

13.1 Operational Definitions of the Principal Radiometric Concepts

The mode of development of the chapter on radiometry in the present work (Chapter 2) was designed to emphasize at the outset the operational aspects of the definitions of the radiometric concepts used in radiative transfer theory, and in hydrologic optics in particular. Therefore we need not undertake an extensive reformulation of these concepts for the purposes of the present chapter. Instead, we shall take the opportunity to collect the principal definitions of the radiometric concepts together in a relatively compact summary form 273

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and to state some of their interrelations which have been found useful in practice.

Radiant Flux

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The concept of the general radiant flux $(D(S, D, t, F))$ was defined operationally and discussed in detail in Sec. 2.1 through Sec. 2.3. We shall use this concept as a base for the present discussion. Thus- we can define the spectral radiant flux $P(S, D, t, \nu)$

at time t of frequency ν , falling on a surface S through a set of directions D , by writing:

$$P(S, D, t, \nu) = \lim_{\Delta F \rightarrow 0} \frac{D(S, D, t, F) - D(S, D, t, F - \Delta F)}{\Delta F}$$

The indicated mathematical operation of going to the limit has its practical counterpart in the selection of a relatively narrow band $F(\nu)$ of frequencies comprising the interval $(\nu - \Delta\nu/2, \nu + \Delta\nu/2)$, so that $\Delta F(\nu)$ ($=\Delta\nu$) is as small

as can be practically obtained. Then the operational definition of $P(S, D, t, \nu)$ is obtained by writing:

$$P(S, D, t, \nu) = \lim_{\Delta F \rightarrow 0} \frac{D(S, D, t, F) - D(S, D, t, F - \Delta F)}{\Delta F}$$

for

$$D(S, D, t, F) = \int_{\nu - \Delta\nu/2}^{\nu + \Delta\nu/2} P(S, D, t, \nu) d\nu$$

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Initial calibrations of radiant flux meters measuring $D(S, D, t, F(\nu))$ are carried out in practice with the help of either manufacturers' data tables on photoelectric devices (cf., e.g., Sec. 2.1) or tabulations of radiant flux output of standard devices and sources, or a combination of these.

For the remainder of the discussion we shall assume that the measurements are made at a fixed time t and for a fixed frequency ν , so that in the interests of brevity, " t " and " ν " may be dropped from the notation.

Irradiance

We turn next to the concept of irradiance, introduced in Sec. 2.4. The irradiance at a point x on a surface S with area $A(S)$ produced by radiant flux falling on S through a set of directions D is defined by writing:

$$H(x, D) = \lim_{\Delta A \rightarrow 0} \frac{D(S, D) - D(S - \Delta A, D)}{\Delta A}$$

In particular, if \hat{n} is the unit inward normal to S at x and if D is now $W(\hat{n})$, the hemisphere of all directions V such that $\hat{n} \cdot \hat{v} > 0$, then $H(x, W(E))$ is the usual form of irradiance used in practice. We can denote $H(x, W(E))$ more briefly by

" $H(x, Q)$ ". In practical work S may be selected as a relatively small collecting surface, say $S(x)$, fitted over the photoelectric element of the radiant flux meter, and we write:

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"H(x, ~" for F(S(x), .)) / A(S(x))

The irradiance meter can be calibrated to read H(x,~) directly. Precautions must be taken, however, to have S(x) collect radiant flux in accordance with the cosine law which holds for irradiance (re: (15) of Sec. 2.4, and (8) of Sec. 2.8), otherwise theoretical calculations of H(X, E) from independent radiance measurements (as in (8), and (17) of Sec. 2.5) may not agree with direct measurements of H(x, ~) using irradiance meters.

