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THE OPTICAL PROPERTIES OF LAKE PEND OREILLE

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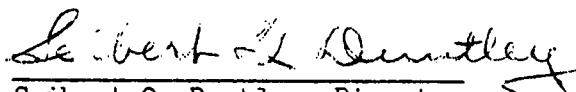
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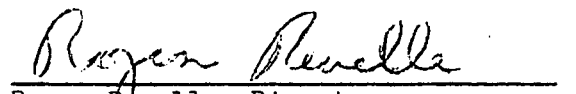
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The Optical Properties of Lake Pend Oreille^{*}

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ABSTRACT

Eight important optical properties have been obtained for Lake Pend Oreille, Idaho, using the method of measurement of radiance distribution as a function of depth. The method for computing the properties from radiance distributions is given in detail. These properties are associated with a narrow band of wavelengths at 480 m μ and are given for a single depth. They should be characteristic of the lake water in the absence of plankton or excess contamination.

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INTRODUCTION

In order to deal exactly with problems involving the penetration and transfer of light through water it is necessary to specify not only the magnitudes of the optical properties but also the depth at which they are measured, and whether they depend on the downwelling (-) or upwelling (+) streams of radiant flux. This is necessary because some of the optical properties of homogeneous water vary with depth although the variation may be small. In order to be exact in meaning, the notation for the optical property or function is followed by the symbol (z) which means we are speaking of the property at a specific depth, z , and with the symbol (-) referring to properties associated with the downwelling stream of radiant flux or (+) referring to the upwelling stream of radiant flux.

In general the variation of the properties tends to diminish with increasing depth and with increasing cloud cover and overcast. When the water is nonhomogeneous the usual variation with depth of the optical properties may be obscured by the further variations caused by this nonhomogeneity.

The important optical properties considered in this paper are shown in Figure 1 which also indicates in flow chart form one possible method of their determination from two basic physical measurements which were made at Lake Pend Oreille, Idaho (Tyler 1958). The two measurements were:

- (a) Radiance distribution as a function of depth, $N(z, \theta, \phi)$
- (b) Beam transmittance, T_r

MEASUREMENT OF RADIANCE DISTRIBUTION

The instrument used for the measurement of radiance distribution is shown in Figure 2. (Duntley, et.al. 1955). When in use, this instrument is suspended on a single cable and powered by an electric cable looped into the underside at the vertical axis of rotation. Rotation about the vertical axis (azimuth position) is controlled by means of a servo system consisting of a gyrosyn compass assembly in the instrument with control transformer and indicator on the main control panel. The error signal resulting from an azimuth mismatch between the control transformer and the gyro heading is used to drive a propeller which rotates the instrument so as to minimize the error signal thus keeping the instrument oriented in azimuth. The optical system is designed to accept light through a cone of acceptance having apex angle of 6.6° and the instrument thus measures relative values of the physical quantity radiance, in the direction specified by θ (the angle from the zenith) and ϕ (the azimuth angle).

Using data obtained with this instrument and the method of integration outlined in the next sections the apparent optical properties $R(z,-)$, $K(z,-)$, $K(z,+)$, $D(z,-)$, and $D(z,+)$ for Lake Pend Oreille were computed, along with the inherent optical property $a(z)$ (volume absorption function).

COMPUTATION OF $H(z,-)$ and $H(z,+)$ FROM $N(z,\theta,\phi)$

It is clear from Figure 1 that the functions $H(z,-)$, $H(z,+)$, $h(z,-)$ and $h(z,+)$ are essential to the determination of the functions $R(z,-)$, $K(z,-)$, $K(z,+)$, $a(z)$, $D(z,-)$ and $D(z,+)$. The basis for obtaining irradiance on a horizontal plane from radiance distribution data is expressed in Equation 1:

$$H(z, \pm) = \int_{\pm} N(z, \theta, \phi) |\cos \theta| d\Omega \quad (1)$$

$$\text{where } \int_{-} = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \quad ; \quad \int_{+} = \int_{\theta=\frac{\pi}{2}}^{\pi} \int_{\phi=0}^{2\pi}$$

$$d\Omega = \sin \theta d\phi d\theta$$

In order to perform the numerical integration, expressed in Equation 1, and thus obtain numerical values for $H(z,-)$ and $H(z,+)$, the radiance distribution solid was enclosed in a spherical grid divided by equal increments of θ and ϕ . The values of radiance at the centers of these increments were then weighted and summed in accordance with Formula (2).

