13.5 A General Theory of Perturbed Light Fields, with Applications to Forward scattering effects in Beam transmittance Measurements

A close study of the preceding three sections would suggest that the problem of the measurement of the optical properties of a given medium is complicated by the fact that the act of measurement perturbs the distribution of radiant flux in the immediate vicinity of the measuring apparatus. Consequently, the numbers derived from a measurement process may not faithfully reflect the inherent optical properties of the medium under study, but rather contain along with the information sought the effect of the presence of the measuring apparatus. In the present section we shall develop a general formulation of the equation of transfer for a perturbed radiance field--in an arbitrary optical medium. The resultant theory is then applied to the problem of the measurement of the volume attenuation function and the beam -transmittance in natural waters, The treatment is sufficiently general to hold in any natural optical medium, in particular the various natural hydrosols of the earth and the atmosphere.

The theory we shall develop below leads to several new measuring techniques for the volume attenuation function a which take into account the perturbation effect on the light field of a standard measuring apparatus used for the determination of a. In addition, the theory provides a means of consistently estimating the relatively elusive forward scattering value o_o, of the volume scattering function o. Finally two criteria are given for estimating the order of magnitude of the forward scattering effects encountered in beam transmittance measurements.

Introduction

It is a cardinal axiom of experimental physics that the act of observing a given phenomenon necessarily disturbs the phenomenon under observation. It follows that the "true" nature of the observed is obscured by some such disturbance generated by the observer. This axiom holds in particular in the field of experimental radiative transfer. An important illustration of this is afforded by the operational procedures, such as those discussed in Sec. 13.4, for the determination of the volume attenuation function a. By way of introduction to the present methods of circumventing the effects of these perturbations, the procedures for finding a will be briefly reviewed. The general theory of a perturbed light field is then formulated and applied to the case of the determination of a, which results in the perturbed light field counterparts to the classical procedures. The theory developed below yields five distinct approaches to the problem of the determination of a, each of which may be transformed into an operational procedure.

The procedures for determining a discussed in Sec. 13.4 assume that the light field is unperturbed or perturbed in an inessential manner as the probing for the relevant information goes on. By ignoring such perturbations an investigator

is rewarded with analytical formulations of ohm-law simplicity, as we saw., e. g. , in (5), (8), and (9) of Sec. 13.4. The price for this is paid by having the resulting prediction .curve for o~ more often than not pass unconcernedly through an array- of nonconformist data points. The two main. techniques now in use may be classified as the bright-target and dark target techniques.

Figure 13.7 depicts the essential geometrical elements of each technique. In the bright-target technique, T is a. self-luminous target viewed by a radiance meter G at a distance r. It is assumed that the target is angularly small-in fact, of zero solid angular subtense--when viewed at each point of the path Cd_r between T and G: In addition, the effect of the ambient light field is removed by either a direct shielding of P from its surrounds or by taking the difference of the G-readings found by turning T on and then off. This then is the bright-target method, of Sec. 13.4, resulting in (10) of Sec. 13.4._ The assumption is made that this on-off procedure does not perturb the ambient light field. Finally, G is assumed to be an ideal collector: any flux entering G and not on
is not recorded. All these assumptions combine to reduce the general equation of transfer:

FIG. 13. 7 Illustrating the geometry of the perturbed light field. T is a generalized target, G is the Gershun tube of a radiance meter. Both T and G induce a perturbation of the
radiance distribution about a point p on the path 9. In the general case, T need not be on the axis of G. The subsets of the direction space M about p which are occupied by the directions of the target, the Gershun tube, and the perturbation are indicated in the fgure .

$dN/dr = - aN + N_*$ (1) to the particularly simple form:

$dN/ dr = -- aN$

for the radiance along i . Thus if N_o and N are the inherent and ¯apparent. -radiances of T along $.Cd_r$, then the operation,

(which follows from the preceding approximate form of the equation of transfer) on the measurable quantities r, N_r, N_o is taken to. yield the required value of a.