The difference between H(x, ~) as measured by an irradiance meter and as computed from a radiance distribution is readily estimable if the meter is calibrated in the laboratory

so as to learn beforehand its response characteristic to arbitrary incident beams of flux. Thus, let D(¹): be a relatively narrow bundle of directions whose magnitude i-s fixed and on the order of a thirtieth of a steradian. Then a plot of H(x, D (~')) can be made versus ~' as ~' varies from the inward normal ~ of the collector surface to directions normal to , and during this variation the flux content of the beam is held fixed in such a way that its radiance is unity. Thus write:

1 ~~ for H.x,D(~')

if f (~ ~) . E ~ Q D E

If the collector is a cosine collector with given x and ~, then f(g',~) will be constant and of unit value with respect to ~', and independent of the shape of D (Ei) as long as Q(D(V)) is kept small, as agreed above.

The quantities f [, ') may be used as follows to predict the departure of irradiance readings from their values computed using knowledge of N(x,~').

Suppose that the radiance distribution N(x, • } is given at x, and that the irradiance meter has an inward unit normal E. Then, according to (8) of Sec. 2.5 the actual, irradiance is

N(x, V) V - tdQ (t')

According to (2b) of Sec. 2.5 this representation may be -rendered into an approximating summation formula of the type:

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where Ni is the radiance related to H(x,Di) by the usual operational connection (re: (2) of Sec. 2.5).6N i = H(x,, Di) IQ (Di)

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and X (~, ~') is defined in (20) of Sec. 2.5. On the other hand

H(x, E) ma§ be estimated by using the irradiance meter. Thus, by (3)

H(x,D(~')) = N(x ~ ') f (. , ') ' • (D(')) • (5)

Using the partition solid angles Di employed in (4), the equation (5) may be written

H(x,D(Ei)) = N(x ~ ') f C ~ i) i

Hence:

is the anticipated reading of H(x,~) using the irradiance meter. Comparing (4) and (6) we see that the total difference AH(x, ~) between (4) and (6) resides in the following sum of weighted differences .

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$$AH(x,t) = \sum_{i=1}^m [f(E_i) - 1] \cdot \int_{\Omega_i} N(x, \omega) d\omega$$

Summarizing, we may say that the departure of an irradiance meter from its ideal cosine-collecting characteristics is measured by the values $f(\omega, E)$ defined in (3); and that the corresponding total error $AH(x, E)$ between the readings of the irradiance meter irradiated by a radiance distribution $N(x, \omega)$ and the actual irradiances calculated from $N(x, \omega)$ may be estimated by means of (7). Alternate means of estimating $AH(x, E)$ can be based on slight variants of f , as defined in (3). Specific examples of such error estimates have been studied by Tyler [299]. It is useful in practice to have a specific partition of the hemisphere $W(E)$ through which the radiant flux is collected prior to an irradiance calculation of the general type indicated in (4). Figure 13.1 depicts one such partition used in actual computations. To be specific, we let E be k , the vertical direction in terrestrial coordinate systems (re Sec. 2.4) and consider the computation of the upward irradiance $H(x, k)$ at a point x in the medium. The hemisphere $W(E)$ is now the upward hemisphere of directions ω_+ and this hemisphere is partitioned into a grid of spherical rectangles by equally spaced longitude and latitude circles as follows. We divide

into $m+1$ zones by means of latitude circles. There is, to begin with, a zone in the form of a spherical cap about the direction k . The half-angle θ_0 width of the cap is $\theta_0 = \theta_0/2$ where θ_0 is a fixed increment defined by the relation:

$$\theta_0 = \frac{2A_0}{m}$$

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FIG. 13.1 A useful decomposition of direction space ω_+ for use in computing irradiances.

In other words, by choosing m , we fix θ_0 ; or, conversely, suitably fixing θ_0 determines an integral m , whichever best suits the needs of the computations. Once the magnitudes of m and L have been decided on we can fix the location of the main $m-1$ zones of ω_+ by writing

for $i = 1, 2, \dots, m-1$. The angles B_i are the central latitudes of the $m-1$ zones depicted in Fig. 13.1. The cap and the half zone of width $\theta_0/2$ (the equatorial zone) whose lower boundary is the equator of ω_+ , make up the remaining two zones of ω_+ .

