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THE MEASUREMENT OF LIGHT IN NATURAL WATERS

RADIOMETRIC CONCEPTS AND OPTICAL PROPERTIES

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CONTENTS (Parenthetical numbers indicate equations)

The Measurement of Light in Natural Waters

Radiometric Concepts and Optical Properties

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ABSTRACT

The object of this note is two-fold: (i) To define and discuss those recently developed concepts of geometrical radiometry which are of greatest use in the experimental study of the light field in natural waters. (ii) To present a systematic development and discussion of the inherent and apparent optical properties of natural waters which are used in modern hydrological optics.

The most useful radiometric concept is that of radiance. A complete documentation of the light field in a natural hydrosol by means of radiance distribution measurements supplies, in principle, all the radiometric information required to solve every practical problem centering on questions of image and flux transfer in natural waters. Where the determination of radiance distributions is impossible or unfeasible, the use of an alternate set of radiometric concepts is proposed. This set consists of

four irradiance quantities which require less effort to obtain than the radiance distributions. They supply enough information about the depth dependence of the light field, its local angular structure and the overall flux transmitting and reflecting properties of the medium to allow many practical problems to be solved with satisfactory precision, and completeness.

This basic quartet of irradiances gives rise, by means of certain well defined operations, to a set of quantities, each of which possesses strikingly regular and reproducible features even though each depends in part on the ephemeral submarine light field. These regularities allow the quantities to be given the status of optical properties and, as such, they considerably simplify the classification of the optical structure of natural hydrosols. These apparent optical properties play key roles in the engineering solutions of image and flux transfer problems and provide powerful empirical checks of theoretical models of the light field in natural hydrosols.

INTRODUCTION

In the field of underwater light measurements early initiative belongs largely to the biologists, who, with little if any help from photometrists devised instrumentation and made measurements to discover the main features of the submarine light field and to correlate biological activity with the light that was measured.

During the period 1935 to 1945 many types of measurements were made in lake and ocean waters. Unfortunately the instruments used by different workers varied somewhat in important respects and as a result the light measurements cannot be directly intercompared. The measurements are of value chiefly to the experimenter who obtained them, and cannot be used for general mathematical or physical applications.

As a result it has become clear that standardization of measuring technique and conformity to radiometric concepts is desirable.

The purpose of this paper is to illustrate some recently developed applications of radiometric concepts to light measurements in the underwater light field and to define the important optical properties of natural waters associated with image and flux transmission through water. Only monochromatic light will be considered at this time; this will provide the basis for the discussion of heterchromatic light at a later time. The radiometric terminology used here follows where possible, and extends where necessary, the terminology recommended by the Committee on Colorimetry (1944a, 1944b).

SOME GENERAL OBSERVATIONS ON UNDERWATER LIGHT-FIELDS

Various aspects of the submarine light field have been noted and described in the literature. M. Minnaert (1940) describes the "large disc of light above your head" (sometimes called the "manhole"), which is the entrance port for all direct sun and skylight reaching a point below the surface. For flat calm water, this man-hole is determined by Snell's law:

$$n \sin\theta = n' \sin\theta', \qquad (1)$$

where n and n' are the refractive indices of air and water respectively, θ and θ' are the angles that the ray makes with the normal to the air-water surface in air and in water, respectively. If the index of refraction of water is taken as $\frac{4}{3}$, then when $\theta = 90^{\circ}$, $\sin\theta' = .75$ and $\theta = 48.6^{\circ}$, which is the angle at which one would expect to see the horizon through a flat calm surface from an underwater vantage point. Actually, little if any light will get through the interface at this specific angle because of the high reflectance predicted by Fresnel's law for $\theta = 90^{\circ}$. As the upward looking line of sight is swept from $\theta = 0^{\circ}$ to $\theta = 48.6^{\circ}$, one can observe the gradual, successively greater compression of objects toward the skyline. On days when the surface is windblown, one can observe within and around the manhole, the glitter pattern produced by the sun on the water. This glitter pattern has been described for example by Cox and Munk (1954). Beyond the edge of the man-hole, at angles θ' greater than 48.6° and less than 90°, one can see the totally reflected and upside down images of fish as they swim by, and the reflected back-scattered light welling up from below (Tyler, 1958a).

If, in horizontally stratified water, the point of observation is moved horizontally, no essential difference in the light field will be observed at the new observation point because over a substantially infinite extent every area of the surface above is being illuminated by the same sun and sky, and in the same way.

As one descends very slightly from the surface, the manhole, which in principle continues to subtend the half angle $\theta' = 48.6^{\circ}$, loses its relatively sharp edges. If there is a clear sunny sky, the sun's image becomes progressively dimmer while scattered light quickly fills in the darker areas, especially in relatively turbid water, softening the initially high contrast of the manhole, and partially replacing the sharply collimated light of the sun with a brilliant field of diffuse, less directional light. At these very shallow depths the decrease in the sun's share of the light seems more than compensated by this spontaneous flood of scattered light. Descending further, the amount of diffuse light begins to fall off sharply, and the light from the sun even more quickly.

Still lower, at very great depths, the direct influence of the sun and sky light seem all but lost, the diffuse field appears to settle down to a fixed angular pattern whose only change with further increase of depth is an overall diminuation in brightness at an unmistakably exponential rate.

CONCEPTS USED IN THE MEASUREMENT OF LIGHT

All the radiometric concepts discussed below, which form the scientific basis for the phenomenological description of light, are based on the single physical idea of radiant flux; that is, the idea of radiant energy in motion. The basic distinction between them arises from the appropriate geometrical channeling of the collected or emitted radiant flux.

Radiant Flux

Radiant flux is the time rate of change of radiant energy. The term "time rate" may in general be interpreted two ways: The first interpretation is associated with a region of space which is producing radiant energy; its time rate of production of radiant energy may be described as its radiant flux output. The second interpretation conceives of radiant energy per unit time crossing a given surface and is described as the radiant flux across the surface (in the appropriate direction). The present paper will limit itself to the second interpretation.

It is usually sufficient to have an operational definition of radiant flux by means of some instrument which can sense and record incident radiant energy. The basic elements of such an instrument (depicted schematically in Figure 1a) are a small flat flux collector of size A, and a recorder. When radiant flux falls on the collector, the recorder reads a response R . The radiometrist takes care to make the response R directly relatable in some known way to the amount P of radiant flux actually incident on A . This means, in particular, that he tries to make the response R to a given amount of flux P <u>independent of the direction of incidence of P</u> . For example, if there are two streams of radiant energy on the collecting surface, one incident perpendicularly, say, and the other at some fixed oblique angle, as shown in Figure 1a, and each stream has the same radiant flux P (as determined e.g., by the electromagnetic picture of light) then each stream must give rise to the same response R on the recorder. When the collecting surface satisfies this requirement, it is referred to as a <u>Lambert collector</u>. In what follows we will always assume that the radiant flux determinations have been made using a Lambert collecting surface. With these remarks in mind, we may write, in symbolic form, the operational definition of <u>radiant flux:</u>

$$P = \Phi R \, , \tag{2}$$

where Φ is the known response characteristic of the instrument which relates R to the incident radiant flux P , and determines the units of P . In practice the recorder is calibrated to read P directly. The dimensions of P are: energy per unit time. In the m.k.s. system the units of P are in terms of watts.

Irradiance and Radiant Emittance

In our present interpretation of radiant flux as the flow of radiant energy across a surface, we consider separately two possible directions of flow. If the surface encloses a region of space within which radiant energy is generated, and this energy flows outward across the surface, it is useful to specify the outward flow across each unit area of the surface. The concept that supplies such information is called <u>radiant emittance</u>. It is defined as the radiant flux emitted per unit area at a given point of the surface, and is denoted by the symbol W. To obtain W operationally, it suffices in principle to place the collecting surface of the flux-recorder over the area of interest on the emitting surface, and to observe the resultant reading P. Then, by definition,

$$W = \frac{P}{A} \, . \tag{3}$$

More sophisticated means of obtaining W are implicit in the discussions below. Evidently, the dimensions of W are: radiant flux per unit area; and its units: watts per square meter.

To complement the idea of radiant flux away from a surface, we have the notion of radiant flux <u>onto</u> a surface; more specifically, the amount of radiant flux incident per unit area at a given point of a surface. The concept used for this purpose is called <u>irradiance</u>. Dimensionally and unit-wise, irradiance and radiant emittance may be considered identical, but they are held conceptually distinct in the sense that irradiance refers to incident flux on a surface, while radiant emittance refers to flux emitted from a surface. The symbol for irradiance is H, and its operational definition is:[1]

$$H = \frac{P}{A} . \tag{4}$$

Radiance

In the preceding discussion of irradiance, the energy flow was allowed to arrive at a point of a surface from all directions within a hemisphere defined by a plane tangent to the surface at the point. Quite often, the individual amounts of flux arriving from <u>each</u> of these directions is of more importance than their total. In order to measure the flow arriving from a particular direction, some kind of "blinder" must be put on the Lambert collector. The blinder serves to block off the incoming flow of radiant energy in all but a small solid angle of directions. A flux collector with a set of blinders is schematically depicted in Figure 1b. This device is conventionally referred to as a <u>Gershun tube</u> (or radiance tube) (Gershun, 1939). In practice, the collecting area is circular, and the blinder is in the form of a long narrow cylinder containing baffles whose surfaces have been treated with matte black paint. The Gershun tube is constructed so that the flat circular collecting area at the base of the cylinder is centered on and perpendicular to the axis of the cylinder.

Consider a Gershun tube whose associated solid angle is Ω. That is, each point of the collecting surface of area A has access to the radiometric environment through a solid angle of magnitude Ω. Of course there will exist in any material Gershun tube a slight variation of solid angle opening from point to point on the collecting surface. However, if the ratio of the length of the tube to its radius is 10 to 1 or greater, this variation is negligible. We will assume that all Gershun tubes used in the arguments below have this property. Now point the Gershun tube in a given direction and suppose the reading of the flux-recorder is P. Then the Gershun tube determines a <u>radiance</u> N for this particular direction, whose magnitude is defined by the rule:

$$N = \frac{P}{A\Omega} \quad (5)$$

From Eq. (4), $P/A = H$, so that the radiance may also be characterized by the formula:

$$N = \frac{H}{\Omega} . \tag{6}$$

In practice Gershun tubes are usually designed so that the flux recorder reads N directly, the quantities A and Ω being fixed characteristics of the assembly. From the definition of radiance, Eq. (5), we see that it has the dimensions: radiant flux per unit area per unit solid angle. Its units are: watts per square meter per steradian.

In the discussion of irradiance and radiant emittance these concepts of radiant flux across a surface were associated with particular directions of flow. A similar useful distinction can be made with radiance. Consider Figure 2a. Radiant flux is passing perpendicularly across a surface S at point p on S. The flow is constrained within a solid angle of magnitude Ω (the solid angle of some Gershun tube.) If the tube were oriented so as to collect the incoming flux, then an irradiance H would be induced on the Gershun tube's collecting plate, and the radiance

$$N = \frac{H}{\Omega} \tag{6}$$

would be reported for this incoming pencil of energy. However, this same bundle of radiant energy could be thought of as giving rise to a radiant emittance W of S at p . The magnitude of the solid angle in which the emitted flux is constrained to flow is still Ω . Hence the radiance may also be characterized by:

$$N = \frac{W}{\Omega} . \tag{7}$$

To distinguish between these two ways of looking at radiance, we call the radiance as given in (6), the <u>field radiance</u>, and the radiance given in (7) as the <u>surface radiance</u>. Field radiance is of greatest use in experimental work in conjunction with the use of Gershun tubes, and surface radiance is used to greatest advantage in theoretical work.[2]

Scalar and Spherical Irradiance

Spherical and scalar irradiance are the last two of the major radiometric concepts to be discussed here. Scalar irradiance gives a quantitative measure of the total radiant flux arriving at a point from all directions about the point. Scalar irradiance, in essence, is a measure of the amount of radiant energy per unit volume of space at a given point; the individual amount coming in from each direction about the point is unimportant, only the total is of interest.

Scalar irradiance, (as defined in the analytic relations section below), can be determined if the field radiance is known for all directions around the point of interest. Such a determination, however, involves a somewhat tedious numerical procedure. A <u>spherical</u> Lambert collector, (Figure 1c) provides a simple experimental means of obtaining scalar irradiance directly. Measurements with a spherical collector systematically differ from theoretically computed scalar irradiance values by a constant factor of 4. (See section on analytical relations, below). All other things being equal, the spherical collector readings are less by a factor of 4 than the scalar irradiance values. Since this difference is known and invariable, a spherical collector can be used to obtain both scalar and spherical irradiance.

The distinction between scalar and spherical irradiance can be stated as follows: scalar irradiance arises naturally in theoretical analyses and has a simple analytic definition in terms of the angular distribution of field radiance about a point in space; spherical irradiance is the associated quantitv measured by a small spherical Lambert collecting surface.

The operational definition of spherical irradiance is as follows: consider a small spherical surface of radius r . Let the surface be a Lambert collector (i.e., each tiny area on the surface acts like a <u>plane</u> Lambert collector). Let P

be the recorded amount of radiant flux incident on the sphere. Then the spherical irradiance $h_{4\pi}$ associated with this flux is defined as:

$$h_{4\pi} = \frac{P}{4\pi r^2} \; . \tag{8}$$

Evidently the dimensions of spherical irradiance are: radiant flux per unit area; its units are: watts per square meter.

SOME THEOREMS OF GEOMETRICAL RADIOMETRY

In this section a few key theorems in geometrical radiometry, will be discussed. The discussion by no means exhausts all of the theorems of this discipline, but rather presents those theorems which will be of greatest use to persons engaged in optical oceanography.