The-dark-target approach, on the other hand, assigns zero inherent radiance to T. The same. assumptions adopted above remain in force . In addition, is chosen so that N* is constant along b. . As we saw in (8) of Sec. 13,.4, the solution of (1) leads ʳto the following operation,

N -N
1 gn ~.r = a
r N
q

on the measurable quantities $N_q = N^*/a$, N_r, and r, and is taken to yield. the required value of a.

General Representation of a Perturbed Light Field

In actuality, the placing of a target T of inherent radiance N_o in the light field perturbs the light field. Further, the target, being a ,material object, occupies a finite volume
of space so that it fills a finite subregion (r') of direction space E as viewed at each distance r' on o"'. From this vantage point the presence of T causes a perturbation sensibly extending over a subset, say $_p$(r') of 2. Finally the tube of the radiance meter, being a material object of finite dimensions, will also contribute its

share to the perturbation and, in addition, will record flux entering G. which is not strictly on (9r; the collection of-such directions which carry accept able flux will be denoted in general by "W_g (r')",

With these observations in mind, the general equation of transfer is replaced by the following form for the new context:

r I ~ = - a (r') N' (r' , ~ + N' (r' ~ ~ a (r ' ; ; ') dQ (')

Here, and in the sequel, the symbols "N", and "N" will denote the perturbed and unperturbed radiance functions, respectively. Thus the preceding equation governs the rate of change of N', as it is actually measured by the real radiance meter in the perturbed light field. The equation may be rewritten more compactly as

where in the case of an instrumental perturbation we write:

"a' ($_r$ I) I t

for a(r') - fW W) a (r '-~s~')dQ(~')

and where, as usual:

r

N_* (r') j N(r' C')a(r^z .,

This is the general equation of transfer for a perturbed light field. Its domain of applicability is quite wide. The notion of radiance meter is here intended to cover all types of radiance detectors, including such organic detectors as human eyes. Since all material radiance detectors occupy finite regions in space, they always give rise to a perturbed a, namely the a' of (4). In view of this fact one can raise the question: Is it meaningful to talk about a "true a" in practical contexts? Any operational procedure designed to determine the a of a medium must necessarily be made through the intermediation of a physical recording apparatus. I t is, therefore, meaningful to talk or think only about the a' for that instrument, or collection of a' values relative to a given collection of radiance detectors. The "true a" is therefore a constitutive rather than an operational concept, a useful fiction with which one may create the theory of attenuation and about which one may conveniently cluster for reference the operationally obtainable a' values.

An examination of (4) indicates that the problem of the determination of a is intimately connected with the determination of the forward scattering values a (r' ; ~; ~) (which will henceforth be denoted by "$_{ao}$ (r')" or ' t a_4") of the volume scattering function a. Here again the physical limitations of the relevant instruments, in this case the a-meters (cf., Sec. 13.6),

prevent an exact determination of a_o(r'). Even if an instrument could, by some clever ruse, be forced to look directly down the one-dimensional path to the primary source, what principle will allow the separation of the so-called forward scattered flux from the unscattered , transmitted flux? This raises the question: Is such an attempted separation meaningful in practice in which steady state fluxes are measured? The answer, clearly, is that it is not. For a discussion of this matter, see Sec. 18 of Ref. [251] and the summary below. But yet, even with strict experimental justification absent, there appears to be some unavoidable compulsion to conceptually decompose the forward flux into ~scattered and unscattered components. The motivation for such a procedure is apparently an esthetic requirement: one in which the gap in the experimental definition of the a function for the singular forward direction be closed by the inclusion of the value a_o(r').

The desirability of obtaining a_o clearly stems from the fact that it is an important constitutive construct that is, one which helps bring order and completeness into the classification and theoretical study of turbid media. It is with this in mind that the study of or_b is carried out in conjunction with the study of operational determinations of a.

From the preceding observations, it is seen that even on a phenomenological (or macroscopic) level, the study of light is beset by limitations on the exact experimental determinations of the three basic notions: N, a, and a. The seeming indeterminacy of N .may be sidestepped in principle by defining N operationally as the apparent limit of a sequence f N_n } of radiance functions given by a sequence of radiance meters which approaches as a limit the ideal (a_g = 9) radiance meter. The corresponding operational values of a and 6_o, how-ever, are determined only after more elaborate procedures are specified; their operational definitions are subject, in the sense explained above, to some quite fundamental difficulties. Some relatively simple ways in which these difficulties can be overcome on a practical level will now be considered.