Next we divide ω_+ into $n+1$ lunes by means of $n+1$ equally spaced longitude semicircles of angular width A_0 radians, such that the relation;

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holds. The longitude ϕ_j of the center of the j th lune is $\phi_j = \phi_0 + j \cdot \frac{2\pi}{n+1}$, which we shall denote by ϕ_j . As a result of these divisions, ω_+ is partitioned into $(m+1)(n+1)$ regions each of which can be indexed by a pair (i, j) of integers, $i = 0, 1, \dots, m$, and $j = 0, 1, \dots, n$ such that the parts of the spherical cap are indexed by $(0, j)$ and the regions of the first zone below the cap by $(1, j)$ $j = 0, 1, \dots, n$, and those in the i th zone below the cap by (i, j) , $j = 0, 1, \dots, n$. The coordinates of the centers of the main $(n+1)(m-1)$ regions (i.e., all regions except those on the cap and the equatorial zone)

are $(e^{-\theta_i}, \dots)$, where $i = 1, \dots, m-1, j = 0, \dots, n$. The weighted solid angle content of the $(m+1)(n+1)$ regions of the present partition of S_+ are given below:

$$A_{Wi} = \sin \theta_i \cos \theta_i A_6 A_{\sim} \text{ for } i = 2, 3, \dots, m$$

$$\sin \theta_i \sim \cos \theta_i [\sin \theta_i (4)]$$

$$0 \leq \theta_i \leq \pi/2$$

$$A_{\sim} = A_2' = \int_0^{\pi/2} \sin \theta_i \cos \theta_i d\theta_i \quad (12)$$

Here " θ_i ", $i = 0, \dots, m$ denotes the solid angle content of the i th of the $m+1$ regions (in an arbitrary lune) weighted by the cosine of the angle between the central direction of the region and k . Let us write " N_{ij} " for the radiance $N(x, \theta_i, \dots)$, where $(\theta_i, 0)$ is the direction of the center of the j th region in the i th one for $i = 1, \dots, m-1, n = 0, 1, \dots, n$, and let the radiances be assigned to the cap regions and the equatorial regions as shown in Fig. 13.1. Then (4) reduces to:

$$H(x, k) = \sum_{i=0}^m \sum_{j=0}^n N_{ij} A_{Si} \quad (13)$$

An example of the use of (13) is displayed in Ref. [306]. Further practical means of computing irradiance are discussed in Examples 14 and 15 of Sec. 2.11.

Spherical Irradiance and Scalar Irradiance

Spherical irradiance and scalar irradiance are both measures of the radiant energy per unit volume at a point in an optical medium. The definitions and connections among h_4 , h and u are given in detail in Sec. 2.7; and may be summarized as

$$v(x) u(x) = h(x) = 4h_{4,r}(x) \quad (14)$$

where $v(x)$ is the speed of light at x , $u(x)$ the radiance density, $h(x)$ the scalar irradiance, and $h_{4,ff}(x)$ the spherical

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irradiance at x . As demonstrated in Sec. 2.7, it is $h_{4,ff}(x)$ which is operationally meaningful while $u(x)$ and $h(x)$ are constructs derived from the theory of geometrical radiometry (radiative transfer theory in a vacuum) and related to $h_{4,ff}(x)$ by theoretical arguments. The quantity $h_{4,ff}(x)$ may be given the following operational definition. Let $S_r(x)$ be a spherical collecting surface of center x and radius r ; Suppose that about each point y of the boundary surface of $S_r(x)$ the surface is a cosine collector. Let $P(S_r(x))$ be the radiant flux recorded as incident on $S_r(x)$ in some radiometric environment. Then we write:

$$P(S_r(x)) = 4\pi r^2 h_{4,ff}(x) \quad (15)$$

When r is sufficiently small, the radiance distributions over the set of points that would be occupied by $S_r(x)$ are independent of location over that set, and also $h_{4,ff}(x)$ is sensibly independent of r . Hence in this sense " r " need not enter into the notation for the spherical irradiance.