Cosine Law

Imagine a small arbitrarily shaped plane area of magnitude A in a uniform stream of radiant energy (Figure 3a). Suppose that when the area is broadside to the stream, so that its normal makes a zero angle with the direction of the stream, the amount of flux across the surface is $P(0)$. Suppose, in general, that when the normal to the area makes an angle θ with the stream, the amount of flux across the area is $P(\theta)$. We may then ask: what is the relation between $P(0)$ and $P(\theta)$? It seems reasonable that the amount would be directly proportional to the projected area $A(\theta)$ that the surface presents to the stream of energy. That is, if the projected area were, for example, just one half A , the amount of intercepted flux is just one half $P(0)$, and so on. In general, we then would expect that

$$\frac{P(\theta)}{P(0)} = \frac{A(\theta)}{A(0)} , \qquad (9)$$

where, of course, $A(0) = A$. Hence, we may express $P(\theta)$ as:

$$P(\theta) = P(0)\,\frac{A(\theta)}{A}\,. \tag{10}$$

So far we have used physical reasoning: no amount of pure mathematics could ever give a relation of the kind summarized in Eq. (9). Relation (9) is, in the last analysis, an experimental fact. However, the next step is purely mathematical: the relation between $A(\theta)$ and A is given by a theorem in geometry which states:

$$A(\theta) = A\cos\theta\,. \tag{11}$$

Hence the answer to the question posed above may be written in the form:

$$P(\theta) = P(0)\cos\theta\,. \tag{12}$$

The principal cosine law in geometrical radiometry is the statement of the dependence of irradiance on θ. If $H(\theta)$ is the irradiance on the surface of Figure 2a when its normal makes an angle θ with the stream of radiant energy, then by definition,

$$H(\theta) = \frac{P(\theta)}{A}\,. \tag{13}$$

By substituting the value of $P(\theta)$ from Eq. (12) into this expression, we have the desired <u>cosine law for irradiance</u>:

$$H(\theta) = H(0) \cos\theta, \tag{14}$$

where, by definition,

$$H(0) = \frac{P(0)}{A} . \tag{15}$$

This cosine relation for irradiance has been derived in detail in order to emphasize that it is, in the last analysis, an experimental relation, or a relation which incorporates assumptions based on experimental fact.

Cosine Law for Surface Radiance

In this section we will derive a useful alternate expression for surface radiance. The derivation will be of particular value in the discussion of the volume scattering function in a later section. In the preceding section entitled "Radiance," the notion of surface radiance was introduced by considering a narrow pencil of radiation leaving a surface in the direction perpendicular to the surface. Of course, pencils of radiation can be emitted from surfaces at arbitrary angles θ with respect to the surface normals. Such a situation is depicted in Figure 2b.

Consider a small region of area A on the surface S which is emitting an amount $P(\theta)$ of radiant flux. In particular suppose that at each point p of S there is a narrow pencil of radiant energy emitted from p in the direction of the arrow, and that the radiant emittance of the surface into each of these directions has some fixed magnitude $W(\theta)$. Therefore $P(\theta) = W(\theta) A$. Let each pencil, which is inclined at an angle θ with the normal to the surface, have a solid angle opening of Ω . Now project the area A on a plane perpendicular to this common direction of the pencils and let the projection have area $A(\theta)$. We suppose that A is sufficiently small so that the following assumption holds: All the radiant flux that leaves A crosses $A(\theta)$ Then by definition of surface radiance in the direction θ , we have: (See Eq. (5))

$$N(\theta) = \frac{P(\theta)}{A(\theta)\,\Omega} \,.$$

From the geometric fact summarized in Eq. (11), this radiance expression may then be written in the following equivalent form:

$$N(\theta) = \frac{P(\theta)}{A\,\Omega\,\cos\theta} \,, \tag{16}$$

which is the desired <u>cosine law for surface radiance</u>.

Furthermore, since

$$W(\theta) = \frac{P(\theta)}{A},$$

we may write

$$N(\theta) = \frac{W(\theta)}{\Omega \cdot \cos\theta}. \tag{17}$$

It should be noted that no such alternate expressions exist for field radiance, since the latter is by definition associated with radiant flux which crosses the collecting surface in the direction of its normal. There is no need of complicating the notion of field radiance by allowing flux to be incident in any other direction on the collecting surface. It is this fact that makes the notion of field radiance conceptually simpler than surface radiance and of key importance in experimental work. For example, all the information about the structure of the light fields in natural hydrosols can be based on the systematic use of field radiance: it is a well-defined quantity obtained by direct and systematic use of a Gershun tube.

The General Relation Between Surface Radiance and Field Radiance

The dual relation between surface radiance and field radiance at a point has already been discussed (see section on "Radiance," and Figure 2 (a)). However, an experimenter working with a Gershun tube in a natural hydrosol is confronted with the following question: When the line of sight of a Gershun tube is directed through water—which scatters and absorbs radiant energy—how does the field radiance N_r of a surface S of radiance N_0 depend on the distance r at which S is viewed?

This question can be resolved into the following two problems: (i) How much flux is transmitted directly from S to G after suffering possible losses by the actions of scattering and absorption? (ii) How much flux is added to the signal arriving at G, which has been contributed by the scattering of ambient light into the intervening space between S and G? We begin by considering in detail the first of these questions.

Suppose a Gershun tube G (Figure 4a) is directed at some surface S, a distance r from G. Suppose further that the field of view of the Gershun tube is of size Ω_0 and that its collecting area is of size A_0. Let N_r be the field radiance induced by the flux transmitted directly from S to G across the distance r. At this distance the field of view of the Gershun tube determines a region on S whose projected area normal to the line of sight is designated by A_r (Figure 4b). Suppose that the field of view of the tube is sufficiently small so that at all points of A_r the surface radiance of S is essentially N_o in the direction of the line of sight. Furthermore, at each point of A_r let N_o be essentially uniform over the solid angle Ω_r subtended by the collecting plate of the Gershun tube. Finally, let P_o be the radiant flux emitted by A_r into the solid angles Ω, and let P_r be that part of P_o transmitted from S to G. Then, we have, by definition,

$$
\begin{aligned}
P_r &= N_r A_o \Omega_o \,, \\
P_o &= N_o A_r \Omega_r \,.
\end{aligned}
\tag{18}
$$

But now observe the following two facts: First, the geometrical fact that

$$A_o \Omega_o = A_r \Omega_r = \frac{A_o A_r}{r^2} \,, \tag{19}$$

and second, the physical fact that

$$P_r = T_r P_o \,. \tag{20}$$

Here T_r is the factor (the <u>beam transmittance</u>) which determines how much of P_o gets through to G. It takes into account the losses suffered by P_o due to scattering and absorption all along its travels from S to G. T_r is actually of the form:

$$T_r = e^{-\alpha r}, \tag{21}$$

where α is the volume attenuation coefficient to be defined in detail below. From statements (18), (19), and (20), we have the conclusion:[3]

$$N_r = T_r N_o. \tag{22}$$

By viewing the preceding arguments in a suitably general way, we can immediately solve the second problem (ii). Consider S as the hypothetical surface of a small volume V on the path of sight at a distance $r-r'$ from G, where r' may take any magnitude between O and r (Figure 4c). Furthermore, the surface radiance N_* of S is now defined as the radiance generated by scattering of ambient flux per unit length of path into the direction of G. (The exact nature of N_* will be determined in the section on the volume scattering function). It follows immediately from (22) that the amount of radiance transmitted from V to G is:

$$T_{r-r'} N_* dr'. \tag{23}$$

Clearly, the total amount of radiance generated in this way and received at G is obtained by summing the above contributions over all distances r' between O and r : i.e.,

$$N_r^* = \int_0^r T_{r-r'} N_* \, dr', \tag{24}$$

where N_r^* is called the path radiance. Thus the answer to the main problem posed in this section is expressed in the following formula:

$$N_r = T_r N_o + N_r^* . \tag{25}$$

It follows that the field radiance N_r of an object viewed along a path of length r generally consists of two parts: the transmitted surface radiance $T_r N_o$ of the object, and the path radiance N_r^* which represents the "space light" generated by scattering in the intervening distance between the object and the Gershun tube.

As a special case of the above formula, suppose the intervening distance between S and G were through a void, then of course $T_r = 1$, and $N_r^* = 0$, so that:

$$N_r = N_o .$$

Thus, the observed field radiance N_r is, in this case, equal to the surface radiance N_o .

The term "field radiance" for the quantity N_r is occasionally replaced by the more suggestive term <u>apparent radiance</u>. These terms are of course to be considered completely synonymous. The latter term is usually employed when a particular object is under view, so that we may speak of the "apparent radiance of an object" in the field of view. In like manner, N_o is usually referred to as the <u>inherent radiance</u> of the object. According to (25), then, the apparent radiance N_r of an object generally consists of the sum of its directly transmitted inherent radiance N_o , and the path radiance associated with the path of sight.

Inverse-Square Law

The inverse-square law is conventionally associated with the irradiance produced by a point source. More explicitly, the law states that the irradiance on a surface produced by a point source varies inversely as the square of the distance between the point and the receiving surface.

The customary proofs of the inverse-square law require the notion of radiant intensity, that is the radiant flux output of a point source. The following discussion, however, considers finite sources of radiant flux and handles the radiometric quantities by means of radiance. This is a more meaningful approach since it deals with measurable quantities.

Consider a plane area of arbitrary shape (Figure 3b). Let the area be of magnitude A. View this area with a Gershun tube so that the line of sight is perpendicular to the area. (If a given area is not normal to the line of sight, let A represent its projected area normal to the line of sight.) Suppose that at each point of the area, the surface radiance is some fixed value N in the direction of the line of sight. If the surface is viewed through a void from a distance at which the surface completely fills the field of view of the tube, then by the preceding arguments, the field radiance determined by the Gershun tube is also N. As the distance from the surface is increased, there will be a distance, say r, at which the entire surface is just within the field of view of the tube; and for all distances greater than r, the emitting surface will be contained wholly within the field of view. The flux from the surface now arrives at the collecting plate of the tube through a solid angle of magnitude,

$$\Omega_r = \frac{A}{r^2},$$

which is not greater than the Ω_r of the tube. It is clear, however, that the field radiance of each point of the emitting surface is still N (as would be verified if a Gershun tube with a solid angle sufficiently smaller than Ω_r were to be used). It follows that the irradiance on the tube's collecting surface must be

$$H_r = N\Omega_r .$$

If these expressions are combined, we have the following form of the <u>inverse-square law for irradiance:</u>

$$H_r = \frac{NA}{r^2} . \tag{26}$$

By including the factor T_r in the above formula, we can describe the transmitted irradiance through a scattering-absorbing medium. Furthermore, an analogous term to N_r^* can also be included when needed.

In this way the inverse-square law (26) is defined in terms of the directly observable quantities N , A , and r . We note in passing that the product NA takes over the role of radiant intensity, having the dimensions of radiant flux per unit solid angle; however, the geometric entity with which this quantity NA is associated is a surface of finite area and arbitrary shape. These considerations lead us to a useful <u>operational</u>

definition of the term "point source." As the preceding discussion shows, the idea of a point source is actually relative to the angular opening of the Gershun tube. (Recall that radiometrically good tubes will have length to radius ratios of 10 to 1 or larger; see, e.g., (Tyler, 1958c)). Thus a source of radiant flux in the radiometric environment of a given Gershun tube may be said to be a point source if it can be completely contained within the field of view of that tube. It is of interest to observe that if we were to indulge in an exact mathematical analysis of the accuracy of the above statement of the inverse-square law, we would find that the above estimate of H_r differs from the exact amount by not more than one per cent of the exact amount. Since our approach is purely operational, the preceding form of the inverse-square law and the definition of point source are evidently the ones that are most natural to adopt in any experimental study of the light field.

Lambert Collectors and Emitters

The definition of a Lambert collector given during the discussion of the measurement of radiant flux can be cast into several alternate forms. The characterization chosen for discussion here is not only applicable to Lambert collectors, but also can be turned around, so to speak, and be used to characterize the complementary notion of a Lambert emitter.

First, as regards the Lambert collector, consider a plane area of arbitrary shape and of area A . Let the area be irradiated at each point of its surface by incoming flux which has a field radiance N in all directions over the incoming hemisphere. Then the radiant flux $P_i(\theta)$ incident on the area (see footnote 1) , through a small solid angle Ω inclined at an angle θ with the normal, is given by:

$$P_i(\theta) = N \Omega A \cos\theta = P_i(0) \cos\theta , \quad (27)$$

and the surface, being a Lambert collector, records a response equal to $P_i(\theta)$, i.e., the recorder exhibits a cosine response to such incoming flux.

Conversely, suppose the area A now emits (or reflects) radiant flux, and suppose that the total flux $P_o(\theta)$ is emitted (or reflected) in a manner described by (27), where the flux being observed is contributed by each point of the surface radiating through a fixed small solid angle Ω , i.e., $P_o(\theta)$ is of the form:

$$P_o(\theta) = N_o \Omega A \cos\theta = P_o(0) \cos\theta . \quad (28)$$

A surface which exhibits such a radiation characteristic is called a Lambert emitter (or reflector). From this, it follows immediately that a Lambert emitter (or reflector) necessarily has a uniform surface radiance N_o in all directions at each of its points.

Analytical Relations Between the Radiometric Concepts

The radiometric concepts are gathered together for convenient reference in Table 4 at the end of the paper. In this section, we wish to analytically tie together the various concepts introduced so far. Observe that all the concepts have been defined in terms of realizable physical operations. However, the various interrelations between the notions are most conveniently brought out by using their analytical representations. The concept of radiance will be singled out as the most basic as far as mathematical operations are concerned. For, from knowledge of N, all the other radiometric quantities are easily determined.

Scalar and Spherical Irradiance. The operational procedure for obtaining spherical irradiance $h_{4\pi}$ has already been outlined. It remains to define scalar irradiance and show the connection between these two irradiances. To this end, let $N(p,\theta,\phi)$ be the field radiance at point p arriving from the direction (θ,ϕ) where θ and ϕ are measured from some fixed reference system (Figure 5). Then the <u>scalar irradiance</u> $h(p)$ at point p is defined as:

$$h(p) = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} N(p,\theta,\phi)\, d\Omega \,, \tag{29}$$

where

$$d\Omega = \sin\theta \, d\theta \, d\phi \,. \tag{30}$$

We can obtain an analytic expression for $h_{4\pi}(p)$ in the following way: consider a small spherical Lambert collector of radius r with center at p. Then the amount $P(p,\theta,\phi)$ of radiant flux intercepted by the spherical surface from a unit solid angle in the direction (θ,ϕ) is (using the cosine law)

$$P(p,\theta,\phi) = N(p,\theta,\phi) \int_{\text{hemisphere}} \cos\psi \, dA \,, \tag{31}$$

where the hemisphere of integration is determined by the plane of the great circle C which is perpendicular to the direction (θ,ϕ).