Linearized Representation of Slightly Perturbed Light Fields

As an illustration of the use of the general representation (4) of a perturbed light field, we reconsider the problem of experimentally determining the volume attenuation function a. The discussions which follow apply to arbitrary optical media, e.g., natural hydrosols or aerosols. The equipments used in the usual procedures have been designed so as to minimize, within reasonable limits, the induced perturbations. Nevertheless, small but detectable perturbations are encountered. We shall assume that such perturbations may be represented by certain linearization conditions imposed on the general structure of (4). Towards this end we postulate three conditions for a linearly perturbed Light field:

(i) The functions a (r' ; E , -) and a (r' ; • ; ~) are constant over g (r') and _E_t (r') , respectively, for each r' , 0 < r' < r

(cf., Fig. 13.7).* Both have the fixed value o

(ii) The radiance functions N' (r' , •) and N (r r , •) are constant functions over W_p (i-') for each r' , 0 _< r' < r; the fixed values will be denoted by t^1N r (r !) " and r'N (r $_1$) rr , respectively.

(iii) $=_p$ (r r) - = y t (r r) for each r' , 0 s r' < r.

Under these assumptions the general equation (4) is transformed into the-following relatively simple statement:

dNr r' = - a+a0(Qg(rr) + Qt(rr)) Nr (rr)-N(rr)o0Qt(rr) + N * (r r)

where we have written:

I f $_{og}$($_r$I) «

for do (E) $~_g$(rr)

r
for do (9)
~, ($_r$r) rl
The general solution of (5) is readily obtained:
Ito t(r1)11

og(r1t) dr" + f ot(rI1) dr"
0 0
These formulations point the way to several novel measuring techniques for a. While the formulations are admittedly approximate in the sense made clear above, they take due
cognizance of the major features of a perturbed light field and the more offensive nonideal features of real radiance meters. In this way an essential advance beyond the classical a-measuring procedures seems possible. Despite the simplified character of the linear theory, its resultant formulations lie *The salient consequence of this assumption is that a_o will be an average of o(rr; ;•) over [77] (r') (rather than
a (r' ; E; E)) as found by means of experiments 3 to 5 described below.

just on this side of practical utility. That is to say, within the present experimental framework, the slightest relaxation of the linearizing conditions pushes the resultant theory close to, if not beyond the borderline of reasonable tractability.However, this observation need not hold when it-comes to using more general approximations based on (4) as a theoretical tool.

Application to Bright-Target Technique

For the case of the bright-target technique for measuring a, (6) takes a particularly simple form which is based on the following considerations:
(i) The realization of the bright-target technique in practice is in the form. of a so-called- a-meter (or beam-tran8mittance meter) which consists essentially in the optical system depicted in Figure 13.8. The most important feature for the present discussion is the condition 0_g(r') = Sg_t (r') which this optical system is designed to impose on the two solid angles defined in the general theory. This equality of solid angles follows from the effective placement of the source at the focal point of lens A so that the source is essentially imaged on lens B. Thus the solid angle 92 (r') (or $\sim 2_t$(r')) is necessarily the solid angle subtended by the farther lens from the point at distance r' from lens A.

Cylindrical envelope of rays To Photoelectric element
FIG. 13.8 The optical system of a a--meter (or beam-transmittance meter), in particular, one in which Q_t (r')
= Sa_g fir'.

