13.10 Theory of Measurement of Local and Global R and T Properties

In this section we invert the usual way of looking at the principles of invariance, and more generally the interaction principle, and show that, by so doing, we encounter new ways in which to measure the inherent and apparent properties of, optical media. We have already used to advantage this point of view throughout all the preceding sections to find operational definitions of such local properties as a, a, a, s, the K functions, R(z,±) and D(z,±). The results of Sec. 7.8 are also pertinent to the present discussion.. Now we wish to show how these procedures can be extended to the complete

set of R and T factors (or operators) for plane-parallel media. As a consequence we shall be able to solve many, problems of the second class in radiative transfer theory (ro: Sec. 2 of Ref. [251]), i.e., problems which require. the determination of the apparent and inherent optical properties (either local or global) of an optical medium, given the radiometric field throughout the medium or on its boundaries (or both). This problem is also referred to more descriptively as the inverse problem of radiative transfer theory. We now go on-to consider some examples of inverse 'problems involving local and global optical properties.

Example 1: R and T Factors in Homogeneous Polarity-Free Settings

To show the. basic point of view taken in the formulation of inverse problems consider a plane-parallel medium X(a,b) in which we can measure the upward and downward irradiances H (y, t) at depths y in X(a,b). Can we make enough measurements, and of the right kind so as to be able to compute the reflectance and transmittance R (x, z) and T (x, z) of an internal submedium X(x,z)? An examination of the principles of invariance for general irradiance settings ((I), and (2) of Sec. 8.1) : - .

$$H(y,+) = H(.z,+)T(z,y) + H(y,-)R(y,z) \qquad (1)$$

$$H(y,-) = H(x,-)T(x,y) + H(y,+)R(Y,x) \qquad (Z)$$

shows that we should first measure the upward and downward irradiances at levels x and z in X(x,z). The relations (1) and (2) reduce in this case to:

$$H(x,+) = H(z.,+)T(z,x) + H(x,-)R(x,z) \qquad (3)$$

$$H(z,^-) = H(x,-)T(x,z) + H(z,+)R(z,x) \qquad (4)$$

in which x <z. Next if it is possible to invoke 'the symmetry conditions:

$$R(x,z) = R(z,x) ='R(Iz-xj) \qquad (5)$$

$$T (x, z) = T(z,x) = T(I z^-x f) \qquad (6)$$

then (3) and (4) clearly permit the determination of R (I z-x I) and T(Iz-xl). This is tantamount to adopting a one-E theory for irradiance (Sec.--8.6)., Thus, assuming that (5) and (6)

hold, (3) and (4) imply:

R(I -
z-xl) H x z + •(7)
= '⁻)H(x,+) H(z, -)H

H^2 (x.,-) - H2 (z,+)

T(i z-xj) H ` H
T H(x,⁻ z,⁻ x,,+)H(z,) (8)

$H2 (x,-) - H^2 (z,x+)$

Observe that in deep natural media, by letting z } co holding. x fixed, (7) implies

and

$\lim_n$ R(I z-x) = .$\lim_{y„}$ R(x, z) = R(x, -)

and

$\lim_{z\_,,}$.T (I z-x)_ = limz-*OT (x, z) = 0

(10)

Furthermore, by letting z -+ x in (7) we have

lim } R z_ x = limn+x R z ~ = b (x) z x x

where "b (x)" denotes the common .value of b (x, --) and b (x,+) , which exists by (5) above and (3) and (4) of Sec. 8.2 (see also (15) of Sec. 1.4 and (11) and (12) of Sec. 8.3). Further, by letting z-*-x in (8) we have

lim 1-T z-~x _ lim} 1-T x z = a(x)D - f (x) [I2)

z}x zx~ z x Z _x1

where "f (x)" denotes the common value of f (x,-) and f (x,+) implied by (6) and (3) and (4) of Sec. 8.2 (see also (16) of Sec. 1.4 and (11) and (12)' of Sec. 8.3). Further, B is the fixed value of the distribution functions. Equations (11) and (12) show that we can estimate f (x) and b(x) by measuring appropriate irradiances in situ along with a. However, some attention. should first be given to the restrictive assumptions about the shape-of the upward and downward radiance distributions. This we shall do in the next example.

Example 2: Homogeneous Media with Polarity

As a second example of the inverse problem in hydrologic optics we reconsider the problem of determining the R and T factors in homogeneous media. We postulate for the present discussion that the symmetry conditions (5) and (6) do not hold. This therefore simulates the empirical setting of the two-n theory of Sec. 8.5, which as we saw is a quite important setting in_ hydrologic optics.

We begin by observing that for each submedium X(x,z) there are generally four R and T factors to be determined. In the present case, therefore, the principles of invariance (3) and (4) may best be used in the form:

(H(x,,+) .,H(z,-)) = (H(z,+),H(x,-))M(x,z)(13)

where M (x, z) is the operator defined in Sec. 7.4, now adapted to the' irradiance context.