The integral is easy to evaluate because it simply represents the projected area of the hemisphere on the plane of its great circle, so that

$$P(\rho,\theta,\phi) = \pi r^2 N(\rho,\theta,\phi). \qquad (32)$$

The amount $P(\rho)$ of flux intercepted by the sphere from all directions is:

$$P(\rho) = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} P(\rho,\theta,\phi)\, d\Omega = \pi r^2 h(\rho). \qquad (33)$$

Finally, the average flux per unit area on the collecting sphere is, by definition:[4]

$$h_{4\pi}(\rho) = \frac{P(\rho)}{4\pi r^2} = \frac{1}{4} h(\rho). \qquad (34)$$

<u>Irradiance and Radiant Emittance</u>. The irradiance $H(\rho)$ produced by a distribution of field radiance $N(\rho,\theta,\phi)$ at ρ on a surface is obtained from Equations (6) and (14):

$$H(\rho) = \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} N(\rho,\theta,\phi) \cos\theta \, d\Omega . \tag{35}$$

On the other hand, if $N_o(\rho,\theta,\phi)$ represents the surface radiance at point ρ then by (17):

$$W(\rho) = \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} N_o(\rho,\theta,\phi) \cos\theta \, d\Omega . \tag{36}$$

Radiant Flux. If a surface S is being irradiated at each point p by a certain radiance distribution, then we can calculate at each point p the irradiance $H(p)$, and it follows that the flux incident on the whole surface S is:

$$P(S) = \int_S H(p) \, dA . \tag{37}$$

Radiant Energy. The radiant energy content $U(R)$ of a given region R of space is:[4]

$$U(R) = \int_R u(p)\,dV = \frac{1}{v}\int_R h(p)\,dV. \quad (38)$$

The unit of radiant energy is a joule.

PROPERTIES WHICH GOVERN THE TRANSFER OF LIGHT

Introductory Remarks

The passage of light through regions containing matter c an be studied on several levels: on the microscopic level using the tools provided by quantum theory and relativity theory; on a band of levels between the microscopic and macroscopic level by means of Maxwell's equations, the equations of relativity or the quantum theory, or a combination of all three; and finally, on a macroscopic level in which the main tools are the radiometric concepts and the simple devices used to measure them, namely the Gershun tube, flat-plate and spherical collectors. We will continue our studies of light by remaining on this macroscopic, or phenomenological level. It is on this level that we can reach, in the quickest and most natural way, the solution of the important problems dealing with the visibility of underwater objects, and the solutions of those problems of marine biology which require detailed knowledge of the quantity and quality of the light field in natural hydrosols.

By adopting the phenomenological approach, we automatically preselect certain macroscopic physical properties of the optical medium which govern the passage of light through and within the medium. In other words we automatically eliminate to a great degree the need to come to grips with the intricate details of the interaction of light with individual atomic systems, nor need we determine various electrical or magnetic properties of the substance comprising the natural hydrosol before we can make meaningful predictions of the passage of light through the hydrosol. By adopting the present approach, we necessarily limit ourselves to small but finite volumes of matter and to the direct observation of how these absorb and scatter the radiant flux through them. By a systematic study of the interaction of light with such small but finite volumes the experimenter can detect and classify the phenomena of absorption and scattering in an exhaustive and detailed manner, and subsequently go on to erect a phenomenological theory of light whose mathematical framework is just as rigorous and internally consistent as those associated with the study of light on the alternate descriptive levels mentioned above.

As the phenomenological classification of the scattering and absorbing properties proceeds, we will see that these properties can be divided quite naturally into two classes: the class consisting of the <u>inherent</u> optical properties of a medium, and the class consisting of the <u>apparent</u> optical properties of the medium. The former class includes such quantities as the volume attenuation function, the volume scattering function, and the absorption function. These summarize certain intrinsic physical actions of the medium on a given beam of light as the beam passes through the medium. This action is generally independent of the orientation of the beam, and of the existing lighting conditions within the medium. The second class contains such quantities as the K-functions, the distribution functions, and the reflectance functions. These describe the behavior of light fields as they exist at the moment of the experiment: they are properties which depend jointly on the inherent properties of the medium and the geometrical structure of the light field.

The apparent optical properties depend in a rather complicated way on the inherent properties and at the same time are at the mercy of highly variable and unpredictable external lighting conditions; however, they are worthy of study and classification because of the following three facts: First, the gross behavior of these properties as determined by actual experiments are strikingly regular. While these gross features do indeed depend upon external lighting conditions, their observable regularities are, as we shall see, amenable to generalizations which apply to all real scattering-absorbing media. Secondly, there are useful theoretical relationships between the inherent and apparent optical properties. These relationships can be deduced from the exact equations of radiative transfer theory, independently of any further experimental considerations. These relationships hold irregardless of the lighting conditions that exist inside or outside of the medium. Finally, the use of apparent optical properties reduces to a practical level the solution of underwater visibility problems and pertinent problems of marine biology. These problems are simplified in the following sense: It is possible, by mathematical procedure, to obtain an exact determination of the light field throughout an optical medium having given only the inherent optical properties and the external lighting conditions (i.e., the radiance distributions at the boundaries of the medium). However, this is a prohibitively complex and lengthy procedure at the present time. By experimentally determining the apparent optical properties of real media, we are in effect solving on a practical level certain particularly difficult parts of this analytical procedure.

Inherent Optical Properties

We will introduce the inherent optical properties by means of three hypothetical experiments. Besides drawing out the basic nature of the optical properties, these experiments outline actual operational procedures for the determination of numerical values for these properties. However, the experiments carried out below are ideal experiments. They are deliberately not complicated by the precautions that must be taken in actual practice in order to avoid the obvious pitfalls accompanying imperfect instruments, perturbations of light fields by measuring equipment, et cetera.[5]

1. Volume Attenuation Coefficient. Consider the experimental arrangement shown in Figure 6a. The source S has a surface radiance N_0 as measured by the Gershun tube G when the latter is at zero distance from the source. The Gershun tube is now moved away from the source but in such a way that it always looks into the beam and in the direction of the source. Let N_r be the radiance measured by G when at a distance r from S. If the intervening region between S and G were a vacuum, then we know from our earlier discussions that for any distance r, we would have $N_r = N_0$. But now the intervening region is assumed to be uniformly filled by the material of some natural hydrosol and the values of N_r are observed to decrease with increasing r. When we plot the following quantity:

$$\ln\left(\frac{N_r}{N_0}\right)$$

for each r, we see that the resultant plot over a certain range is a straight line with negative slope (depicted schematically in Figure 6b). Let the absolute value of the slope of this line be designated by α. Then, the relation between N_r and N_0 in this range is evidentaly of the form:

$$N_r = N_0 e^{-\alpha r}, \tag{39}$$

We perform this experiment several times, each time setting N_o to some new value and observing the resultant N_r values over the same path. Each time a plot is made, another line with the same slope is obtained. By taking measurements along other paths, the same value α is obtained again. In the terminology defined above,[5] we are working in a homogeneous region of the medium.

In these experiments, we have taken precautions to measure only the light that has come directly from the source, and also not to take readings at extreme distances so as to stay within the region of linearity.

In order to fully understand the meaning of the value of α, let us write (39) in differential form:

$$dN_r = -\alpha N_r \, dr \, . \tag{40}$$

The first observation we can make is that for each increment dr of distance away from S, the corresponding increment dN_r of the observed radiance is negative. This reflects the observation that N_r decreases with increasing distance from S. The second fact we may observe is that this increment dN_r is linearly proportional to N_r and to dr. But since we performed the experiments for several values of N_o and variously oriented paths, and obtained the same α each time, we conclude that α must be some inherent property of the medium independent of the amount of flux in the beam and of the beam's orientation. Apparently α has the dimensions of reciprocal length. From the differential statement, we conclude that α gives the attenuation per unit length of a beam of unit radiance. The preceding statement, Eq. (40), being written in differential form, is a statement of the change of N_r over small increments of path length. It is quite possible that the value α may be found to be different at other points of the path and, in general, at other points of the medium. In such cases, as noted above (footnote 5), we will refer to α as the volume attenuation function.

We can make some further observations about α by returning to the original experimental setup. By careful measurement of the beam's radiance from regions just outside of the beam (Figure 7), we detect radiation of the same wavelength as that of the source. This stray light can be positively identified as coming from the beam. From this we may conclude that the attenuation of the beam's radiance is partially due to a scattering of some of its flux out of the main direction of travel of the beam. A critical examination of the scattered flux would soon reveal that scattering alone would not account for the total attenuation of the original beam. We conclude therefore that the medium, in addition to inducing a loss by scattering, also 'absorbs' some of the beam's radiance. Since we have fixed our radiance tube's sensitivity at one particular wavelength, and since the total attenuation of the beam's radiance cannot be accounted for by scattered flux of the same wavelength in the vicinity of the beam, this <u>absorption</u> must manifest itself by a conversion of some of the beam's radiant flux into radiant flux of a different (generally longer) wavelength. This conjecture could soon be verified by suitably probing the immediate vicinity of the beam with Gershun tubes which have been made sensitive to radiant flux of longer wavelengths.

Equations (39) and (40) supply the following alternate operational definitions of α :

$$\alpha = -\frac{1}{N_r}\frac{dN_r}{dr},$$

$$\alpha = \frac{1}{r}\ln\left(\frac{N_r}{N_o}\right). \qquad (41)$$

α is apparently the sum of two generally independent terms: a term, s , which refers to that part of the attenuation due to scattering of flux from the beam without change in wavelength, and a term a which refers to the conversion, or absorption, of some of the flux into flux of different wavelength as that of the original beam. Thus, we may write $\alpha = a + s$. In this way we come to the concepts of the total volume scattering function and the volume absorption coefficient.

2. Volume Scattering Function. In the preceding discussion, during the attempt to estimate the amount of radiant flux scattered out of the original beam, the experimental arrangement shown in Figure 7 was used: The Gershun tube G was directed at a fixed point p in the original beam. The tube was turned so that it successively looked at point p in all directions θ from the direction of the source S. Thus θ was varied essentially from 0 to 180°. For each orientation θ, G was always kept at a small fixed distance r' from p, and the corresponding field radiance $N_l^*(\theta)$ of the beam was recorded. The length $l(\theta)$ of the path of sight through the beam for that particular orientation was also noted. From this information, the flux scattered out of the beam over each unit of path length can easily be computed. We now go through the details of this computation because they lead us directly to (a) the volume scattering function σ, (b) the path function N_*, (c) the (volume) total scattering coefficient s, and (d) a simple derivation of the basic equation of transfer for radiance.

(a) The volume scattering function.

Assume r' is small, so that the surface radiance of the small segment of the beam under view by G is essentially the recorded field radiance $N_l^*(\theta)$. Let $A(\theta)$ be the projected area that the observed element of volume presents to the line of sight of G. Then by (16), the flux per unit solid angle emitted by the volume in the direction of $\mathcal{G}$ is clearly:

$$N_{\ell}^{*}(\theta)\ A(\theta) .$$

But the volume of the observed element of beam can be represented by

$$A(\theta)\ \ell(\theta) ,$$

so that the flux in a unit solid angle in the direction of $\mathcal{G}$ emitted by a unit volume of the medium at p is evidently:

$$\frac{N_{\ell}^{*}(\theta)\ A(\theta)}{A(\theta)\ \ell(\theta)} = \frac{N_{\ell}^{*}(\theta)}{\ell(\theta)} . \tag{42}$$

Let the cross-sectional area of the beam be designated by A . Then the quantity

$$\frac{A\ N_{\ell}^{*}(\theta)}{\ell(\theta)}$$

has the following simple interpretation: it is first of all the amount of flux per unit solid angle scattered in the direction of $\mathcal{G}$; and secondly, it is scattered by an element of volume of the medium which has cross section A and unit length in the direction of the beam. Thus the integral

$$A \int_{4\pi} \frac{N_\ell^*(\theta)}{\ell(\theta)}\, d\Omega \tag{43}$$

over all solid angles about p evidently gives the total flux scattered out of the beam per unit length of travel of the beam through the medium.

While (43) is useful in practical estimates of the rate of loss of radiant flux from the beam through the mechanism of scattering, it also contains the germ of the idea of the volume scattering function. To see this, we first note that the total flux of the beam across the area A is

$$N_r \Omega_r A \,,$$

where Ω_r is the solid angle subtense of the source at point p and N_r is the radiance of the beam at r . Therefore, if we divide (43) by this quantity, the result,

$$\frac{1}{N_r \Omega_r} \int_{4\pi} \frac{N_\ell^*(\theta)}{\ell(\theta)}\, d\Omega \tag{44}$$

has the following interpretation: it is the total amount of radiance lost by scattering per unit length of travel of a beam of unit radiance.

We now may inquire about the directional distribution of the scattered flux. From (44) we see that this distribution is governed by

$$\sigma(\theta) = \frac{1}{N_r \Omega_r} \cdot \frac{N_\ell^*(\theta)}{\ell(\theta)} . \tag{45}$$

When this operation on the observable quantities N_r , Ω_r , $N_\ell^*(\theta)$ and $\ell(\theta)$ is examined in detail, we uncover the following set of experimental facts:

(i) $\sigma(\theta)$ is found to be independent of the amount of irradiation $N_r \Omega_r$.

(ii) $\sigma(\theta)$ is independent of the magnitude of $\ell(\theta)$.

(iii) If θ, r, r', d and N_o are all held fixed and G is swung around the beam, $\sigma(\theta)$ remains fixed.

(iv) $\sigma(\theta)$ is independent of the <u>absolute</u> orientation of S and G about p (medium is isotropic).

These four experimental findings form the basis for the conclusion that $\sigma(\theta)$ is an inherent optical property of the medium. Clearly, on the basis of (i), $\sigma(\theta)$ does not depend on the absolute amount of irradiation on the element of volume of the medium. Furthermore, on the basis of (ii), the <u>relative amount</u> of flux observed to be scattered at a given angle θ

by a <u>small</u> irradiated volume does not depend on the length of the path of sight through that small volume. Finally, according to (iii) and (iv), $\sigma(\theta)$ does not depend on the spatial orientation of the plane formed by the irradiating beam and the direction of observation of the irradiated volume. The function σ , which depends only on θ (in a homogeneous medium) is called the <u>volume scattering function</u>. Its operational definition is given by (45), or by the equivalent form

$$\sigma(\theta) = \frac{N_*(\theta)}{N\Omega} , \tag{46}$$

where

$$N_*(\theta) = \frac{N_\ell^*(\theta)}{\ell(\theta)} , \tag{47}$$

is a quantity independent of the length $\ell(\theta)$ (fact (ii)). N and Ω refer to the radiance and solid angle subtense (at the point p) of the irradiating source. The dimensions of σ are: per unit length per unit solid angle. Both the unit of length and solid angle are in the <u>direction of observation</u> of the irradiated volume.