(ii) The path 6>r between A and B has been designed so as to be shielded from the surrounding natural light field. From
this, it follows that N* (r') = N (r') = 0 on O_r. With these observations, (6) reduces to
N' (r) = TIN' (0)
so that

N' (r) /N' (0) = Tr = exp { - ar + 2wa$_a$0 (r) j (8) where we have written:
for 2 r r 21 _ $_a$2 + r2 1/2 + $_{\cdot a2}$ + (1212 1 /2)
and where a is the common radius of lenses A and B. For ratios a/r <_ 1/10, (8) may be usefully represented by the approximation:
!
T' = exp - ar + 27ra ar (9)
Either (8) or (9) show that the perturbation of the light field leads in this case to an apparently reduced value of a as measured by the instrument, e.g., from (9)
a'
a - 27a
(9a)
An apparent reduction of a will also be found in the dark target case. In fact, the general equation for the perturbed light field shows that this apparent decrease in a is a universal manifestation traceable directly to the simple fact that for all material radiance meters, and all material targets, Sa$_g$ (r) and Q$_t$ (r) are greater than zero.
Application to Dark--Target Technique
In the case of the dark-target technique, the path is chosen so that N$_*$ and N are constant over its extent. In fact, we set N = N$_*$/a = N, and N' (0) = 0. ~2t(r) and 2 (r)
are no longer rigidly coupled as in the bright-target cise, but now assume the forms
$_r$r
- (r-r')
(b^2 + (r-r,)2) I/2

governed by obvious geometrical requirements. Here a and b are, respectively, the radii of the target and the circular collecting surface in the radiance meter (see Fig. 13.7).
Under these conditions, (7) becomes:
Tr, = exp { - ar' + 27rQ$_a$~(r,r')) (10) where we have written:
for 2(r-rl)+b-(b2+(r-r')2)1r2-(a2+r2)1/ 2
) 1/2
+ (a2+r
and (6) becomes
r
N' (r) = N$_q$ (Tr) - f [a-a OQt (r')]Tr, dr' (12) 0
The chore of carrying out the integration of (12) can be considerably reduced, and the utility of (12) considerably enhanced, if the following approximate representation of ~(r,r') is adopted:
~(r,r')
a 2 2r
(13)
which follows from using 2r'-r instead of
- $_{(b}$2 + ($_{r}$-$_r$,)2)1/2 + (a,2 + r, 2) 4~2
and
instead of
$_{(a}$2 + r2)1/2

The justification for these approximations, especially that for 2r'-r, may be illustrated by considering a numerical example in which the quantities have magnitudes which are typical for dark target experiments. Thus let a/r = b/Q = 1/30, where Q = 0.30 meters and Z is the length of the radiance meter's collecting tube, and r = 10.70 meters. The graph of Fig. 13.9 shows a plot of 2r '-r and the graph that it is to approximate.

By means of (13) , the form for N' (r) becomes

N' (r) = N_q exp 2 T r o $_{a1}$ b
r
- a2 (1 - $_e$- ar$_)$, Q e a (r_ r') S~ ($_r$ t) dr f
2r o
)] L a t
(14)

FIG. 13.9 Illustrating the approximation which simplifies the analytical formulation of the dark-target technique in typical applications,

which is still a formidable analytical haystack from which one must extract a ando_o.

Some ways in which this may be done will now be considered.

An Outline of Possible Experimental Procedures of a in Perturbed Light Fields

In the task of measuring a and ain light fields which have been measurably perturbed by the-probing instrument, we can distinguish five possible procedures yielding a:

I f CF. is known and a is sought

Experiment I. Single use of an a-meter. Experiment 2. Single dark-target experiment.

If both a and a_o are sought:

Experiment 3. Two a-meters used simultaneously. Experiment 4. Two dark-target experiments conducted simultaneously.

Experiment 5. Simultaneous a-meter and dark-target experiments. We consider each experiment in turn, but outline only the principal analytical steps required in-each case.

Experiment 2. The procedure is straightforward: Knowledge of N' (r) /N' (9) and o_a allows an immediate estimate of a from (8) or (9). Foreknowledge of 6_o may be obtained by
using the o-recovery method (Sec. 13.6).

Experiment 7. Equation (14) may be 'written as

N -N' (r)
= F(a;a,b,r;a_o) (15)
q

The function F has a as main argument, with a,b,r and a_o as parameters fixed during a given experiment. It is-therefore possible to mathematically compute and plot the values F(a;a, b, r,o_o) (Fig. 13.10(a)) from which the a may in principle be found.