.It is clear that we cannot reach our goal of finding the four factors R(x, z) , T (x, z) , R(z,x) , T (z,x) by measuring only the incident and response irradiances at levels x and z, for (13) represents only two equations. We must therefore find two more equations which have as unknowns the same four

factors in M(x, z) . Now one way in which this may be done is to measure the irradiances at the boundary of another slab

X(x', z') for which z' -x' = z-x, i.e., which has the same thickness as X(x, z) . Then, since the medium is homogeneous,
M(x, z) = M(I z-x I)' = M(I z' -x' 1) = M(x' ,z [1]) (see (19) and (2D) of Sec. 8.7). Let us follow this lead to see where it takes the discussion. The matricial statement of the principles of invariance for X(x', z') is
(H(x :+) ,H(Z' ,-)) = (H(z',+) ,H(x' ,⁻))M(x, z) (14)
Now let us write:

for
H(z,+) H(x,-) H(z'., +) H(x

and

J H(x,+) H(z,-)

H (x' ,+) H(z' -)
By means of these definitions (13) and (14) may be represented as:
$H_x = H_z$ M (x, z) (17)
It follows that if H_z has an inverse, we may determine M(x, z)
by means of the relation:

M(x, z) = H- Hx
(18)
What is the physical requirement placed on H_z in order that H_z exist.
First of all, we require that the two vectors:
and
(H(z,+) ~H(x.,-))
(H(z' ,,+) .,H(x' ,,-))
not be linear combinations of one another. That is, there should exist no nonzero real number c such that:
c(H(z,+).%H(xj-)) + (H(z' .,+) sH(x' ,-)) = U
In other words, we must have no c such that:
c H (z,+) + H(z',+) = Q
c H(x, -) + H(x',-) = Q
An equivalent way of stating this is that:
H z + H (x,-)
z' ,+ H 7 x
,
(19)

Thus, whenever (19) holds, we may use (18) to solve for the R and T factors for x(x, z) . How likely is it that the condition (19) holds? A perusal of (22)_ and (23) of Sec. 8-.6 shows that we must stay away from one-D settings in optically deep media. On the other hand, the results of Example 1 of Sec. 8.7 show that two-D settings in deep or shallow media will often give rise to the condition (19). In these latter settings the theory may take over nicely. Thus (18) may be .used to exactly complement the one-D and 'two-D theories in the task of determining the reflectance and transmittance factors for irradiance.

The net result of Examples 1 and 2 is to provide the experimenter with two complementary means of determining the four R and T factors for submedia y(x, z) in a given homogeneous plane-parallel medium X(a,b). The associated local optical properties are then found using (3) and (4) of Sec. 8.2 and their companions leading to (7) of Sec. 8.2. (Note also (11) and (12) of Sec. 8.3.3 Observe that the R and T factors and their local counterparts discussed in the preceding two examples are apparent optical properties and that it was possible to determine these factors only after establishing a one-D or two-D assumption.

Example 3: Forward and Backward Scattering Functions

we consider next the inverse problem which requires determination of the forward and backward scattering functions f (z, ±) , b (z, ±) , when the irradiance flows in a stratified plane-parallel medium are given.

It is at once clear from (9) and (10) of Sec. 8.3 how one can go about finding f (z , ±) and b (z , ±) . We can measure H(z,±) over -a small depth range about various depths z in order to obtain all radiometric terms in (9) and (10) along with their derivatives. However, it is also clear that there are more unknowns than equations which govern them. There is only one possible way out of this difficulty: the measurements must be taken at a minimum of two depths, say x and z, and we require in addition two small miracles to take place simultaneously, namely

f(x~-) - a(x,-) = f(zf_) - a(z,-) (=T(-))

(i}

b(x,-) - b(z,-) (_

and

H(x,+) H(z,+)

det 0 H(x,-) H(z,-)

For, suppose (i) and (ii) hold. Then, by (i) above and (10) of Sec. 8.3 we have:

H (x, +) p (+) + H (x, -) .r(-)

H(z,-+)p(+) + H(z,-)T(-) = dH z'.- z

This may be written:

H(x,+) H(z,+)1

H (z,

dH (x -) dH z

x z

By (ii) we then have

dH z,- H(x,+) z H(x,-)

H(z,+) _, H(z,-)

(2D)

In this way $p(+)$ and $T(-)$ are determinable. A similar equation holds for $p(-)$ and $T(+)$, using (9) of Sec. 8.3. Since
$a(z, \pm) = a(z) D(\pm)$ and a is determinable by beam transmittance, and since $D(\pm)$ are generally known (Sec. 8.5)., $b(+)$ and $f(-)$ then follow from (11) and (12) of Sec. 8.3.
It should be noted that conditions (i) and (ii) above incorporate rather stringent requirements on the irradiance field. Before using the present method, condition (ii), which is the more critical of the two, must be verified. It is clear from the asymptotic radiance theorem that in deep media and for depths far from the surface, condition (ii) is not likely to hold. As in Example 2, we must work near the surface of deep natural hydrosols, or in shallow hydrosols, preferably those whose lower boundaries are visible or at least whose presence is detectable by not having exactly fixed exponential decrease of irradiance with depth throughout the medium.

