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The Contrast Transmittance Distribution as a Possible Tool in Visibility Calculations*

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INTRODUCTION

The purpose of this note is to introduce the concept of the contrast transmittance distribution $\mathcal{T}_1(x, \cdot)$ and to point out its potentially useful application to the general visibility problems in natural aerosols and hydrosols. In particular, we will show that the contrast transmittance distribution:

(a) Can quite generally be assigned the status of an (apparent) optical property of scattering-absorbing media. Furthermore, $\mathcal{T}_1$ together with the radiance function N, may serve to completely document an optical medium for the purposes of solving the problems of the apparent contrast of objects.

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(b) Possesses certain useful analytical properties which are precisely analogous to those of the well-known beam transmittance function T_1.

(c) Can eliminate the need for the explicit determination of the volume attenuation function α insofar as α is required in the customary solution procedures of apparent contrast problems.

OUTLINE OF CURRENT PROCEDURES

The current procedure for determining the apparent contrast $C_r(\underset{\sim}{x}, \underset{\sim}{\xi})$ of a given object is outlined in reference 1. Here $C_r(\underset{\sim}{x}, \underset{\sim}{\xi})$ stands for the apparent contrast of an object at distance r in the direction $\underset{\sim}{\xi}$ as seen by an observer at $\underset{\sim}{x}$. In essence, the current procedure is based on the general contrast transmittance formula:

$$\frac{C_r(\underset{\sim}{x}, \underset{\sim}{\xi})}{C_o(\underset{\sim}{x}, \underset{\sim}{\xi})} = \frac{{}_bN(\underset{\sim}{x}_t, \underset{\sim}{\xi})}{{}_bN(\underset{\sim}{x}, \underset{\sim}{\xi})} T_r(\underset{\sim}{x}, \underset{\sim}{\xi}) . \qquad (1)$$

The beam transmittance $T_r(\underset{\sim}{x}, \underset{\sim}{\xi})$ for a path of sight (of length r, oriented in the direction $\underset{\sim}{\xi}$, and with initial point at $\underset{\sim}{x}$) is determined by first explicitly measuring the attenuation length $L(\underset{\sim}{x}') = 1/\alpha(\underset{\sim}{x}')$ at each point $\underset{\sim}{x}'$ of the path of sight and then combining these L-quantities in the manner shown in equation (16) of reference 1. Knowledge of the radiance distribution $N(\underset{\sim}{x}, \cdot)$ at each point $\underset{\sim}{x}$ of the medium supplies the remaining information to evaluate the right-hand side of (1) which, along with information about the reflectance properties of the given object, finally allows a determination of $C_o(\underset{\sim}{x}_t, \underset{\sim}{\xi})$ and hence the required $C_t(\underset{\sim}{x}, \underset{\sim}{\xi})$.

The weakest link in the presently employed chain of computations leading to $C_t(\underset{\sim}{x}, \underset{\sim}{\xi})$ is the one concerned with the determination of $L(\underset{\sim}{x}')$: the present method requires that $L(\underset{\sim}{x}')$ be determined by a particular case of the following formula:

$$L(\underset{\sim}{x}) = \frac{N_q(\underset{\sim}{x}, \underset{\sim}{\xi})}{N_*(\underset{\sim}{x}, \underset{\sim}{\xi})} \qquad (2)$$

(equation (11) of reference 1). Specifically, it is required to find a direction $\underset{\sim}{\xi}_0$ along which $N_*(\underset{\sim}{x}, \underset{\sim}{\xi}_0)$ (and hence $N_q(\underset{\sim}{x}, \underset{\sim}{\xi}_0)$) is independent of $\underset{\sim}{x}$. Strictly, in real media, no such direction $\underset{\sim}{\xi}_0$ generally exists; however, the current procedure makes use of the fact that any horizontal $\underset{\sim}{\xi}_0$ in horizontally stratified atmospheres or hydrospheres comes very close to satisfying the above requirements for most practical purposes. But the attendant effort required to continuously and accurately align airborne or submarine-borne instruments precisely in these distinguished directions $\underset{\sim}{\xi}_0$ becomes exceedingly difficult. Other means of determing $L(\underset{\sim}{x}')$ also exist. But these, too, are made to depend on various assumptions about the atmosphere (such as optical standard atmosphere, special lighting conditions, etc.).

