sufficient number of terms of its representing infinite operator series (in practice two terms beyond the identity I should be sufficient)

R_ (X)R+(S)]- = I + I T~ (S$X) (25) ' =I

where we write:

"TJ +I (5 $_P$X) ~~ for T (S, X) Tj (S, X) (26)

and

T' (5, X) tt or "T (S, X) " for R_- (X) R+ (S) (2 7)

The result is a time--averaged downward radiance .distribution to be operated on by R_ (X) T° (S). The reader may deepen his understanding of (24) by referring back to a related but more readily visualized example in terms of irradiance. Thus, the exact irradiance counterpart to (19) of Sec. 12.13 is (2) of Sec. 12. 2 and, in particular, the interacting counterpart to (24) is the second term (2) of Sec. 12.2. This shows that the algebraic structures of both the irradiance and radiance cases (in either the s tat is or dynamic context) are identical. The practical difference between these cases is that in the irradiance case one works with numbers; in the radiance case one works with functions and, in addition to extensive numerical details, the order of operations on functions in this case must be scrupulously observed.

Concluding Observations

one of the two main goals of this chapter has been the pair of equations (18) and (19) of Sec. 12.13. They describe the time-- averaged radiance distributions N_+ (S) , N_ (S) directed into the aerosol and hydrosol, respectively, above and below the dynamic air-water surface. The main purpose of these equations is to form a solid mathematical and physical foundation for practical techniques of describing and predicting the average radiance distributions of wind--blown seas, lakes, and other natural hydrosols. The preceding discussion of the hierarchies of approximate theories leading from (18) and (19) of Sec. 12.13, shows that this purpose has been adequately fulfilled. Thus we were able to deduce the two classical formulas for the time-averaged apparent radiance and apparent contrasts of submerged objects as in (18) and (21). However, (18) and (19) of Sec. 12.13 allow us to go beyond these classical formulas and establish (7) and, more completely, (24) which can take into account the shielding effects of wave slopes and wave elevations from sky- and sunlight without the complications of radiometric self-interactions of the dynamic air-water surface.

The complete description of the averaged light field, with self-interactions, shieldings, and hydrosol light-fields, all in concert, as given in (18) and (19) of Sec. 12.13, has not yet been subject to numerical applications. Before this can be done, further study of the operator R- (X) in (9) of Sec. 12.13 must be made. To a first approximation we may use the R(0,co) operators (with kernels R(9,o.o; E' ; Q) in Sec. 7.6

instead of R_ (~) .   However, interested students after careful study will see what more can be done here.

The theory of the optical properties of the air-water surface has thus been brought to a level of completion summarized by (18) and (19) of Sec. 12.13.