(b) <u>The path function N_*.</u>

$N_*(\theta)$ as defined in (47), is interpreted as follows: it is the radiance per unit length in the direction of the line of sight, generated by the scattered light of the beam. N_* is called the <u>path function</u>. It plays an important role in the general theory of radiative transfer and in the solution of visibility problems. By (46) we may write

$$N_*(\theta) = \sigma(\theta)\, N\, \Omega . \tag{48}$$

It is easy to generalize this formula to the following form:

$$N_*(p,\theta,\phi) = \int_{4\pi} \sigma(p;\theta,\phi;\theta',\phi')\, N(p,\theta',\phi')\, d\Omega(\theta',\phi') , \tag{49}$$

where the point p is now being irradiated by flux from <u>all</u> directions about p. An example of the calculation of $N_*(p,\theta,\phi)$ in real media is given in (Preisendorfer 1956). $\sigma(p;\theta,\phi;\theta',\phi')$ is the value of the volume scattering function at point p for light incident in the direction (θ',ϕ') and scattered off in the direction (θ,ϕ). (θ,ϕ) and (θ',ϕ') are measured with respect to some fixed reference frame (in the derivation (θ,ϕ) was taken as $(0,0)$ and $\sigma(\rho;\theta,\phi;\theta',\phi')$ was conveniently abbreviated to $\sigma(\theta)$). This generalized form $N_*(\rho,\theta,\phi)$ has the same general interpretation as $N_*(\theta)$ above, but now the radiance per unit length in the direction of observation is generated by light scattered into the line of sight from all directions about the point ρ. By property (iv), $\sigma(\rho;\theta,\phi;\theta',\phi')$ for any pair (θ,ϕ), (θ',ϕ') is known from the determination of $\sigma(\theta)$ at point ρ, as defined in (46).

c. The volume total scattering coefficient.

From the above arguments we now have an explicit expression for the term, s, which arose in the discussion of the volume attenuation coefficient α. For this term is evidently none other than that given in (44) which, by (45), may be written

$$s = \int_{4\pi} \sigma(\theta)\, d\Omega = 2\pi \int_0^{\pi} \sigma(\theta) \sin\theta \, d\theta . \tag{50}$$

The second expression follows from facts (iii) and (iv). This is the volume total scattering coefficient. In non homogeneous media, it may change from point to point, but in any event, s does not depend on the direction of the irradiating beam (facts (iii) and (iv)). Closely related to s are the (volume) forward scattering and (volume) backward scattering coefficients f and b defined by the following formulas:

$$f = 2\pi \int_0^{\pi/2} \sigma(\theta) \sin\theta \, d\theta \, , \tag{51}$$

$$b = 2\pi \int_{\pi/2}^{\pi} \sigma(\theta) \sin\theta \, d\theta \, , \tag{52}$$

so that

$$s = f + b . \tag{53}$$

As in the case of s, both f and b may vary with position, but they do not in any event depend on the direction of the irradiating beam.

d. Equation of transfer.

From the preceding interpretations of α and N_* it is easy to verify that the equation of transfer for field radiance (or surface radiance) N_r in a source-free medium is expressible as:

$$\frac{dN_r}{dr} = -\alpha N_r + N_* . \tag{54}$$

The first term on the right gives the space rate of loss of N_r by attenuation; the second term gives the space rate of gain of N_r by scattering. It is of interest to point out that Eq. (25) is the formal solution of Eq. (54).

3. The Volume Absorption Coefficient. During the discussion of the volume attenuation coefficient α we found that α summarizes two distinct types of action by the medium on the beam: absorption and scattering. The preceding discussion of the volume scattering function resulted in an explicit formula for s, (50). Thus from (53) we may obtain a by subtraction:

$$a = \alpha - s . \tag{55}$$

There exists another way of obtaining a . This method requires no previous knowledge of α or s . It is exact, and completely general. Furthermore, it is particularly simple to use in natural hydrosols. This is the method which makes use of the divergence relation of the light field (Preisendorfer 1957a) and yields the equation:

$$\frac{d\overline{H}(z,+)}{dz} = a(z)\,h(z). \tag{56}$$

To understand the physical significance of the terms occurring in (56), consider the experimental arrangement in Figure 8. $\overline{H}(z,+)$ is the net upwelling irradiance measured at depth z, i.e., $\overline{H}(z,+) = H(z,+) - H(z,-)$. Here $H(z,+)$ is the irradiance at depth z on a flat plate collector which receives the upward moving flux. $H(z,-)$ is the irradiance at depth z due to downward moving flux. $h(z)$ is the scalar irradiance at depth z, and $a(z)$ is the value of the volume absorption function at depth z. According to (56), to obtain $a(z)$ one performs the following operation:

$$a(z) = \frac{1}{h(z)}\,\frac{d\overline{H}(z,+)}{dz} \tag{57}$$

on the measurable quantities $\overline{H}(z,+)$ and $h(z)$. If the medium is homogeneous (footnote 5) then (57) will automatically yield the value of the volume absorption coefficient. In the determination of $a(z)$ by this method, it is clear that in order to evaluate the derivative of $\overline{H}(z,+)$, measurements of $H(z,+)$ and $H(z,-)$ must be made in some interval of depths about the depth z.

The volume absorption coefficient is an inherent optical property of the medium. While this fact may be somewhat difficult to establish from (57), it is readily seen to be true by (55), since we have already shown in detail that both α and s are inherent optical properties. The dimension of a as that of α and s , is reciprocal length. This can be established by inspection of either (55) or (57).

Apparent Optical Properties

The apparent optical properties of natural waters consists at present of a set of seven quantities whose numerical values depend on the angular structure of the light field as well as on the physical composition of the water.

The apparent optical properties can be obtained from four basic measurements. These measurements take the form of two pairs of irradiance quantities: one pair consists of ordinary irradiances, the other of scalar irradiances. In each of these pairs, one member is assigned to upwelling flux, the other to downwelling flux. The reason that there are precisely four such quantities stems from our conceptual decomposition of the flow of radiant energy in any natural hydrosol. (stratified or not) into two streams: an upward flowing stream and a downward flowing stream across each horizontal plane in the medium.[6]

The four basic irradiances are:

$$\begin{matrix} H(z,+) & H(z,-)\,, \\ h(z,+) & h(z,-)\,. \end{matrix} \qquad (58)$$

$H(z,+)$ and $H(z,-)$ are the <u>upwelling (+) and downwelling (−) irradiance</u>, respectively. They are induced by the up and downwelling flux streams at depth z. These quantities may be obtained from field radiance measurements, or they may be measured by flat Lambert collectors exposed to the appropriate hemispheres (Figure 8). In like manner, $h(z,+)$ and $h(z,-)$ are the <u>upwelling (+) and downwelling (−) scalar irradiances,</u> and refer to up and downwelling flux, respectively, at depth z. They may be obtained from field radiance measurements. Alternatively, spherical Lambert collectors may be used to measure these quantities. A possible experimental arrangement is shown in Figure 9. Observe that the collectors are complete spheres in each case. The sphere that measures $h(z,-)$, for example, should be shielded from the upwelling flux by some device which at the same time impedes as little as possible the interchange of flux across the horizontal plane at depth z. In analogy to our earlier discussion of the relation between h and $h_{4\pi}$, we can show that the <u>downwelling spherical irradiance</u> $h_{4\pi}(z,-)$ actually measured by the shielded sphere shown schematically in Figure 9a is related to $h(z,-)$ by:

$$h_{4\pi}(z,-) = \frac{1}{4} h(z,-) . \qquad (59)$$

Similarly, the <u>upwelling spherical irradiance</u> $h_{4\pi}(z,+)$ measured by the other shielded sphere shown schematically in Figure 9b, is related to $h(z,+)$ by:

$$h_{4\pi}(z,+) = \frac{1}{4} h(z,+) . \qquad (60)$$

The connection between $h_{4\pi}$ and the spherical irradiances defined above, assuming ideal shielding, is straightforward:

$$h_{4\pi}(z) = h_{4\pi}(z,-) + h_{4\pi}(z,+) . \qquad (61)$$

Furthermore

$$h(z) = h(z,-) + h(z,+) . \qquad (62)$$

1. The Reflectance Functions. The reflectance functions are defined by:

$$R(z,-) = \frac{H(z,+)}{H(z,-)} ,$$

$$R(z,+) = \frac{H(z,-)}{H(z,+)} . \tag{63}$$

The physical interpretation of $R(z,-)$ is straightforward: it represents the ratio of the upwelling irradiance at depth z to the downwelling irradiance at depth z, so that $R(z,-)$ may be thought of as the reflectance, with respect to the downwelling flux, of a hypothetical plane surface at depth z in the medium. For completeness, we have included the reflectance $R(z,+)$ for the upwelling stream. However, this is simply the reciprocal of $R(z,-)$. In actuality, $R(z,-)$ depends on the scattering properties of the entire medium above and below this level. It will also depend in part on the reflectance properties of the upper and lower boundaries of the medium if these are within sight of the flux collectors. $R(z,-)$ is not an inherent property of the medium, for experiments and theory show in general that for a given medium and a given depth in that medium, the value $R(z,-)$ changes with the external lighting conditions.

Definition (63) is completely general: it applies to any medium, be it deep or shallow, irradiated by the sun in a clear sky or by any type of overcast. Because of this generality, very little can be said about exactly how the values of $R(z,-)$ should depend on depth. No pat statement can be made which asserts that $R(z,-)$ should always increase with depth, or that it should always decrease with depth, or that it should go through maxima or minima at certain depths, remain constant with depth, and so on.

Despite this unwillingness of $R(z,-)$ to have its characteristics typed in very fine detail, there are certain gross characteristics which make it an indispensable tool in engineering calculations: in optically deep homogeneous hydrosols, $R(z,-)$ varies very little with depth. Near the surface of these media, it shows relatively high variability with depth which depends on the state of the surface and incident lighting patterns, but soon settles down and approaches a constant value independent of depth. $R(z,-)$ thereby takes on the status of an apparent optical property of the medium. Furthermore, in media that have no self-luminous organisms, $R(z,-)$ behaves as any respectable reflectance should: it is never greater than 1. In fact, in most natural hydrosols the values of $R(z,-)$ are usually found to be somewhere in the neighborhood of 0.02, give or take 0.01. In media containing self-luminous organisms distributed throughout some layer, it is quite possible, however, for the values of $R(z,-)$ to approach 1 as this layer is approached, and even become greater than 1 just before it enters the layer.

Some examples of $R(z,-)$ are given in Table 1.

While the problem of the fine detail of the depth dependence of $R(z,-)$ is mainly of academic interest, we note that there is no dearth of theoretical approaches to this interesting problem. One model of the light field which is particularly useful in the study of this problem is the so-called two-D theory (Preisendorfer 1957b). This model is particularly simple to use, and is still sufficiently detailed to supply a multitude of examples of the depth dependence of $R(z,-)$: it supplies cases in which $R(z,-)$ can increase or decrease over preselected depth ranges. In all cases, however, the model states that there is some value R_∞ which $R(z,-)$ approaches asymptotically with depth in optically infinitely deep media. This asymptotic value depends in a calculable way on both the inherent optical properties of the medium and on the limiting lighting conditions. Further remarks on the behavior of $R(z,-)$ at great depths are made in the closing section of this paper.

2. <u>The Distribution Functions.</u> A particularly simple means of characterizing the depth dependence of the shape of radiance distributions, without resorting to an actual measurement of the radiance over all directions at each depth, is given by the <u>distribution functions:</u>

$$D(z,-) = \frac{h(z,-)}{H(z,-)} ,$$

$$D(z,+) = \frac{h(z,+)}{H(z,+)} . \tag{64}$$

It is easily seen from the definitions of h and H that if the shape of the radiance distribution changes with depth, then $D(z,-)$ and $D(z,+)$ will change with depth; and conversely, if the values of the distribution functions vary with depth, the radiance distributions must be changing shape with depth. It is clear from the definitions that $D(z,-)$ gives an index of the shape of the radiance distribution in the upper hemisphere (i.e., for the downwelling flux), and $D(z,+)$ does a similar job of characterizing the shape of the radiance distribution in the lower hemisphere (i.e., for the upwelling flux).

Detailed experimental studies of the light field in Lake Pend Oreille show that both $D(z,+)$ and $D(z,-)$ exhibit relatively little change with depth (Tyler,1958a). Furthermore, this independence of depth is found whether the external lighting conditions are sunny or overcast. Under either of these conditions, the values $D(z,-)$ hovered very closely in the neighborhood of 1.3, while the values $D(z,+)$ clustered around 2.7. Examples of $D(z,-)$ and $D(z,+)$ are given in Table 1. It appears at present that these values should be typical of the values that one may find in many natural hydrosols.

Of course, as in the case of $R(z,-)$, the quantities $D(z,-)$ and $D(z,+)$ will obstinately refuse to have any sweeping generalizations made about the fine structure of their depth dependence. However, as in the case of $R(z,-)$, simple theoretical tools exist which can be directed toward such problems if the need ever arises to discuss depth dependence in detail (Preisendorfer 1957c). Furthermore, the ultimate depth dependence of $D(z,-)$ and $D(z,+)$ in deep media is quite regular and predictable (see closing section).

The observed constancy of the distribution functions with depth has important practical consequences. In homogeneous media exhibiting this type of behavior a few well-selected measurements of the inherent optical properties together with radiance distributions near the surface would suffice as the basis for an estimate of the quantity and quality of the light field for <u>all</u> depths in the medium. Such estimates could be made by means of the two-D model (Preisendorfer 1957b) or the simple radiance model (Preisendorfer 1957c).

In addition to characterizing the depth dependence of the angular structure of radiance distributions, $D(z,-)$ and $D(z,+)$ play indispensable roles in the equations of applied radiative transfer theory, particularly in those equations which link the inherent and apparent optical properties of a medium. These roles will be illustrated as a matter of course in the discussions below.

3. The K-Functions. The reflectance function gives a running account of the relative magnitudes of the irradiance of each stream of radiant flux. In this section we now discuss the quantities which characterize the individual depth dependence of the up and downwelling irradiances and of the scalar irradiance. These are called the K-functions. The motivations for the definitions of these functions are supplied by both theoretical and experimental precedent extending back over at least fifty years of applied radiative transfer theory.

The theoretical motivation for the K-functions for irradiance and scalar irradiance defined below stems from an attempt to increase the usefulness of the Schuster equations for the two-flow analysis of the light field. The detailed development of this approach and its practical applications recently have been completed (Preisendorfer 1958a).