The mechanics of the integration process at a given depth can be expressed in symbols as follows:

$$H = \sum_{i=0}^m \sum_{j=0}^n N_{ij} \Delta \Omega'_{ij} \quad (2)$$

where $N_{0j} = N_{00}$ for $j = 0, 1, \dots, n$; since the radiance in the zenith direction does not change with azimuth; and where the $\Delta \Omega'_i$ are weighted solid angles defined for the indicated zones ($i = 0, 1, \dots, m$) as follows:

$$\Delta \Omega'_i = \sin \theta_i \cos \theta_i \Delta \theta \Delta \phi ; \text{ for } i = 2, 3, \dots, m-2$$

$$\Delta \Omega'_0 = \Delta \Omega'_m = \frac{3}{8} \sin\left(\frac{\Delta \theta}{4}\right) \cos\left(\frac{\Delta \theta}{4}\right) \Delta \theta \Delta \phi$$

$$\Delta \Omega'_{i+1} = \Delta \Omega'_{m-i} = \left[\sin \Delta \theta \cos \Delta \theta + \frac{\sin\left(\frac{\Delta \theta}{4}\right) \cos\left(\frac{\Delta \theta}{4}\right)}{8} \right] \Delta \theta \Delta \phi$$

and

$$m = \frac{\pi}{2 \Delta \theta} ; n = \frac{2 \pi}{\Delta \phi} - 1$$

$$i = 0, 1, \dots, m ; j = 0, 1, \dots, n$$

$$\theta_i = i \Delta \theta ; \phi_j = j \Delta \phi$$

The preceding Formula (2) may be used for either $H(\mathbf{z}, -)$ or $H(\mathbf{z}, +)$. Figure 3 shows the geometrical arrangement and identifies centroid values of radiance for $H(\mathbf{z}, -)$. To use (2) for the determination of $H(\mathbf{z}, +)$, reflect the sphere in its equatorial plane.

COMPUTATION OF $h(z, -)$ and $h(z, +)$ FROM $N(z, \theta, \phi)$

The procedure for obtaining scalar irradiance from radiance distribution data is expressed by:

$$h(z, \pm) = \int_{\pm} N(z, \theta, \phi) d\Omega \quad (3)$$

The mechanics of numerical integration to obtain hemispherical scalar irradiance, $h(z, -)$, $h(z, +)$, are expressed in Formula (4) which employs the same general notation as Formula (2) and Figure (3), and is used in the same general manner.

$$h(\text{hemispherical}) = \sum_{i=0}^m \sum_{j=0}^n N_{ij} \Delta\Omega_i \quad (4)$$

where

$$N_{0,j} = N_{00}$$

and where the $\Delta\Omega_i$ are weighted solid angles defined for the indicated zones ($i = 0, 1, \dots, m$) as follows:

$$\Delta\Omega_i = \sin\theta_i \Delta\theta \Delta\phi \quad ; \text{ for } i = 2, 3, \dots, m-2$$

$$\Delta\Omega_0 = \frac{3}{8} \sin\left(\frac{\Delta\theta}{4}\right) \Delta\theta \Delta\phi$$

$$\Delta\Omega_1 = \left[\sin\Delta\theta + \frac{\sin\left(\frac{\Delta\theta}{4}\right)}{8} \right] \Delta\theta \Delta\phi$$

$$\Delta\Omega_{m-1} = \left[\cos\Delta\theta + \frac{\cos\frac{\Delta\theta}{4}}{8} \right] \Delta\theta \Delta\phi$$

$$\Delta\Omega_m = \frac{3}{8} \cos\left(\frac{\Delta\theta}{4}\right) \Delta\theta \Delta\phi$$

In evaluating Formulas (2) and (4) each hemisphere was divided into 8 increments of $\Delta\theta$ and 2 increments of $\frac{\Delta\theta}{2}$ and into 18 increments of $\Delta\phi$. By substituting the values of

$$\Delta\theta = \frac{\pi}{2m} = 10^\circ = 0.1745 \text{ rad}$$

$$\Delta\phi = \frac{\pi}{n} = 20^\circ = 0.3491 \text{ rad}$$

into formulas (2) and (4) a table of factors, $\Delta\Omega$ and $\Delta\Omega'$ was obtained (Table I).

For each hemisphere the sum of the 18 values of N_{ij} for each i was multiplied by the corresponding $\Delta\Omega'_i$ or $\Delta\Omega_i$ and the sum of the results gave the irradiance or scalar irradiance, respectively, for the hemisphere.