Experiment 3. Suppose two a-meters, each having a distinct ~-function (8), simultaneously measure beam transmittance in a given medium. Then, if r_2;
and Trip

T'$_2$, represent the appropriate quantities of each instrument, tKe two equations for a and a $_o$
T' = exp - ar . + 2 zra
r. ~ o

have the solutions:
ao = (r$_2$ Ln Tr - r$_1$ Ln Tr) /2n(r$_Q$ ~ - r$_1$ ~ 1
1 91
2
a = ~~$_2$ Xn Tr - ~~ An Tr)/(r$_2$ 0$_1$ - r$_1$ 0$_2$) 1
Experiment 4. Equation (14) is the basis for this experiment. The general idea is to obtain two readings of N'(r) from two distinct geometrical arrangements, and then solve two simultaneous equations for a. and a$_o$. No simple closed expressions for a and a$_s$ may be obtained as in Experiment 3. Recourse to numerical solutions is generally the only way out. The numerical procedure will be considerably simplified if each geometrical arrangement is so made that b-(a^2/2r) = 0. This may be done in the following way. Let X and b be, respectively the effective length and the radius of the radiance tube (see Fig. 13. 7). Then the radius a of the dark target is given by.a = rbk/k, where k ? 1 (k is introduced to insure the fulfillment of the condition that the target must at least fill the field of the radiance tube). Then the condition b-(a^2/2r) = 0 requires that the range of the target be r = 252/bk2.
In illustration, let b/Z = 1/30, L = 0.30 meter. Choose two k-values, e . g. , k,= ,7, k$_z$ = 2. These numbers now fix a and r for each choice of k:
= 29, ^{2}/bk^2 = 9.00 meters
a$_1$ = r$_1$ bk$_1$/k = 0.432 meters
4.50 meters
a = 0.300 meters.
2
Now returning to (14) in which b-(a ^{2}/2r) = 0, we have for two such experimental arrangements:

N $_q$ - N'~ r N$_q$
H a ; a , b , r , c $_o$)
FIG. 13.10 Two numerical procedures for determining . Part (a) : Experiment 2; Part (b) : Experiment 4 (see text).
c (r$_1$) = (N $_q$- N' (r,))IN $_q$ = e ' + a0I ($^{a, a}$1 , r,)
-ar
c (r~) _ (N-N' (r))IN = e + aI (a, a, r
q 2q o 2 2)
ri -a(r. -r')

e I n t (r ') dr'
This set of equations may be arranged to read:

-~ ar

Each side represents a computable function of a (the ai and ri being known and fixed during the experiment). Hence both A and B can be graphed over a certain domain of a values. If the curves are graphed over the same set of axes (Fig. 13.10 (b)) the point of intersection of the graphs defines the required a. The graph of Fig. 13.10(b) merely represents the idea of this solution procedure; it need not represent an actual set of A and B graphs.

Experiment 5. Equations (8) and (14) are the basis for this experiment. Equation (8) is solved for a and the resultant expression for a_e is substituted in (1~). After this is done, the remaining procedure is in principle covered by the analytical steps outlined for Experiment 2.

Order of Magnitude Estimates

A given bright or dark-target arrangement can be given a quick preliminary analysis by means of equations which approximate (8) and (14). The appropriate equation for (8) is given by (9). Turning to (14) for the purpose of obtaining a useful approximation, we see that a lower bound on the a_o effect may be obtained by setting Si_t (r') = Si_t (r) so that (14) reduces to:

N' (r) = N_q exp 27T6_0 (b - a^2)] . (1 - $_e$- ar 1 - '6 e (a 2 2r) a r '

(16)

Thus if (16) predicts a measurable deviation from the unperturbed radiance N (1-e^-ar), the actual a_4-effect produces an even greater deviation (an illustration is given below). Unfortunately, a correspondingly good upper bound is not found in such a simple way, so that no general bracketing expression can be simply given for N' (r) in the dark-target case. Each problem is best handled separately using (14), or its variant (15).

From the preceding analyses it is evident that the a-meter technique (i.e., the bright-target technique) appears to be the more simple to handle analytically. From (9a) one can estimate the percent difference between a and a':

(17)

the minus sign denoting that a' i s always less than a.