The experimental motivation for the K-functions rests in early empirical relations of the kind:

$$I_z = I_0 e^{-Kz} , \tag{65}$$

which simultaneously were to characterize the depth dependence of I_z and define its depth-rate of decay, K. In the above relation I_z took many forms: in some studies it was downwelling irradiance, in others it was a scalar irradiance-like quantity; in still others, its exact nature was not quite clear. Therefore, there was no universal agreement as to what radiometric quantity it should represent. As a result, there was no agreement as to what it really measured. A plot of I_z on semi-log paper with depth as abscissa yielded $-K$ as the slope of the streight line. K could thus be defined operationally as:

$$K = -\frac{1}{z} \ln \left[\frac{I_z}{I_0} \right] . \tag{66}$$

It suffices to observe here that these early theoretical and experimental approaches to characterize a K-like optical property of natural hydrosols were inadequate to the subsequent needs for precision and completeness in modern hydrological optics. In current basic research I_z is replaced by the three precisely defined irradiances $H(z,-)$, $H(z,+)$, and $h(z)$. Furthermore, it has become necessary to distinguish not only between the magnitudes $H(z,-)$, $H(z,+)$, and $h(z)$, but also their logarithmic rate of change with depth. Careful measurements (Table 1) show that their logarithmic rates of change are generally different, and the difference far exceeds the range of experimental error. In general, semi-log plots of $H(z,-)$ $H(z,+)$, and $h(z)$ also exhibit noticeable departures from linearity, especially in near-surface regions. This fact, of course, is part of the folklore of the study of hydrological optics which has been extant for many years, but this non-linearity has been considered more of an annoyance than a source of enlightening information. In particular this non-linearity made it impossible to define a single unambiguous K, of the kind appearing in (66) which otherwise could be used to help classify the optical properties of the medium.

The current views in hydrological optics are such that the departures from linearity by semi-log plots of $H(z,-)$, $H(z,+)$ and $h(z)$ are a source of extremely useful insight into the intricate structure of light fields in natural hydrosols. Far from being ignored, these departures from linearity should be welcomed as harbingers of new and deeper understanding. The logarithmic slopes of the $H(z,-)$, $H(z,+)$, and $h(z)$ plots are defined in general as follows:

$$K(z,\pm) = -\frac{1}{H(z,\pm)}\frac{dH(z,\pm)}{dz}, \tag{67}$$

$$k(z) = -\frac{1}{h(z)}\frac{dh(z)}{dz}.$$

Some Relations Between Inherent and Apparent Optical Properties

We continue our present discussion of optical properties of natural hydrosols by exhibiting a few general relations between the inherent and apparent optical properties discussed above. These relations have been found helpful in collating the data of basic experimental research and have provided, in some instances, deeper insight into the whys and hows of the fine structure of the depth dependence of the apparent optical properties. The derivations of these relations need not concern us here. These details, and some further relations may be found elsewhere (Preisendorfer 1958a).

The most important of these connecting relations is the following:

$$R(z,-) = \frac{K(z,-) - a(z,-)}{K(z,+) + a(z,+)}, \tag{69}$$

where

$$a(z,\pm) = D(z,\pm)\, a(z)\,. \tag{70}$$

Thus (69) links together the K -functions for irradiance, the R -functions and the D -functions, i.e., the main apparent optical properties, with the inherent optical property a .

There also are available the following useful inequalities:

$$a(z,-) \leq K(z,-) \leq \alpha(z,-) \tag{71}$$

or equivalently:

$$a(z) \leq \frac{K(z,-)}{D(z,-)} \leq \alpha(z) . \tag{72}$$

Similarly,

$$a(z,+) \leq -K(z,+) \leq \alpha(z,+) \tag{73}$$

or equivalently:

$$a(z) \leq -\frac{K(z,+)}{D(z,+)} \leq \alpha(z), \tag{74}$$

where

$$\alpha(z, \pm) = D(z, \pm)\, \alpha(z). \tag{75}$$

The right-hand sides of all these inequalities hold without qualification. However, the left-hand side of (71) holds whenever $0 \leq K(z, +)$. The left-hand side of (73) holds whenever $K(z, -) \leq 0$. While our treatment of the downwelling and upwelling streams has been deliberately kept symmetrical whenever possible, nature takes a hand in the matter at this point and clearly shows a preference to the downwelling stream in the following sense: the condition $0 \leq K(z, +)$ almost always holds, so that the inequalities of (71) for downwelling stream almost always hold. However, the condition $K(z, -) \leq 0$ almost never holds, so that the left side of (73) for the upwelling stream almost never holds. The condition $K(z, -) \leq 0$ means that the downwelling stream is constant or growing with increasing depth, a situation which occurs, if at all, only in regions of very shallow depths in the hydrosol, or in regions where there are self-luminous sources distributed throughout some layer.

Some further inequalities which are helpful in checking experimentally obtained optical properties and which aid in the understanding of the mutual interactions between the up and downwelling streams of radiant flux are:

$$K(z,+)\,R(z,-) \leqslant K(z,-)\,, \tag{76}$$

or equivalently

$$\frac{dH(z,-)}{dz} \leqslant \frac{dH(z,+)}{dz} \tag{77}$$

These relations hold for arbitrarily stratified source-free media. The same is true for:

$$\frac{dR(z,-)}{dz} = R(z,-)\left[K(z,-) - K(z,+)\right]. \tag{78}$$

The quantities $a(z,\pm)$; $\alpha(z,\pm)$ defined in (70) and (75) are <u>hybrid</u> optical properties: they are the result of simple combinations of the inherent and apparent optical properties. Eq. (70) gives the volume absorption function for each stream, and (75) gives the volume attenuation function for each stream.

These quantities by definition do not fall directly into either the inherent or apparent class.

To round out and complete the picture of the hybrid optical properties, we mention the <u>(volume) forward scattering functions</u>:

$$f(z, \pm), \tag{79}$$

the <u>(volume) backward scattering functions</u>:

$$b(z, \pm), \tag{80}$$

and the <u>(volume) total scattering functions:</u>

$$s(z, \pm), \tag{81}$$

<u>for each stream</u>. Detailed definitions and discussions of these quantities may be found in the references (Preisendorfer 1957b). The hybrid optical properties play important roles in the exact theoretical discussions of the two-flow analysis of the light fields. They also are of use in collating experimental data on inherent and apparent optical properties. Examples of such uses may be found in the references (Preisendorfer 1958a).

The Behavior of the Apparent Optical Properties at Great Depths

It was emphasized repeatedly during the introduction and discussion of the apparent optical properties that they exhibit certain useful, regular behavior patterns. One of the most striking of these patterns occurs at great depths in optically deep natural waters. We briefly summarize here some of the more important of these facts. Proofs of these results, their historical background, and practical consequences are given elsewhere. (Preisendorfer 1958b, 1958c, 1958d).

For simplicity, we consider an infinitely deep source-free homogeneous natural hydrosol. In actuality the results cited below hold in all natural hydrosols in which the ratio σ/α becomes independent of depth with increasing depth.

In analogy to $K(z,+)$, $K(z,-)$, and $k(z)$, we can define one more K-function. This is associated with radiance $N(z,\theta,\phi)$:

$$K(z,\theta,\phi) = -\frac{1}{N(z,\theta,\phi)}\frac{dN(z,\theta,\phi)}{dz}. \quad (82)$$

It can be shown that

(i) $k(z)$ approaches a limit as, $z \to \infty$. Let this limit be denoted by k_∞.

In symbols:

$$k_\infty = \lim_{z \to \infty} k(z) . \tag{83}$$

It can be shown that k_∞ does not exceed α . In symbols:

$$0 \leq k_\infty \leq \alpha .$$

(ii) For each fixed (θ, ϕ) , $K(z, \theta, \phi)$ approaches a limit as $z \to \infty$, and this limit is independent of (θ, ϕ) . This common limit for all directions (θ, ϕ) is k_∞ . In symbols:

$$\lim_{z \to \infty} K(z, \theta, \phi) = k_\infty$$

for all (θ, ϕ) .

(iii) $K(z, -)$ and $K(z, +)$ approach limits as $z \to \infty$ and these limits are equal to k_∞ . In symbols:

$$\lim_{z \to \infty} K(z, -) = \lim_{z \to \infty} K(z, +) = k_\infty .$$

(iv) The distribution functions $D(z, +)$ and $D(z, -)$ approach a limit as $z \to \infty$. Let these limits be denoted by $D(+)$ and $D(-)$. In symbols:

$$D(-) = \lim_{z\to\infty} D(z,-) \, ,$$

$$D(+) = \lim_{z\to\infty} D(z,+) \, . \qquad (84)$$

(v) The reflectance function $R(z,-)$ approaches a limit as $z \to \infty$. Let this limit be denoted by R_∞. In symbols:

$$R_\infty = \lim_{z\to\infty} R(z,-) \, .$$

Then it follows from (69) that

$$R_\infty = \frac{k_\infty - D(-)\,a}{k_\infty + D(+)\,a} \, . \qquad (85)$$

We conclude with a few observations: Property (i) shows that the depth dependence of the amount of light (more precisely, radiant density, or scalar irradiance) in a natural hydrosol eventually becomes exactly exponential in behavior. The value k_∞ is uniquely determined by σ and α. Property (ii) states that the radiance distribution eventually assumes a fixed angular structure (the asymptotic radiance distribution) at great depths. This limiting angular structure is readily found in principle; it is independent of the external lighting conditions, and depends only on the angular structure of σ . Property (iv) is an equivalent assertion to (ii), but now phrased in terms of the distribution functions. The quantities $D(+)$ and $D(-)$ are readily obtained from the limiting form of the radiance distribution functions. Properties (iii) and (i) show that the logarithmic derivatives of irradiance and scalar irradiance eventually coincide as depth increases indefinitely. It may easily be shown that the logarithmic derivatives of $h(z,+)$ and $h(z,-)$ also approach k_∞ as $z \to \infty$ (Of course, then so do the logarithmic derivatives of $h_{4\pi}$, and $h_{4\pi}(z,\pm)$ approach k_∞ as $z \to \infty$). Finally, property (v) states that $R(z,-)$ approaches a fixed value as $z \to \infty$, and this value is characterized in terms of k_∞ , $D(\pm)$, and α , as shown in (85).

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FOOTNOTES

Footnote 1. In radiometric discussions requiring extreme care with respect to dimensions, it is sometimes necessary to make an explicit distinction between outward and inward types of radiant flux. In such events, it is customary to use P_o for outflowing flux and P_i for inflowing flux, so that $W = P_o / A$ and $H = P_i / A$ become symbolically distinct.

Footnote 2. For example, in radiative transfer theory, just as in fluid dynamics and neutron transport theory, the equations are most easily formulated by adopting a Lagrangian approach: the investigator follows in imagination the flow of material in its natural path through space, and tallies up its gains and losses all along the path. This tally takes the form of a continuity equation; in the case of radiative transfer theory, it is the equation of transfer for radiance in which the surface radiance (7) is most conveniently used .

Footnote 3. For complete generality, we must also take into account the possibility that the index of refraction of the medium differs at S and at G . This situation is encountered, for example, when the line of sight has one end in air and the other in water. To this end, suppose that the index of refraction at S is n_o, and at G is n_r. Then the appropriate form of (22) is $N_r = (n_r/n_o)^2 T_r N_o$. T_r retains its interpretation as the transmittance for the path. Its general form is $T_r = exp\{-\int_0^r \alpha(r')\, dr'\}$ which reduces to (21) if α is a constant over the path.

Footnote 4. The relation between scalar irradiance and radiant density u (no. of joules of radiant energy per unit volume) is: $h = v u$ where v is the speed of light in meters per second.

Footnote 5. A preliminary word about notation and terminology at this point would be desirable. The present discussion is concerned with the phenomena of scattering and absorption in optical media, specifically natural hydrosols. The mathematical concepts that handle these ideas are: the volume scattering function σ, the volume absorption function a, and the volume attenuation function α. First of all, the word "volume" is used to distinguish these quantities from their "mass" counterparts in astrophysical optics in which the passage of light is principally through gaseous rather than incompressible liquid media. Furthermore, the word "attenuation" is understood to denote the effects of the simultaneous action of scattering and absorption. Finally, the word "function" is used to point up the fact that the quantities a, and α

are functions in the mathematical sense: to each point of the optical medium they assign a real number which--with the appropriate units--has the physical significance described in detail below. If a, α, and σ are independent of position in the medium, then the medium is said to be homogeneous, and to point up this fact throughout a discussion which employs a and α, we will refer to these as the volume absorption coefficient and volume attenuation coefficient, respectively. The quantity σ will always be referred to as a function, because it depends in general not only on position in the medium but on two given directions. In the interests of simplicity all subsequent discussions will deal with homogeneous media.

Footnote 6. We observe that the decomposition of the flow of radiant energy in a natural hydrosol need not be into up and downwelling flows: the two flows could conceivably be thought of as occurring across any arbitrarily oriented plane. Furthermore, it is quite possible to consider decompositions into more than two flows. However, the essentially plane-parallel geometric structure of all natural hydrosols and their almost unanimous propensity toward horizontal stratification of physical properties assigns a particularly high utility to the adopted two-flow decomposition.

Footnote 7. This function is independent of depth in homogeneous media (for definition, see footnote 6).

Footnote 8. This function is generally dependent on depth, even in homogeneous media (cf. Table 2, and appropriate defining equations).

TABLE 1

EXAMPLES OF THE VALUES OF $D(z,\pm)$, $K(z,\mp)$, $a(z)$, $R(z,-)$

z (Meters)	$D(z,-)$	$D(z,+)$	$K(z,-)$	$K(z,+)$	$a(z)$	$R(z,-)$
4.24	1.247	2.704				0.0215
7.33			0.129	0.126		
10.42	1.288	2.727	0.153	0.150	0.115	0.0184
13.50			0.178	0.174		
16.58	1.291	2.778	0.174	0.172	0.118	0.0204
22.77			0.171	0.170		
28.96	1.313	2.781	0.169	0.169	0.117	0.0227
35.13			0.167	0.167		
41.30	1.315	2.757	0.165	0.165	0.117	0.0235
47.50			0.162	0.163		
53.71	1.307	2.763	0.158	0.158	0.112	0.0234
59.90			0.154	0.154		

Explanation of Table 1: Depths and units are in terms of meters. Data is associated with a wavelength of 480 mμ and was derived from radiance information summarized in (Tyler, 1958a). The optical medium (Lake Pend Oreille, Idaho) was found to be essentially homogeneous, the volume attenuation coefficient being α = 0.402/meter. The sky was clear and sunny with the sun at about 40° from the zenith. The values $a(z)$ were obtained by means of (57).