For example, (29) may be used whenever
$K(y,+) \sim K(y,^-)$ over the depth range $x < y < z$.
Alternate, less fundamental approaches to finding f and b may be based on the one-D and two-D models of Chapter 8. For example, in the one-D model, we have the general relation
$k = [aD (aD + b)]1/2$.Measurements of D, a and k will yield estimates of the backward scattering coefficient 'b. The forward scattering coefficient then follows from the relation
$s = f + b$. Figures 1.41-1.45 may facilitate such estimates. Furthermore, limit calculations based on (3) and (4) of Sec. 8.2 using measured values $T(x, z)$, $R(z, x)$, are potential means of finding b and f.

Example 4: R and T Operators for Radiance
In this example we consider the problem of determining the R and T operators for a submedium x (x, z) of an optical medium x(a,b), given sufficient radiance measurements. The principles of invariance for x(x, z) are
$N_+(x) = N+(z)T(z.,x) + N\_ (x)R(x,z)$
$N\_ (z) = N_+ (z) R(z,x) + N\_ (x)T (x, z)$
Using _the operator M (x, z) of Sec. 7.4, now once again in the radiance context, these principles may be written:
$(N_+ (x) ,N\_ (z))$ _ $(N_+ (z) ,N\_ (x))M(x, z)$ (21)
Clearly (21) by itself is not sufficient to determine M(x,z). For it is not generally possible to predict the response of X(x,z) to every radiance distribution on the basis of only one irradiation as indicated in (21). Therefore we must irradiate $X(_{x,z})$ in a sufficient number of ways so as to extract the essential form of M(x,z). One practical way of doing this is to switch from (21) to its matrix approximant. This tactic was employed repeatedly in our earlier studies and so need not be elaborated here. See, for example, Secs. 7.7 and 7.8 for the details of the transition from (2l) to matrix form.
Thus, suppose + and _ are partitioned into n and m pieces, respectively, as in (1) and (2) of Sec. 7.7. These partitions, in turn, induce decompositions of the N+ (z) and $N_+ (x)$ appearing in (21). Thus, $N_+ (z)$ and $N_+ (x)$ go over into n= component

vectors and N- (x) and N_ (z) go over into m- component vectors. As a result we can write:
11N 11 z
for [N_+(z) , N_(x)1
for [N_+ (x) ,N_ (z)]
and thus N_z and N_x are m + n component vectors. Furthermore the component R and T operators of M (x, z) go over into matrices dimensioned as follows:
n .T(z,x) R(z,x) m 'R (x.* Z) T (x., z) n m
(23)
Thus T (z, x) becomes an n x n matrix T (z ,x) and R (x, z) becomes an m x n matrix R(x, z) , and so on.- Let us denote the resultant (m + n) x (m + n) matrix approximant to M(x, z) as ^{11}14(x, zy".

Now let N^1 , N^2, ..., Nz^{+n} be m+n linearly independent vectors of the :kind defined in (22)
, Write:
and:

N
z
N^2
z

"N ff
z for (24)

$_N$m+ n

z

N'

x
N^2
x
f 1 N 1 T
~ !
x for

$_N$m+n x

where Nx is the response vector to Nl as governed by (21). It follows that,(21) may be written down m+n times, once for each pair (N',N'). The resultant system of m+n equations may be given the compact form:

$N_x = N_z M (x, z$
(26)
Since .N_z is invertible, we can in principle determine M(x,z)

N^{-}~ N z x
(27)
The importance of working with inherent optical properties becomes manifest in the present example. That is to say, when we must change the irradiation pattern on x(x,z) in order to obtain more conditions we require the interaction operators to be invariant under the change of. irradiation pattern. This requires that the same operator M(x, z) appear in (23) and (26). Since the R and T operators for radiance are inherent optical properties, this change of irradiation pattern is permissible. Otherwise, if the operators were apparent optical properties, changing the lighting conditions would serve no useful purpose in the solution of the inverse problem, for with each change in the lighting conditions four more new operators appear and-so there will always be two more operators than radiometric equations available for solution.

General observations on Inverse Problems in Hydrologic Optics

observe that the inverse problem of determining the local optical properties p (y) and T(y) is readily solved in hydrologic optics by using the results of Examples 2 or 3 and the theory of Sec. 7.3. Hence the inverse problem in hydrologic. optics is completely solvable by 'appropriately using the general concepts assembled in Chapter 7.

One important reason why the inverse problem presents no novel difficulties in hydrologic optics (in principle) rests on the fact that the optical medium whose optical properties are sought is directly accessible to experimental probing. This fact was used throughout the preceding examples.

In branches of radiative transfer other than geophysical optics, such as the current fields of astrophysical optics or planetary optics, the problem of determining, say, a and a throughout a stellar or planetary atmosphere is much more difficult when the atmosphere cannot be directly probed internally. Indeed, under such a condition, unless some specific laws governing at least the internal depth behavior of a and a are available, or some equivalent information is available or even good. guesses possible, then the general inverse) problems of the second class, are insoluble.