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The K-method of Determining the Path Function*

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1. Introduction - The purpose of this note is to introduce a third method of the determination of the path function N_*. The two methods already extant are the direct method and the operational method.

The direct method[1] makes use of the basic definition of N_*:

$$N_*(\underset{\sim}{x}, \underset{\sim}{\xi}) = \int_{\Xi} N(\underset{\sim}{x}, \underset{\sim}{\xi}') \sigma(\underset{\sim}{x}; \underset{\sim}{\xi}'; \underset{\sim}{\xi}) \, d\Omega(\underset{\sim}{\xi}'),$$

which clearly requires: knowledge of the radiance distribution $N(\underset{\sim}{x}, \cdot)$ at the point $\underset{\sim}{x}$, and the volume scattering function $\sigma(\underset{\sim}{x}; \cdot ; \cdot)$ at the same point, and an integration process over Ξ.

The operational method[2] is derived from the operational definition of N_*:

$$N_*(\underset{\sim}{x}, \underset{\sim}{\xi}) = \lim_{r \to 0} \frac{N_r^*(\underset{\sim}{x}, \underset{\sim}{\xi})}{r},$$

where $N_r^*(\underset{\sim}{x}, \underset{\sim}{\xi})$ is the path radiance generated by some light field (either artificial or natural) over a path of length r. For example, $N_r^*(\underset{\sim}{x}, \underset{\sim}{\xi})$ may be measured in the air path of a meteorological range meter. In natural hydrosols, $N_r^*(\underset{\sim}{x}, \underset{\sim}{\xi})$ may be obtained from a small set of dark-target setups.

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2. <u>The K-method</u> - The K-method makes use of the canonical form of the equation of transfer.[3]

$$N(z,\theta,\phi) = \frac{N_*(z,\theta,\phi)}{[\alpha(z) + K(z,\theta,\phi)\cos\theta]},$$

and has been successfully tested in recent σ-recovery calculations.[4] By rearranging the canonical form to read:

$$N(z,\theta,\phi)[\alpha(z)+K(z,\theta,\phi)\cos\theta] = N_*(z,\theta,\phi),$$

the basic procedure of the K-method is made clear: From knowledge of N, α, and K (each of which is presumed tabulated), the appropriate value of N_* is immediately determinable by performing the indicated operations. The function $K(\cdot,\cdot,\cdot)$ is determinable by means of its operational definition:

$$K(z,\theta,\phi) = \frac{-1}{N(z,\theta,\phi)} \frac{\partial}{\partial z} N(z,\theta,\phi),$$

the operations being performed on a known radiance function N. A sample of some recent N_* determinations using the K-method are appended below. The function $K(\cdot,\cdot,\cdot)$ has also been found of use in the problem of the asymptotic radiance distribution: <u>A necessary and sufficient condition that an asymptotic radiance distribution exist is that K be a constant function</u> (i.e., independent of z, θ, and ϕ). By means of plots of K, the condition for asymptoticity is thus reduced to a simple visual criterion.

3. A Sample of N_* - values Obtained by the K-method -

	$z = 34.2$ ft		$z = 54.4$ ft	
θ	$N(z,\theta,0)$	$N_*(z,\theta,0)$	$N(z,\theta,0)$	$N_*(z,\theta,0)$
30°	2 540 000	193 000	385 000	29 600
60°	55 900	6 819	24 400	2 880
90°	10 600	1 495	4 050	571
120°	2 940	488	1 010	170
150°	1 410	258	502	91.9
180°	1 180	224	418	79.8
210°	1 260	231	462	85.0

	$z = 95.0$ ft		$z = 135.5$ ft	
θ	$N(z,\theta,0)$	$N_*(z,\theta,0)$	$N(z,\theta,0)$	$N_*(z,\theta,0)$
20°	22 100	1 750	1 680	145
50°	5 110	537	510	53.6
80°	799	105	88.9	11.7
110°	191	30.4	21.8	3.45
140°	79.6	14.3	10.2	1.84
170°	56.2	10.7	7.53	1.43
200°	57.3	10.7	7.86	1.47

4. Some Comments on the Tables - The data for the radiance values $N(z,\theta,0)$ were taken from the lake Pend Oreille measurements (spring 1957) by J. E. Tyler. The lake was essentially homogeneous with an

attenuation function value: α =.141/ft. The sky was clear and sunny with the sun at an altitude of about 42° from the zenith. All the entries in the tables are relative magnitudes. To obtain absolute radiance and path function values all entries must be multiplied by 6.85 microwatts/steradian. It must be emphasized that this number is approximate and not to be used in actual computations. It merely serves here to give an order of magnitude estimate of the absolute values of the tabulated quantities. The units of the path function are in terms of per foot, and the associated wavelength of all quantities is about 478mμ. z is depth below the surface. θ is measured from the zenith. Observe that in all entries ϕ = 0, signifying that the numbers refer to radiance and path function values in the vertical plane containing the sun.

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