TABLE 2

HIERARCHY OF OPTICAL PROPERTIES IN HYDROLOGICAL OPTICS

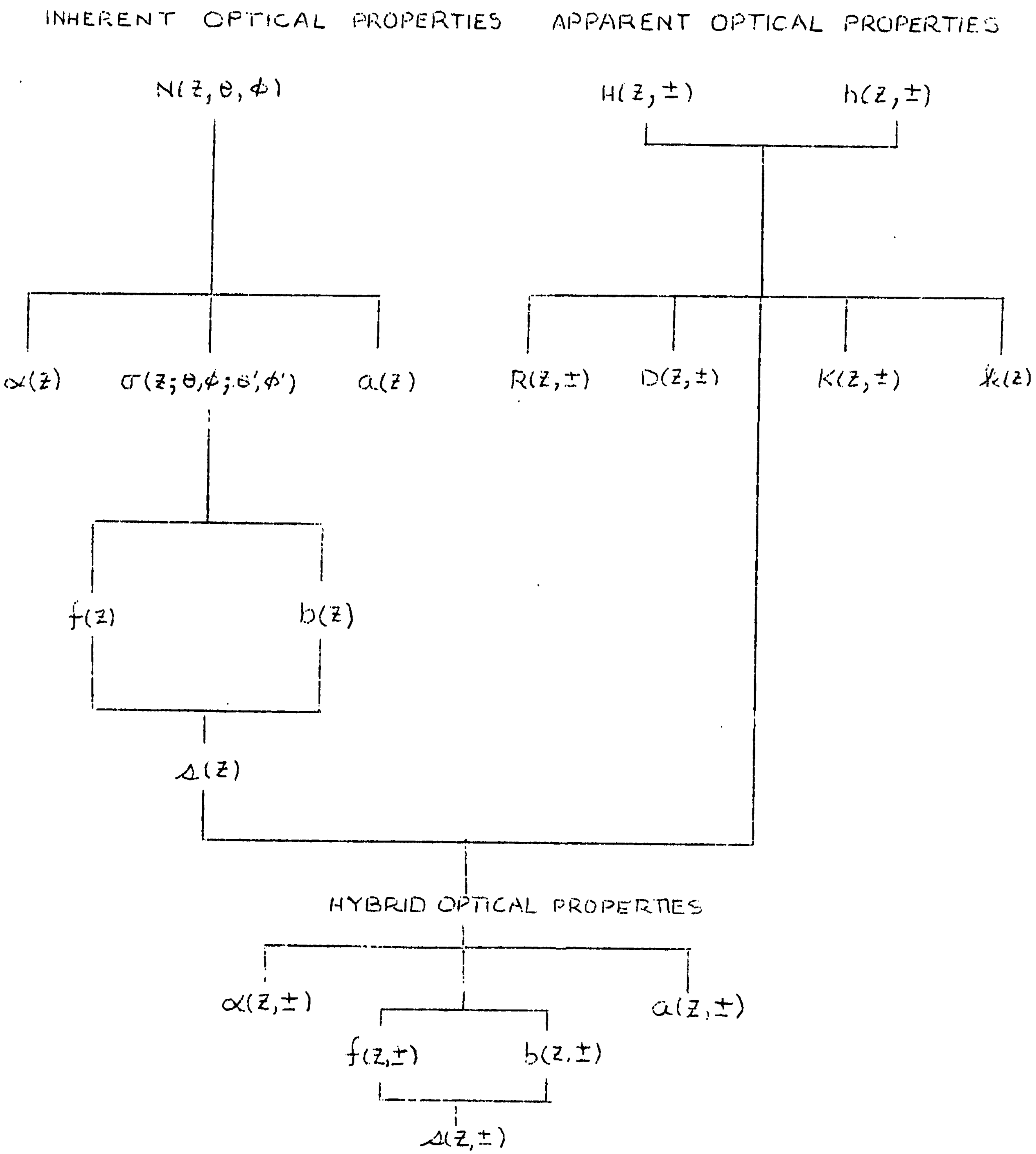

TABLE 3

INDEX OF SPECIAL CONCEPTS USED IN HYDROLOGICAL OPTICS

SYMBOL	DESCRIPTION OF CONCEPT	DEFINING EQUATIONS
$N(z,\theta,\phi)$	Radiance at depth z in direction (θ,ϕ)	(5)
$N_*(z,\theta,\phi)$	Path function at depth z in direction (θ,ϕ)	(49)
$N_r^*(z,\theta,\phi)$	Path radiance of path of length r, with initial point at depth z, direction (θ,ϕ)	(24)
$H(z,\pm)$	Upwelling (+) and downwelling (–) irradiance at depth z	(18)
$h(z,\pm)$	Upwelling (+) and downwelling (–) scalar irradiance at depth z	
$h_{4\pi}(z,\pm)$	Upwelling (+) and downwelling (–) spherical irradiance at depth z	(59), (60)
$T_r(z,\theta,\phi)$	Beam transmittance of path of length r with initial point at depth z, direction (θ,ϕ)	(21)
$\alpha(z)$	Value of volume attenuation function at depth z[7]	(41) *
$\sigma(z;\theta,\phi;\theta',\phi')$	Value of volume scattering function at depth z[7] for incident flux in direction (θ',ϕ'), and scattered flux in direction (θ,ϕ)[7]	(46), (49)
$f(z)$	(Volume) forward scattering function at depth z[7]	(51) *
$b(z)$	(Volume) backward scattering function at depth z[7]	(52) *
$s(z)$	(Volume) total scattering function at depth z[7]	(53) *

* Refer to footnote 5.

SYMBOL	DESCRIPTION OF CONCEPT	DEFINING EQUATIONS
$a(z)$	Volume absorption function at depth z.[7]	(57)
$R(z,\pm)$	Reflectance at depth z for upwelling (+), downwelling (−) flux.[8]	(63)
R_∞	Limit of $R(z,-)$ as $z \rightarrow \infty$	(85)
$D(z,\pm)$	Distribution function for upwelling (+), downwelling (−) flux at depth z.[8]	(64)
$D(\pm)$	Limit of $D(z,\pm)$ as $z \rightarrow \infty$	(84)
$K(z,\pm)$	K-function for upwelling (+), downwelling (−) irradiance at depth z.[8]	(67)
$k(z)$	K-function for scalar irradiance at depth z.*	(68)
k_∞	Limit of $k(z)$ as $z \rightarrow \infty$	(83)
$\alpha(z,\pm)$	Volume attenuation function for upwelling (+), downwelling (−) flux at depth z.[8]	(70)*
$f(z,\pm)$	(Volume) forward scattering function for upwelling (+), downwelling (−) flux at depth z.[8]	(79)*
$b(z,\pm)$	(Volume) backward scattering function for upwelling (+), downwelling (−) flux at depth z.[8]	(80)*
$s(z,\pm)$	(Volume) total scattering function for upwelling (+), downwelling (−) flux at depth z.[8]	(81)*
$a(z,\pm)$	Volume absorption function for upwelling (+), downwelling (−) flux at depth z.[8]	(70)*

* Refer to footnote 5.

TABLE 4

BASIC RADIOMETRIC CONCEPTS

NAME	BASIC SYMBOL	m.k.s. UNITS	DEFINITION
Radiant Flux	P	watt	Basic
Radiant Emittance	W	watt/m^2	$W \equiv P_o/A$
Irradiance	H	watt/m^2	$H \equiv P_i/A$
Radiance (Field) (Surface)	N	watt/(m^2 x steradian)	$N \equiv P_i/A\Omega = \frac{H}{\Omega}$ $N \equiv P_o/A\Omega = \frac{W}{\Omega}$
Scalar Irradiance	h	watt/m^2	$h \equiv \int_{4\pi} N \, d\Omega$
Spherical Irradiance	$h_{4\pi}$	watt/m^2	$h_{4\pi} \equiv P_i/4\pi r^2$ $h_{4\pi} = \frac{1}{4} h$
Radiant Density	u	joule/m^3	$u \equiv h/v$
Radiant Energy	U	watt sec = joule	$U \equiv \int_R u \, dV$

FIGURE LEGENDS

Figure 1a Figure 1b Figure 1c	Schematic diagrams of radiant flux meter, Gershun tube, and spherical irradiance meter.
Figure 2a	Illustrating conceptual duality of irradiance H and radiant emittance W.
Figure 2b	Derivation of cosine law for surface radiance.
Figure 3a	Derivation of the cosine law for irradiance.
Figure 3b	Derivation of the inverse square law for irradiance.
Figure 4a Figure 4b	Derivation of the relation between surface radiance and field radiance.
Figure 4c	Derivation of the formula for path radiance.
Figure 5	Derivation of the relation between scalar irradiance and spherical irradiance.
Figure 6a	Experimental arrangement for the determination of volume attenuation function α.
Figure 6b	Hypothetical plot of experimental results for determination of α.
Figure 7	Experimental arrangement for the determination of the volume scattering function σ.
Figure 8	The experimental determination of the volume absorption function a.
Figure 9a	Schematic diagrams for instruments to measure up- and downwelling spherical irradiance.

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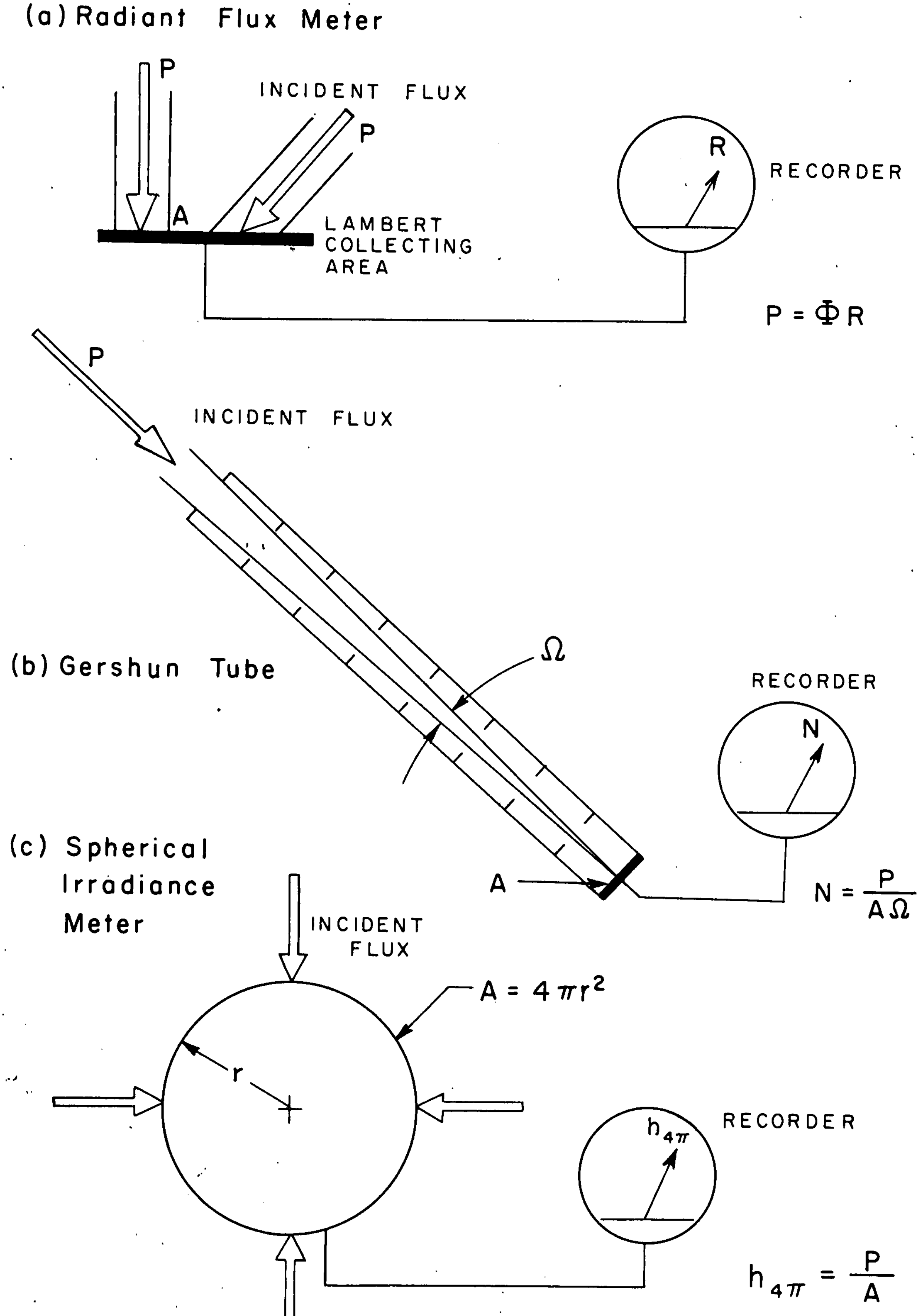
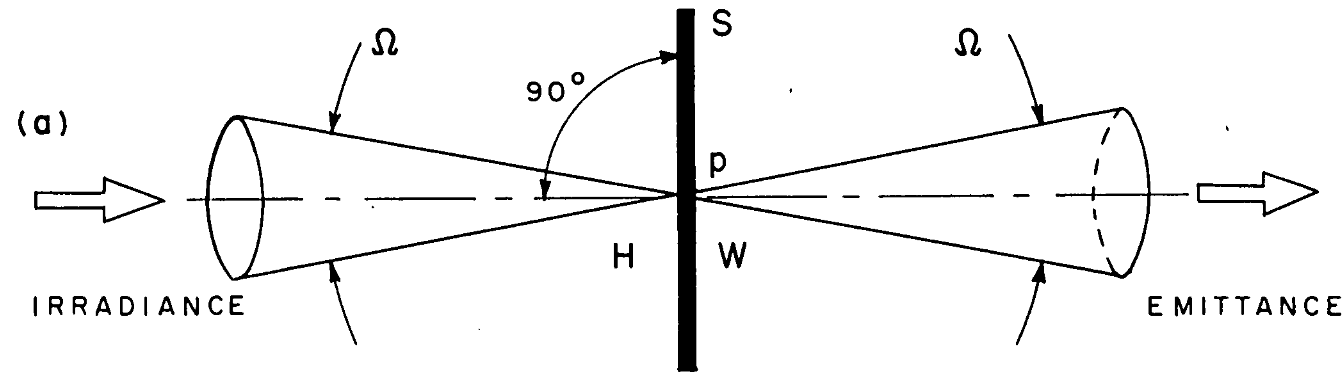