In constructing a realization of $S_r(x)$ (determining S_r with actual instruments) it is impossible to obtain a collecting surface about each point of which the material, exhibits

sine collecting. properties, zero reflectance and unit transmittance, the properties needed. in order that the measurements not, disturb the light field at x. To study an important effect of the departure of the surface of $S_r(x)$ from this ideal, let us suppose that at each point y of , the surface of $S_r(x)$ behaves in accordance with $f(\tilde{n}(y), \tilde{n}')$ as given in (3) when the experiment giving rise to (3) is now repeated for $S_r(x)$. While we may not now have $f(\tilde{n}(y), \tilde{n}')$ independent of \tilde{n} , where $\tilde{n}(y)$ is the inward unit normal to $S_r(x)$ at some given point y on its surface, we should at least be able to manufacture* a surface so that $f(\tilde{n}(y), \tilde{n}')$ is independent of y . Assuming this done, we repeat the general arguments leading to (5) of Sec. 2.7. Thus the radiant flux recorded by the spherical collector, when radiance is incident on $S_r(x)$ over small solid angle Ω_i about \tilde{n}_i (see Fig.. 2.17), is given by:

$$N_i \Omega_i f(\tilde{n}_i, \tilde{n}') \cdot \int_{S_r(x)} f(E(Y), \tilde{n}') dA(Y) \cos \theta_i$$

where $S_r(\tilde{n}_i)$ is that part of $S_r(x)$ such that $\tilde{n}(y) \cdot \tilde{n}_i > 0$, i.e., $S_r(\tilde{n}_i)$ is the illuminated hemisphere of $S_r(x)$. Let us write:

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$$C_r = \int_{S_r(x)} f(\tilde{n}(y), \tilde{n}') \cos \theta dA(Y) \quad (16) \quad S_r \text{ CC.}$$

*If the y -dependence is not eliminable, then the analysis leading to (17) can be extended quite readily by using bounds on the y -variability of $f(\tilde{n}(y), \tilde{n}')$. Hence (17) will

be generalized to bracketing inequalities around $h_4 \tilde{n}(x)$.

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It is clear that, by our hypothesis about f , C_r is independent of \tilde{n}_i . Geometrically, C_r is the effective cross section area of the sphere presented to the beam of light in Fig. 2.17. In the case of $S_r(x)$ being ' an ideal cosine collector, we have

$$f(\tilde{n}, \tilde{n}') = \frac{1}{4} \quad \text{for all } \tilde{n} \text{ in } \Omega_+, \quad \text{so that} \\ C_r = \pi r^2. \quad \text{For real collectors, } C_r < \pi r^2.$$

We now sum the terms $N_i \Omega_i C_r$ over all i to obtain:

$$C_r \int_{\Omega} \tilde{n} \cdot \sum_{i=1}^n N_i \tilde{n}_i d\Omega$$

which we recognize as $C_r h(x)$, the total radiant flux recorded by $S_r(x)$. The recorded spherical irradiance is then, by (15), given as:

$$\frac{C_r h(x)}{4 \pi r^2} \quad (17)$$

which is a generalization of (14). Thus if a spherical collector is used to measure $h_4 \tilde{n}(x)$ the connection with $h(x)$ is still the simple linear type of connection given in

(14) from which the constants may be divided out in practice. That is, as the spherical collector moves down into a lake, say, we have by (17), the following useful relations

$$h_{47r}(Y) = h(Y) = u(Y)V(Y) = u(z) \frac{4 \pi r^2}{z} \\ z \quad h(z) = u(z) v(z) u(z) \quad (18)$$

for every pair of depths y, z in the medium. The last equality follows if the index of refraction is constant with respect to depth. The linearity summarized in (17) is the most important feature required of a spherical irradiance meter or for that matter, any other meter used in applied radiative transfer theory. Thus while it is generally too much to

ask that a spherical collector $S_r(x)$ measure exactly $h(x)$ or $u(x)$, we certainly can obtain these quantities to within known, fixed numerical factors using the general connection between $h_{4f}(x)$ and $h(x)$ given in (17).