These considerations lead us to search for an alternate experimental-theoretical procedure leading to the determination of $C_r(\underset{\sim}{x}, \underset{\sim}{\xi})$. The procedure proposed below eliminates the difficulties centering around the explicit determination of $L(\underset{\sim}{x}')$ by replacing this quantity by another whose determination requires neither the horizontal stratification assumption nor the need to seek out and align the measuring apparatus along certain distinguished directions, nor any other special assumptions about the optical properties or light field structure in the medium.

THE CONCEPT OF THE CONTRAST TRANSMITTANCE DISTRIBUTION

The proposed alternate procedure is also based on formula (1). However, the contrast transmittance C_r / C_o will not be decomposed explicitly into the product of two quantities, namely the product of the radiance ratio ${}_bN(\underset{\sim}{x}_t, \underset{\sim}{\xi}) / {}_bN(\underset{\sim}{x}, \underset{\sim}{\xi})$ and the beam transmittance $T_r(\underset{\sim}{x}, \underset{\sim}{\xi})$. Rather, the contrast transmittance will be considered as a fundamental irreducible observable. In short, it will take the role of an <u>apparent</u> optical property of the medium, just as T_r takes the role of an <u>inherent</u> optical property of the medium. Thus contrast transmittance takes on the same status as the K-functions used in hydrological optics (reference 2): that is, it is dependent upon the simultaneous combination of the ephemeral light field (by virtue of the presence of the ratio ${}_tN(\underset{\sim}{x}_t, \underset{\sim}{\xi})/{}_bN(\underset{\sim}{x}, \underset{\sim}{\xi})$) and the local inherent optical properties of the medium (by virtue of the presence of $\mathcal{T}_r$). However, as in the case of the K-functions of hydrological optics, it possesses sufficient regularities which serve to elevate it to the level of an apparent optical property of the medium and to be used as such in both the experimental documentation of the medium and in the subsequent visibility calculations associated with the medium. This thesis will now be developed.

Definition of $\mathcal{J}_1$

Consider the schematic representation of an experimental apparatus in Figure 1. It consists of a Gershun tube G at point $\underset{\sim}{x}$ and a target at range $r = 1$ meters (say) from G of inherent radiance $N_0 = 0$. The line of sight is along an arbitrary direction $\underset{\sim}{\xi}$. The instrument is devised so that it reads directly the observable ratio

$$\mathcal{J}_1(\underset{\sim}{x}, \underset{\sim}{\xi}) = \frac{C_1(\underset{\sim}{x}, \underset{\sim}{\xi})}{C_0(\underset{\sim}{x}, \underset{\sim}{\xi})} = -C_1(\underset{\sim}{x}, \underset{\sim}{\xi}), \tag{3}$$

which is simply the negative of the apparent contrast of the target against its background as seen at $\underset{\sim}{x}$ from a unit distance away from the target in the direction $\underset{\sim}{\xi}$. We observe that definition (3) need not be restricted to the adopted special value $C_0(\underset{\sim}{x}, \underset{\sim}{\xi}) = -1$ C_0 could actually be any real number other than zero, but for simplicity, C_0 will be taken as -1 .

Now the analytic structure of $\mathcal{T}_1$ is of the same kind as radiance: the complete specification of some value of $\mathcal{T}_1$ is made only after the location $\underset{\sim}{x}$ and direction $\underset{\sim}{\xi}$ of $\mathcal{C}$ have been given. Furthermore $\mathcal{T}_1$, is implicitly associated with a given wavelength of radiant energy. Thus, in analogy to the radiance distribution $N(\underset{\sim}{x},\cdot)$ at $\underset{\sim}{x}$ which is a function assigning to each direction $\underset{\sim}{\xi}$ a radiance value $N(\underset{\sim}{x},\underset{\sim}{\xi})$ at $\underset{\sim}{x}$, we have the notion of the <u>contrast transmittance distribution</u> $\mathcal{T}_1(\underset{\sim}{x},\cdot)$ at $\underset{\sim}{x}$, which assigns to each direction $\underset{\sim}{\xi}$ a contrast transmittance value $\mathcal{T}_1(\underset{\sim}{x},\underset{\sim}{\xi})$ at $\underset{\sim}{x}$. We underscore the observation that the concept $\mathcal{T}_1$, is a <u>local</u> concept in the same sense that N, N_* . N_q, and σ are local concepts which are associated with a given point and direction at that point in an optical medium. The key analytic property of the function $\mathcal{T}_1$, which endows it with a utility favorably comparable to that of the analogous beam transmittance function will now be discussed.

