13.3 Operational Definitions' of Path Radiance's and Path Functions

The path radiance and path function were conceptually formulated by means of the interaction principle 'in Sec. 3.12, and so are firmly grounded in the basic principles of radiative transfer theory. We now turn those conceptual formulations into useful operational definitions o f the path .radiance and path-function.

Operational Formulation of Path Radiance

There are two general ways in which we may obtain the path radiance Nr (z , ~) of a path O (x, ~) in an optical medium, as depicted, e.g., in Fig. 13.-5. Each method is based on the fundamental relation:

N(z.,E) = N(x,E)Tr(x,~) + Nr(z,E)

which is an instance of (5) of Sec. 3.13. Suppose that it is possible ,to set:

then (1) yields Nr (z , ~) immediately. . This is the dark target method of determining N*(z,-~). Workable equivalents to the

FIG. 13.5 A path ~r(x,E) with initial point x, internal point y, and end point z.

condition N(x,E) = o may be obtained by actually viewing a small black light trap placed at x.

The alternative method of finding N^*(z,~) is to first determine the beam transmittance of $\sim_r$ (x j) , as in Sec. 13.2.

Then, (1) allows the rigorous computation of N.** (z, Q in the form:

N*(z~) - N (z., ' N(x,~)Tr(x,)

assuming all three quantities on the right in (2) are measurable. The two radiances on the right in (2) may be as they are found in nature, or as generated by artificial sources.

Operational Formulation of Path Function

An operational formulation of N*(x,~) can be based on (15) of Sec. 3.12. For suppose $()_r$ (x, E) is a path so short that the radiance distributions-over it are independent of location and that for all practical purposes T_r(x,Q = 1. Then, very nearly:

in which N_ff(x,E) may be determined by means of the preceding dark target method or by means of (2). In general, (15) of Sec. 3.12 may be written rigorously as:

Nr (z , ~) = N_* (x, ~-) r + o (r) (4)

where o(r)/r-}O as r -+ o (cf., (2) of Sec. 3.12). From (3) follows for small r. this,,

An alternative means of finding N*(z,~) under special lighting conditions and in homogeneous media is given by (1) of Sec. 4.3. For, then:

over a path 9,(x,E) (as in Fig. 3.33) which lies in a homogeneous, uniformly lighted stretch of optical medium. Knowing Nr (z, C) and a, then yields N_* (x, ~) .

Formulation (5) casts light on what is meant in (3) by "small r." We shall consider r "small 's when ar <1 , i. e. , when r <1/a = L_a, where L is the attenuation length of the medium.

Actual measurement procedures leading to N*(x,~) can be based on a device of the kind depicted in Fig. 13.,6. The device consists of a radiance meter directed into a black target. The ambient radiance distribution N(x,•) at a general

FIG. 13.6 Dark target arrangement for determining path function value N_* (x, C)

point x is scattered into the direction E by the material comprising the cylinder around the short path segment Qr(x,O (shown shaded) between the radiance meter and the black target. Conditions can usually be arranged so that the assumptions leading to (3) are satisfied with reasonable accuracy. An illustration of a path function meter of the kind described here may be found in Ref. [80].

Further, by combining (2) and (3) we have very nearly:

$N(x,E)T_r(x'O$

the setting for which is depicted in Fig. 3.33. We shall adopt (3) as the operational definition of N,jx, ~) .

Still further, if the volume scattering function is known, then it follows at once from (8) of Sec. 3.14 that:

n

N_* x, NCx~ Oc^rCx.$~_i$.C^6 i-1

for each j, j -_ 1, . . . , n over some suitable partition

... , M_n of W. Hence by measuring the functions N(x, •) and

6 [x; • ; •) , N_* (x, •) can be obtained by direct computation. Computations of this kind were explored in Ref. [213].

Finally, the K-method of determining the path function may be used in virtually all real media. The K-method, as

developed in Ref. [219], is based on the canonical form of the equation of transfer. To illustrate the method, we consider an arbitrarily stratified plane-parallel medium. The associated canonical form of the equation of transfer is given by (21) of Sec. 4.5, and is repeated here:

N (z,)

a(z) + K z, cos 9

Solving this for N_*(z,E):

N*'(z.~) = [a(z) + K(z,E) cos 9]N(z,0 (7)

Hence if all quantities on the right side of (7) are known, or determinable, then N_*(z,,E) is determinable. Examples of the use of (7) using real data are given in Ref. [219].