Figure 1
Preisendorfer and
Tyler



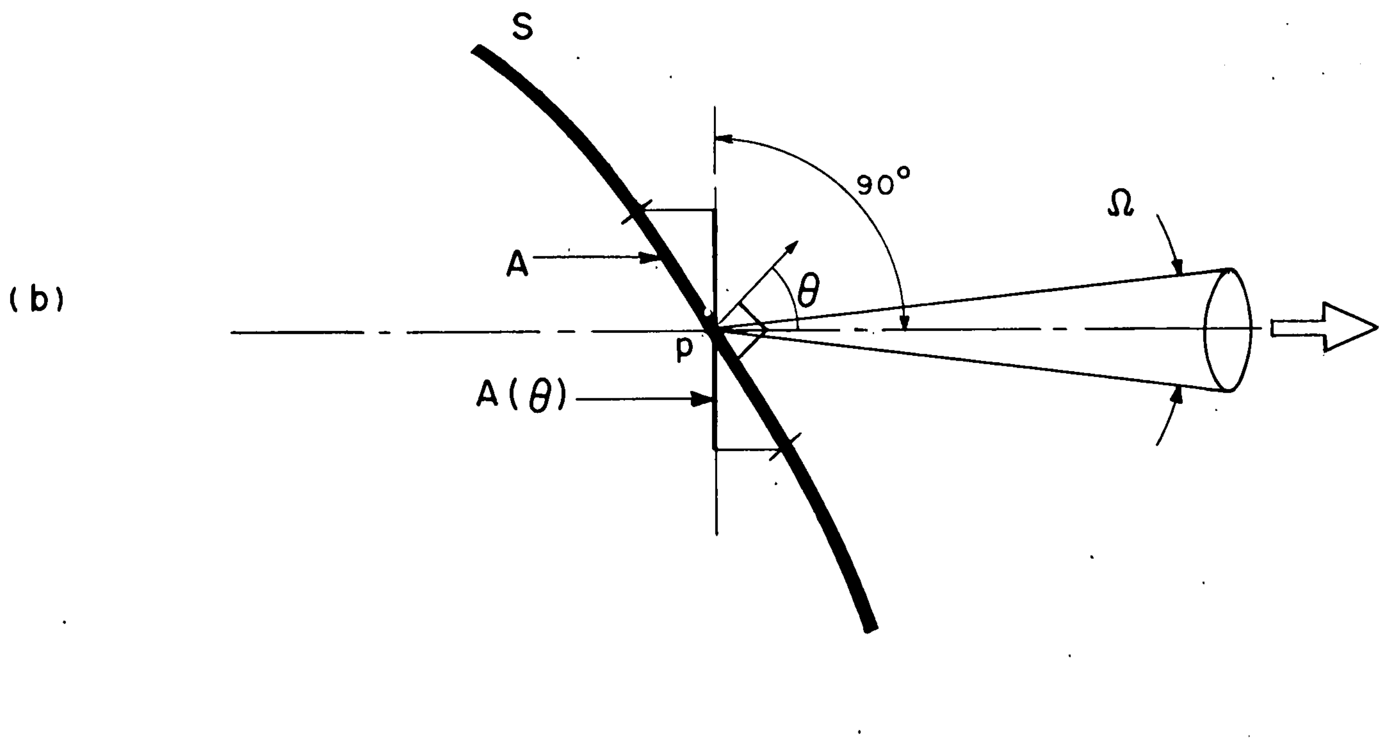


Figure 2

Preisendorfer and Tyler

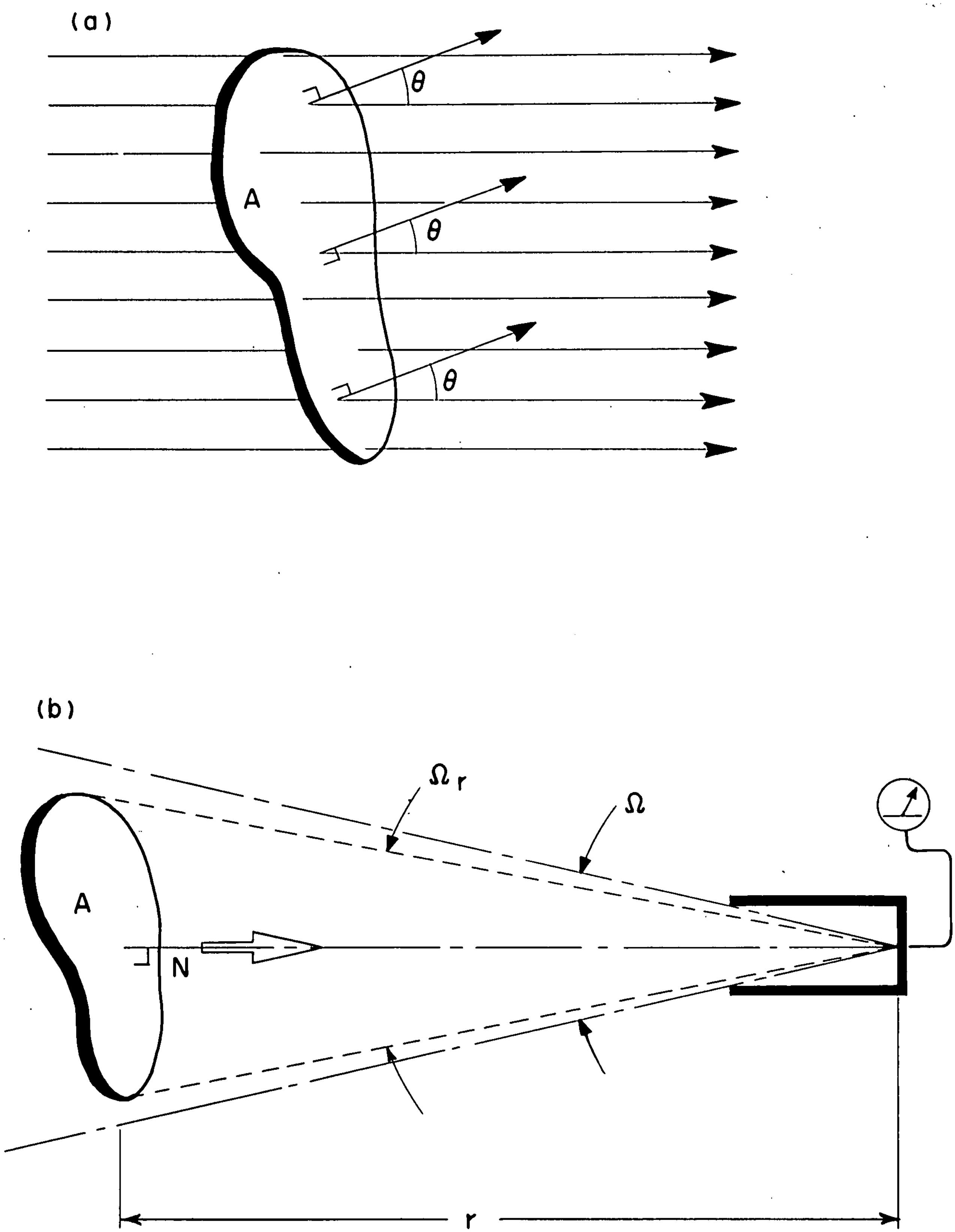


Figure 3

Preisendorfer and Tyler

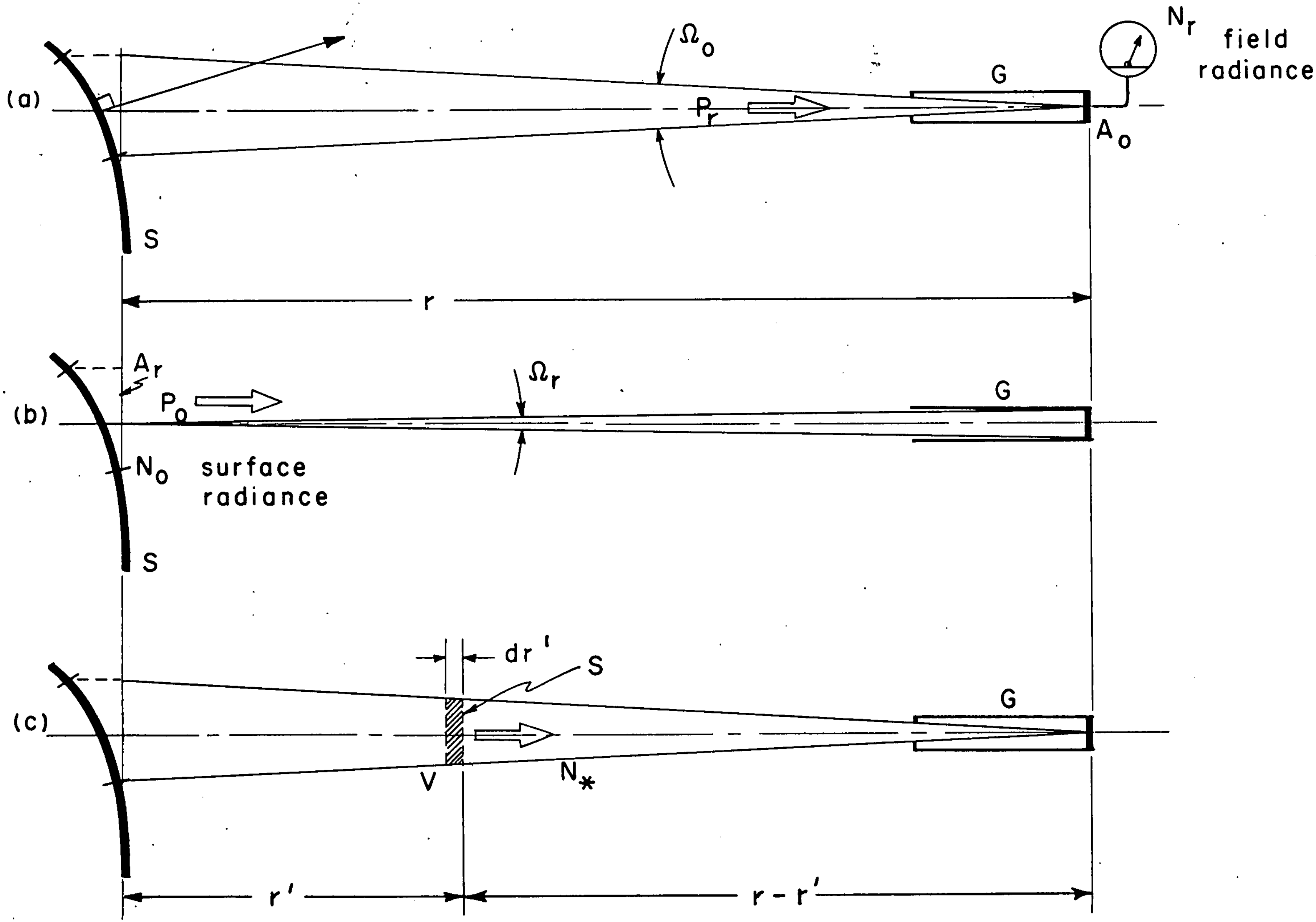


Figure 4
Preisendorfer and Tyler

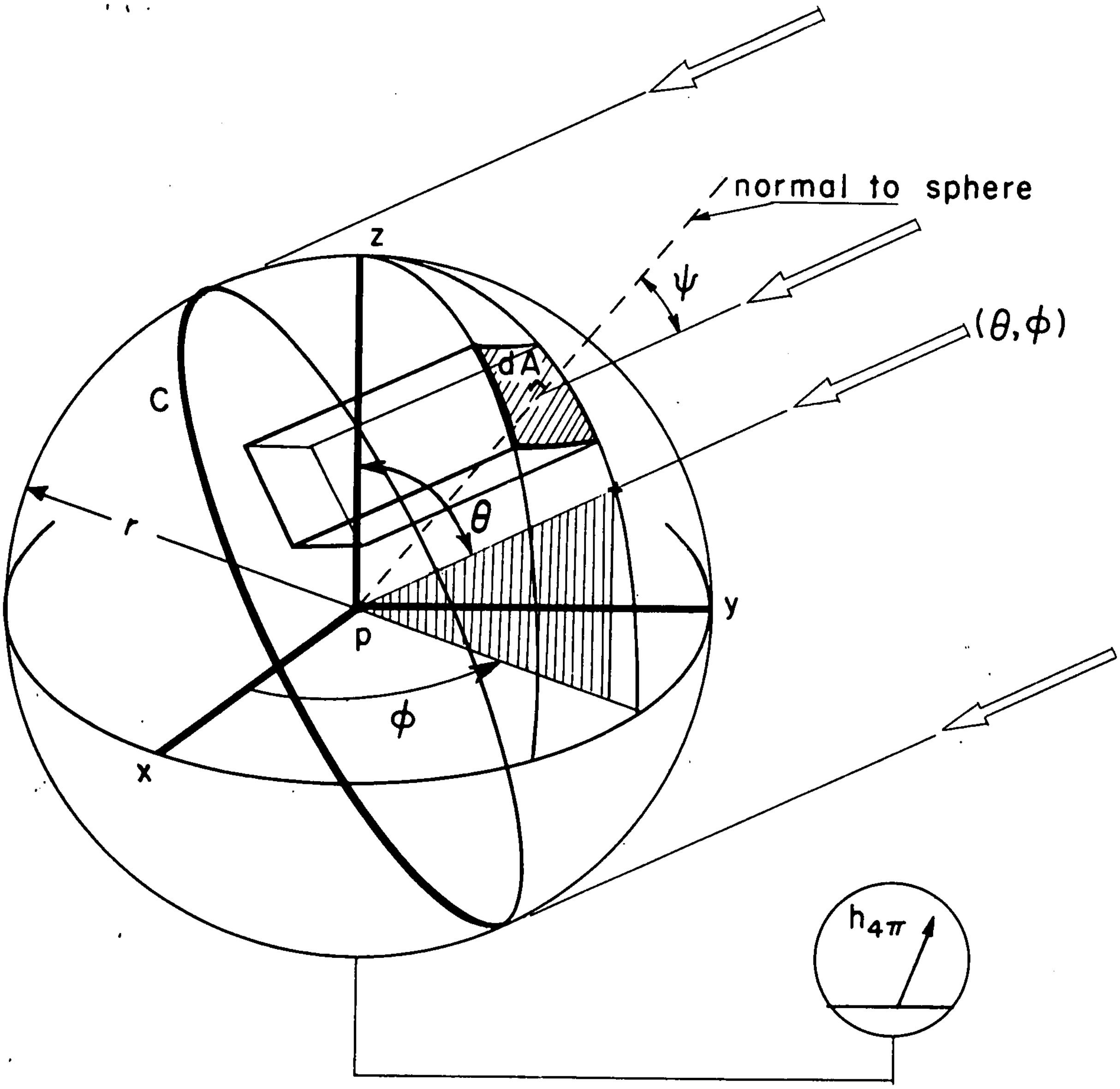


Figure 5

Preisendorfer and Tyler