The Semi-group Property of $\mathcal{T}_r$

The so-called semi-group property of T_r is well known. Suppose we have two contiguous path segments of length r_1 and r_2 along a common natural path P (Figure 2). Then, in its simplest form, the semi-group property of T_r asserts that:

$$T_{r_1} T_{r_2} = T_{r_1 + r_2} . \qquad (4)$$

This property holds for an arbitrary path of length $r = r_1 + r_2$ over which the volume attenuation function α is of arbitrary structure.

To show that the semi-group property is possessed by the contrast transmittance function, we return to (1). In general, for an arbitrary path of length r, define

$$\mathcal{T}_r(\underset{\sim}{x}, \underset{\sim}{\xi}) = \frac{C_r(\underset{\sim}{x}, \underset{\sim}{\xi})}{C_o(\underset{\sim}{x}', \underset{\sim}{\xi})} \tag{5}$$

where $\underset{\sim}{x}$ is the observation point and $\underset{\sim}{x}'$ the (generalized) target point of the path P.

Then, according to (1) and the notation of Figure 2, we have, for each of the two locations $\underset{\sim}{x}_1$, $\underset{\sim}{x}_2$ along the path P, the following particular expressions for $\mathcal{T}_r$:

For the segment $(\underset{\sim}{x}_1, \underset{\sim}{x}_2)$ along P :

$$\mathcal{T}_{r_1}(\underset{\sim}{x}_1, \underset{\sim}{\xi}) = \frac{{}_bN(\underset{\sim}{x}_2, \underset{\sim}{\xi})}{{}_bN(\underset{\sim}{x}_1, \underset{\sim}{\xi})} T_{r_1}(\underset{\sim}{x}_1, \underset{\sim}{\xi}) . \tag{6}$$

For the segment $(\underset{\sim}{x}_2, \underset{\sim}{x}_3)$ along P :

$$\mathcal{T}_{r_2}(\mathbf{x}_2, \boldsymbol{\xi}) = \frac{{}_bN(\mathbf{x}_3, \boldsymbol{\xi})}{{}_bN(\mathbf{x}_2, \boldsymbol{\xi})} \, T_{r_2}(\mathbf{x}_2, \boldsymbol{\xi}) . \tag{7}$$

For the entire path P :

$$\mathcal{T}_r(\mathbf{x}_1, \boldsymbol{\xi}) = \frac{{}_bN(\mathbf{x}_3, \boldsymbol{\xi})}{{}_bN(\mathbf{x}_1, \boldsymbol{\xi})} \, T_r(\mathbf{x}_1, \boldsymbol{\xi}) . \tag{8}$$

Then the product

$$\mathcal{T}_{r_1}(\mathbf{x}_1, \boldsymbol{\xi}) \; \mathcal{T}_{r_2}(\mathbf{x}_2, \boldsymbol{\xi})$$

reduces to

$$\mathcal{T}_{r_1}(\mathbf{x}_1, \boldsymbol{\xi}) \, \mathcal{T}_{r_2}(\mathbf{x}_2, \boldsymbol{\xi}) = \frac{{}_bN(\mathbf{x}_3, \boldsymbol{\xi})}{{}_bN(\mathbf{x}_1, \boldsymbol{\xi})} \, T_{r_1+r_2}(\mathbf{x}_1, \boldsymbol{\xi}) = \mathcal{T}_{r_1+r_2}(\mathbf{x}_1, \boldsymbol{\xi}) \tag{9}$$

in which the semi-group property (4) of T_r has been used. Thus, in the sense made clear in (9), the function $\mathcal{T}_r$ possesses its own semi-group property:

$$\mathcal{T}_{r_1} \mathcal{T}_{r_2} = \mathcal{T}_{r_1+r_2} . \tag{10}$$

Application of the Semi-Group Property of $\mathcal{T}_r$ to $\mathcal{T}_1$

The general property (10) of $\mathcal{T}_r$ is immediately applicable to the basic contrast transmittance distribution $\mathcal{T}_1$ by setting $r = 1$. Suppose that the distribution $\mathcal{T}_1(x, \cdot)$ has been experimentally determined at all points x of an optical medium (e.g., some aerosol or hydrosol). Then its values are known in particular all along some given line of sight P (Figure 3). This information allows the computation of the contrast transmittance $\mathcal{T}_r(x_1, \xi)$ for the entire line of sight as follows:

Divide the line of sight into n segments, each of unit length (i.e., the unit of length associated with $\mathcal{T}_1$). Then since $\mathcal{T}_1$ is a special case of $\mathcal{T}_r$, we have immediately, by successive applications of (10):

$$\mathcal{T}_r(x_1, \xi) = \prod_{i=1}^{n} \mathcal{T}_1(x_i, \xi) . \tag{11}$$

THE N-DISTRIBUTION AND $\tilde{\mathcal{T}}_1$ -DISTRIBUTION AS BASIC EXPERIMENTAL OBSERVABLES

The various observations presented above may be summarized as follows:

(a) The contrast transmittance function $\tilde{\mathcal{T}}_1$ defined in (3) is a directly observable quantity of the light field in natural optical media such as the atmosphere and the sea. It may be viewed, for the purposes of the experimental documentation of real media, as an (apparent) optical property of these media. It possesses the same basic analytic properties as the beam transmittance function T_1 (equations (4) and (10)) which has made T_1 useful in engineering calculations of contrast transmittance, but is free of certain fundamentally difficult intermediate measuring procedures required to determine T_1.

(b) In view of the general relation (1), the basic formula for the apparent contrast $C_r(x, \xi)$ may be written

$$C_r(x, \xi) = \tilde{\mathcal{T}}_r(x, \xi)\, C_0(x_t, \xi) \; ; \tag{12}$$

in which $\tilde{\mathcal{T}}_r(x, \xi)$ is determined from knowledge of $\tilde{\mathcal{T}}_1$ by means of formula (11).

(c) The complete solution of the general problem of determining the apparent contrast $C_r(z, \xi)$ of an object can be made to rest on the experimental knowledge of two basic distributions: the N -distribution, and the $\mathcal{T}_1$ -distribution. That is, the N -distribution will allow the determination of the inherent contrast C_0 of a given object. The required apparent contrast then follows by use of the basic relation (12). In this way the explicit determination of T_r is obviated, and the problem of experimentally documenting any given optical medium may be reduced to the determination of the two functions N and $\mathcal{T}_1$.

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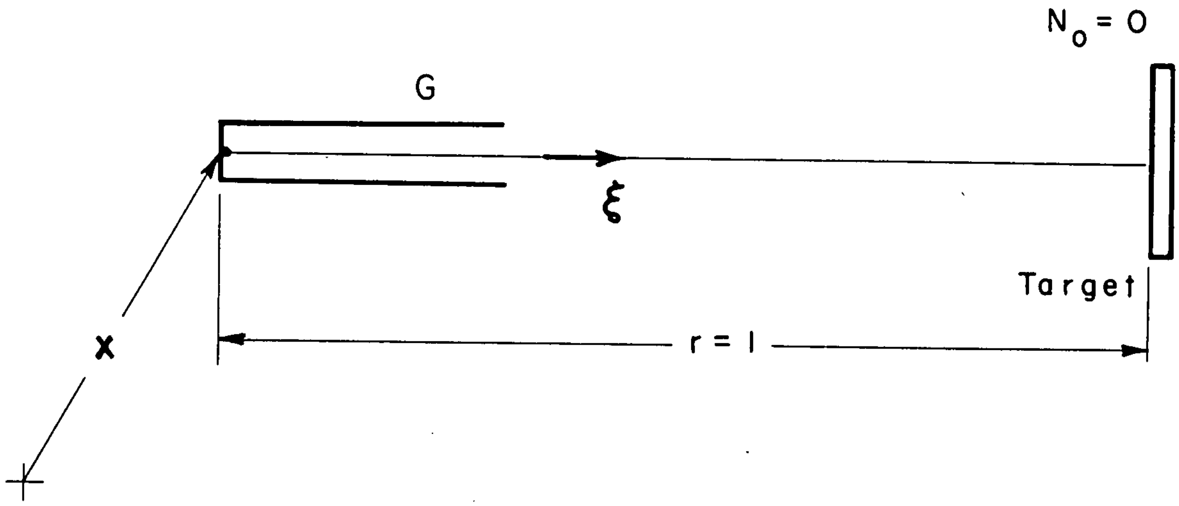