To gain a rough idea of the order of magnitude of the forward scattering effect in a typical hydrosol and aerosol,

we choose for the hydrosol: a_o .= 1.92/meter steradian; a w 0.402/meter; * and for the aerosol, a_o = 9 x 10^{-4}/meter-steradian, a = 32 x IO^- s/meter. ** The corresponding ,values of A for a set of a-meters (characterized by their a/r ratios) are given below.

a/r	- 0 (in percent)	
	Aerosol	Hydrosol
1/200	0.044	0.075
1/100	0.176	0.300

1/50	0.704	1.20
1/25	2.'82	4.80
1.12.5	11.3	19.2

We conclude with an example of the use of (16) for the' case of the present hydrosol. Using the set-up suggested in Experiment 4, and observing that the path lengths were chosen s o that b - $(a^2/2r)$ w 0, we have

-ar_e -ar,

N' (r~) = Nae° (l - e (1 - 0.040) _ (4'. 96)N_q l - e , r = 9 meters.

-ar_2

N' (r 2) = N a e° 1 - e) (1 - 0.080) = (0.92)N $_a$1 _ e-ar2 r w 4.5 meters.

2

It appears that a definitely measurable perturbation of the light field would be induced in the present case. Thus, some radical procedure, such as that outlined in Experiment 4, should be followed in order to obtain an accurate estimate of a.

*The a and a_s are associated with a wavelength of about 478 mp; a and a_o are based on measurements taken by J. E. Tyler in Lake Pend Oreille and are representative of moderately clear lake and near shore ocean water (Sec. 1.6). Depending on the medium, the ratio a_o/a may range over several orders of magnitude.a $_o$ was estimated using the method of a-recovery (Sec. 13.6).

**Based on Waldram's ([177] p. 48) data for industrial haze, c_o/a for clear air is on the order of a seventh of that for industrial haze. The associated wavelength is 574 mu;ao was estimated by extrapolation, a highly perilous operation.

Summary and Conclusions

The general equation of transfer (4) ~ for an arbitrarily perturbed light field is formulated. From this 'is deduced the linearized equation or transfer (5) which is applicable to the study of slightly perturbed light fields such as those induced during the measurement of the volume attenuation function (or beam transmittance) by means of the bright- or dark-target techniques. The general solutions (6) and (7) of the linearized equation lead to analytical expressions which may be used to estimate the true value of a when either the bright-target approach (8) or the dark-target approach (14) is used. The general solution of the linearized equation yields in particular five possible experimental procedures leading to an estimate of a. Finally, two methods are based on (16) and (17) for estimating the order of magnitude of the forward scattering effects encountered in beam transmittance measurements.

There remains still another possibility for determining a and a for a given optical medium, and that is by using radical radiometric techniques based on electromagnetic theory (see, e.g., Sec. 126 of Ref. [251]). Admittedly, this changes the main rule of the radiative transfer game ("to solve .radiative transfer problems using only geometrical radiometry and the interaction principle"). However if the solution of the problem of determining the structure of the graph of a(x;~;•) for ~' near ~, continues to elude the most incisive transfer techniques, then we must not be too proud to call for help from other fields of mathematical physics. Electromagnetic theory and quantum theory are right next door, so to speak, and surely could cast some light on the qualitative and quantitative nature of forward- and near forward scattering in natural optical media. Our current inability to meaningfully measure or meaningfully predict what happens for scattering angles less than 1/2°, indicates one of the more serious shortcomings of an otherwise complete and elegant theory of radiative transfer. The reader who wishes to see the nature of the c_o-problem is asked to complete the graph of a in Fig. 1.72 in a logically defensible way.

In answering this challenge, the reader is cautioned not to use such shortcuts to a (at least not without justification) as may be suggested by the simple superposition of singly scattered radiant flux components from each member of a dense aggregate of particles comprising the scattering volume. (See, e.g., [308].) It is to be specifically noted that the volume scattering function, as i t is correctly used in radiative transfer theory, is itself viewable as the solution of a complex multiple scattering problem defined within the experimenter's irradiated scattering volume (see Sec. 5 of [251], and the closing remarks in Sec. 13.12, below).