The preceding analysis was based on the agreement to use a spherical collector. Actually the theory of Example 15 Sec. 2.11 allows any of a wide class of collectors to measure not only $h(x)$ but $N(x, \cdot)$ as well. Thus, for $h(x)$ one may use, by the argument leading to (72) of Sec. 2.11, the relation $h(x) = \frac{1}{4\pi r^2} \int_{\Omega} H(x, \theta) d\Omega$ (17a) if one already has a flat plate irradiance collector. This may also be extended to practical measurement conditions analogous to (17).

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Radiance

We conclude the present discussion on radiometric concepts with some practical observations on the operational definition of radiance. Following the discussion of Sec. 2.5, we fit a radiant flux meter with a cylindrical tube so that each point on the collecting surface $S(x)$ of the radiant flux meter is exposed to radiant flux incident in a small set $D(\sim)$ of directions about the unit inward normal to $S(x)$. If the radiant flux reading of this assembly is $P(S(x), D(\sim))$ then we agree to write:

$$N(x, \sim) = \frac{P(S(x), D(\sim))}{A(S(x)) Q(D(\sim))} \quad (19)$$

With $S(x)$ and $D(\sim)$ understood, the definitional identity for radiance is simply. - $N = P/ASS = H/Q$

where,
 $H = P/A$

Thus we see that the measurement of N requires a knowledge of the amount P of radiant flux collected, the area on which it is collected, and the solid angle Q within which it is collected. Since the latter two parameters can be instrument constants it is evident that the technique for measuring relative radiance requires:

- 1) a collecting area of fixed size,
- 2) a limited solid angle of fixed size, and
- 3) an electro-optical coupling system whose characteristics are invariant or known under wide ranges of change of the light field levels to be measured.

An important feature of radiance distribution measurements is the detail or resolution with which they are made. This detail is governed by the size of the solid angle of acceptance of the optical system. If the solid angle is small the resolution will be high but the flux collected will be small. Alternatively a large solid angle will collect a large amount of flux but will not accurately report the directional structure of the light field. A natural lower limit for the resolving power of a radiance photometer might be set by the angular subtense of the sun's disc which is 34 min 4 sec of arc (average) in air, or about 24 min from an underwater station (underwater solid angle equals 0.005277r). This level of resolving power would make it possible to describe the underwater light field in the vicinity of the sun with great detail if the air-water interface remained perfectly flat. In a more practical situation, however, the air-water interface would be rippled and the

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image of the sun would be replaced by a glitter pattern whose size would depend on the recent wind history at the surface and to some extent on the angle of observation.

Under these conditions an angular subtense equal to half the angular size of the glitter pattern might give adequate resolving power. Some information on the size of glitter patterns has been given by Cox and Munk [57].

At great depths in optically deep water the glitter pattern will not be observable. The selection of resolving power of the radiance tube then depends only on the accuracy with which one desires to determine the average relative values of radiance at any point along the radiance distribution curve.

As we have seen, the restriction of the solid angle of acceptance can be accomplished by means of a radiance tube or by an appropriate optical system. The radiance tube was suggested in 1936 by Gershun in his paper on the Light Field [98] and is frequently referred to as a "Gershun tube" for this reason. It consists of an internally baffled and blackened tube constructed so that internal reflections from the walls of the tube cannot reach the detector. The angle of acceptance of a radiance tube is defined by a simple geometry as shown in Fig. 13.2. Note that the angle γ_1 is the solid angle within which every point of the area A collects flux. The angle γ_2 establishes the penumbral limits of the device. Rays within the shadowed area will be seen by only a portion of the detector area.