Figure 1

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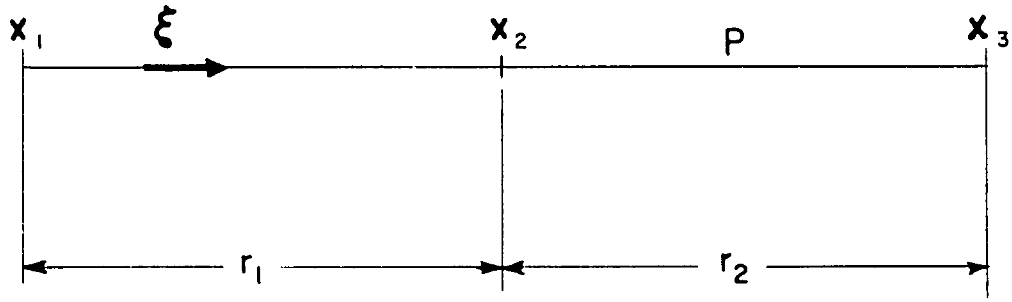

Figure 2

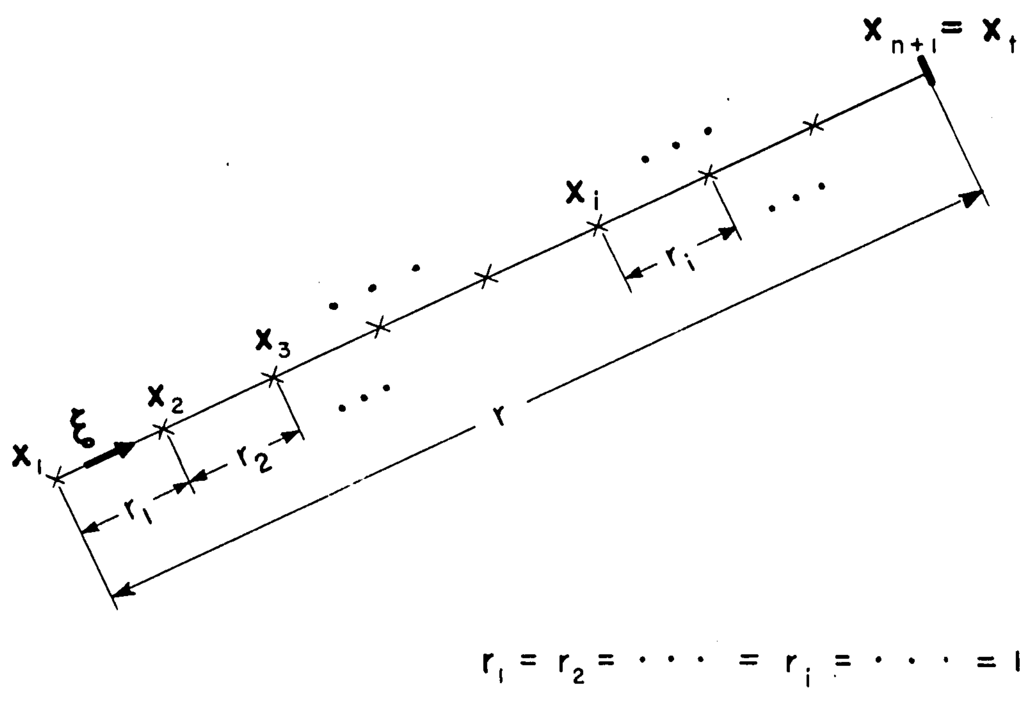

Figure 3

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