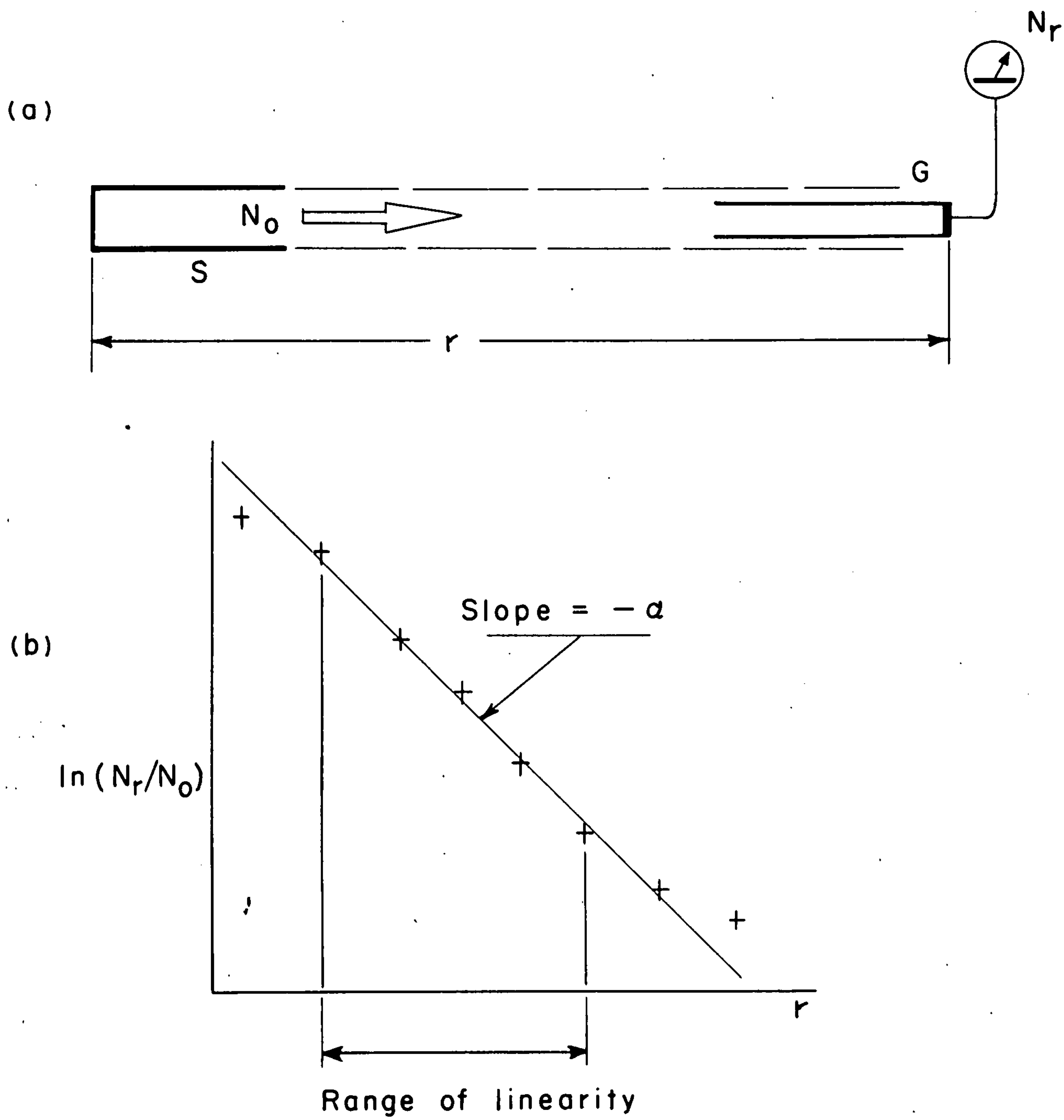


Figure 6
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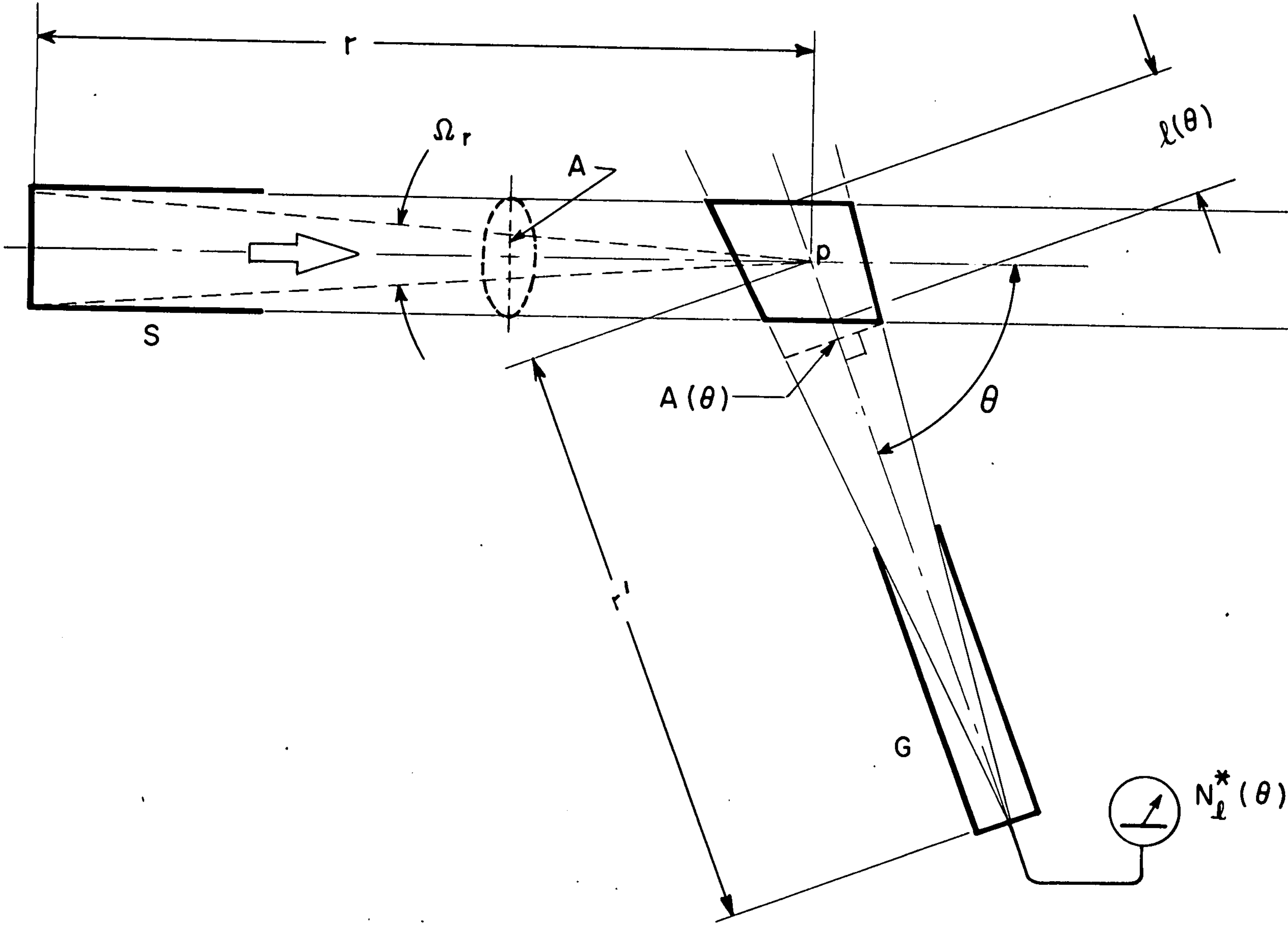

Figure 7
Preisendorfer and Tyler

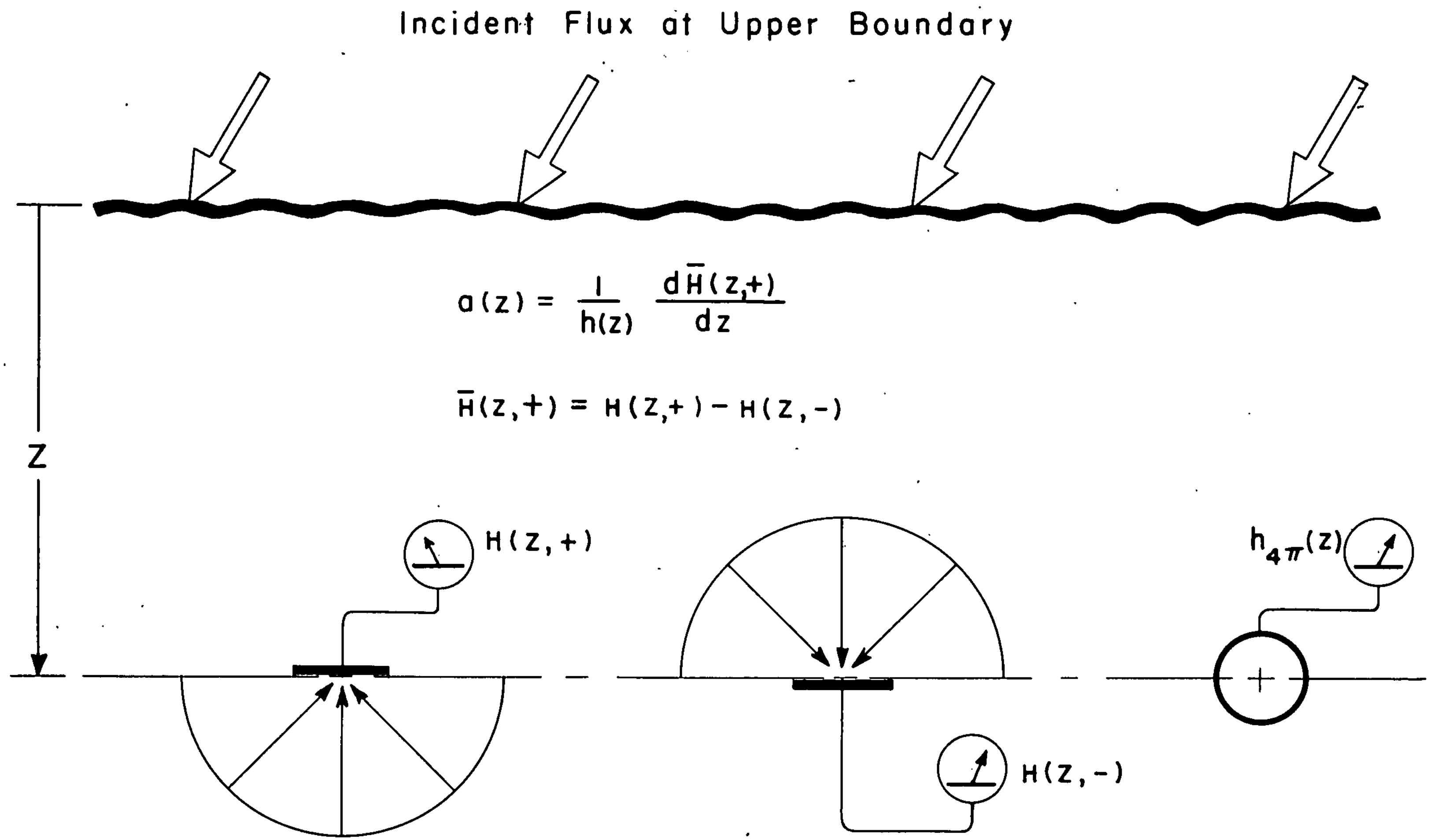

Figure 8
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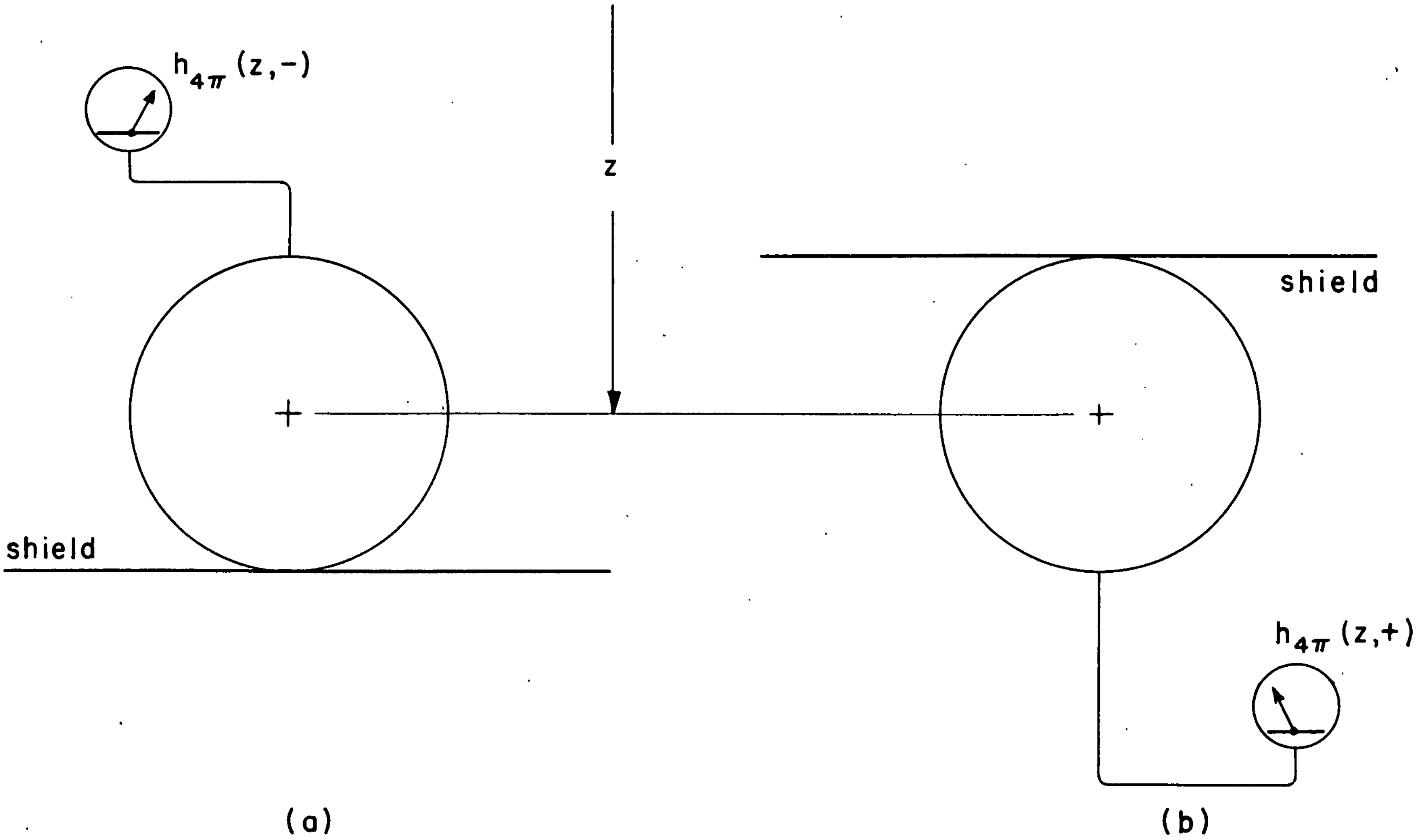

Figure 9
Preisendorfer and Tyler