The theory of the baffle tube is illustrated in Fig. 13.3. For effective baffling it is necessary that the baffles be closer together at the forward end of the tube. At sufficiently large entry angles the rays are not specularly reflected from the black paint, and subsequent multiple reflections from wall to wall of the tube quickly reduce their contribution to the collected flux. To quantitatively describe the requisite baffling, observe that baffles A-deep, spaced B-apart at locations (1) and (2) in Fig. 13.3 will eliminate

the rays between the angles $\tan^{-1}(D/L)$ and $\tan^{-1}(2A/B)$. Now, $\tan^{-1}(2A/B) = \tan^{-1}[D/(L-(B/2))]$ and:

FIG. 13.2 Placement of baffles in a radiance tube.

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FIG. 13.3 Theory of radiance tube baffles.

$$\tan^{-1} \frac{2A}{C} = \tan^{-1} \frac{D}{L - \frac{B}{2}}$$

$$C = L - \frac{B}{2}$$

Hence $\tan^{-1}(2A/C) = \tan^{-1}[2A/B]$, and all rays parallel to ray e-f across the aperture D will be stopped by baffles (1) or (2). Next we observe that:

$$\tan a = \frac{B}{2} \quad \tan b = \frac{A}{C/2}$$

and since $b > a$, we have:

$$A > A$$

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so that:

$$\frac{1}{C} \quad \frac{1}{B} \quad \text{or} \quad C < B$$

which shows how the baffles, in order to be effective, should begin to crowd closer together as they near the opening of the tube.

Some experiments with early models of radiance meters brought out a rather unexpected disadvantage of the radiance tube for underwater measurements when the radiance meter's tube is of great length. To obtain the radiance distribution accurately at a point underwater with a meter which has a relatively long tube, the radiance meter should not be rotated around its detector end, but rather it should be rotated about the forward end of the radiance tube. If, as is usual, the radiance meter is rotated around an axis at the detector end of the radiance tube, then the radiance measured in each direction will be effectively for a different depth, namely the depth of the forward end of the tube. Since the variation of radiance with depth in relatively turbid media is quite marked over short distances and since it is nonlinear and different in every direction, correction of the data to a single depth under such circumstances would be very difficult. The use of a carefully designed optical system for restricting the solid angle of acceptance makes it possible to obtain the same resolution with a shorter tube, thereby alleviating this difficulty. In this connection, appropriate internal baffling and black finish are just as important for an optical system as for the baffle tube. In any case, for very accurate results in water having relatively large volume attenuation function a , measurements with an optical system may possibly have to be corrected for significant depth changes at the end of the measuring tube, as the tube scans its environment.

In closing it may be observed that radical new radiance distribution measurement techniques may be possible by adopting some of the observations of Examples 14 and 15, Sec. 2.11, built around novel collection devices and Legendre polynomial analysis and synthesis. Another interesting possibility is the use of Fourier analysis techniques on the radiance distribution $N(x, \cdot)$ over the compact set Ω of the directions

In particular, the sampling theory [29] coupled with resolution requirements may obviate the need for scanning each point of Ω , but only a discrete set of appropriately placed points in Ω . This of course holds also for the Legendre polynomial analysis, and indeed, for any orthogonal family of functions defined on W .

Finally, it may be noted that the time-consuming sweep over all directions in order to determine radiance distributions may be altogether

eliminated by employing a "fisheye" lens which maps each ray in a fixed incoming 2π solid angle into a unique point in a small circular area at the base of the lens. This lens would allow an instantaneous photo-recording of a hemispherical region of radiance values, which may be analyzed by a specially programmed computer. Such a device has been studied by John Tyler and Raymond Smith at Scripps Institution of Oceanography.