TABLE 1

	Θ_i	$\Delta \Omega_i$	$\Delta \Omega'_i$
0	180	0.000 966	0.000 966
10	170	0.010 918	0.010 747
20	160	0.020 835	0.019 579
30	150	0.030 461	0.026 379
40	140	0.039 160	0.029 997
50	130	0.046 665	0.029 997
60	120	0.052 758	0.026 379
70	110	0.057 247	0.019 579
80	100	0.067 603	0.010 747
90	90	0.022 824	0.000 996

COMPUTATION OF OPTICAL PROPERTIES

In modern two flow theory the functions describing the interaction of light with water must, in general, be associated with the specific stream to which they apply (Preisendorfer 1958a). For horizontally stratified scattering-absorbing media (for example, large bodies of water), there will be one set of functions for the downwelling stream (identified by the symbol -) and another set of functions for the upwelling stream (identified by the symbol +). The functions which have been computed in this experiment from the radiance distribution are as follows:

	Downwelling	Upwelling
Attenuation functions	$K(z,-)$	$K(z,+)$
Volume absorption functions	$a(z)$	$a(z)$
Reflectances of a hypothetical horizontal plane	$R(z,-)$	$R(z,+) = \frac{1}{R(z,-)}$
Distribution functions	$D(z,-)$	$D(z,+)$

COMPUTATION OF $K(z,-)$ and $K(z,+)$

The attenuation functions, $K(z,-)$ and $K(z,+)$ for natural downwelling and upwelling light in the water are determined by the formula:

$$K(z, \pm) = \frac{\ln \frac{H(z - \frac{1}{2} \Delta z, \pm)}{H(z + \frac{1}{2} \Delta z, \pm)}}{\Delta z} \quad (5)$$

This formula is derived from the definition of $K(z, \pm)$,

$$H(z, \pm) = H(0, \pm) e^{-\int_0^z K(z', \pm) dz'}$$

where the primed symbols denote dummy variables of integration:

Then, in particular

$$H\left(z + \frac{\Delta z}{2}, \pm\right) = H\left(z - \frac{\Delta z}{2}, \pm\right) e^{-\int_{z-\Delta z}^{z+\Delta z} K(z', \pm) dz'}$$

If Δz is sufficiently small $K(z, \pm)$ may be treated as being constant over the interval $z - \frac{\Delta z}{2}, z + \frac{\Delta z}{2}$ and:

$$\begin{aligned} \int_{z-\frac{\Delta z}{2}}^{z+\frac{\Delta z}{2}} K(z', \pm) dz' &= K(z, \pm) \int_{z-\frac{\Delta z}{2}}^{z+\frac{\Delta z}{2}} dz' = \\ &= K(z, \pm) \Delta z = \ln \left[\frac{H\left(z - \frac{\Delta z}{2}, \pm\right)}{H\left(z + \frac{\Delta z}{2}, \pm\right)} \right] \end{aligned}$$

The attenuation functions $K(z, \pm)$ generally vary with depth. Their values at any given depth, z , are determined by the operational procedure summarized in (5), which requires knowledge of the irradiances just above and just below z .

COMPUTATION OF $R(z, -)$

The reflectance, $R(z, -)$, of a hypothetical horizontal plane in water with respect to the downwelling flux at a depth, z , below the surface was computed from the defining equation:

$$R(z, -) = \frac{H(z, +)}{H(z, -)} \quad (6)$$

Note that

$$R(z, +) = \frac{H(z, -)}{H(z, +)} = \frac{1}{R(z, -)}$$

COMPUTATION OF DISTRIBUTION FUNCTIONS

Distribution functions, $D(z, \pm)$, are measures of the directional distribution of the upwelling and downwelling radiance. They are defined (Preisendorfer 1958 a) by the equations:

$$D(z, \pm) = \frac{h(z, \pm)}{H(z, \pm)} \quad (7)$$

COMPUTATION OF THE ABSORPTION FUNCTION

The value of the absorption function, $a(z)$, was determined from the formula:

$$a(z) = \frac{\ln \left[\frac{\bar{H}(z - \frac{\Delta z}{2}, +)}{\bar{H}(z + \frac{\Delta z}{2}, +)} \right] \bar{H}(z, +)}{\Delta z h(z)} \quad (8)$$

where $\bar{H}(z, +) = H(z, +) - H(z, -) = -\bar{H}(z, -)$, net upwelling irradiance,

$h(z) = h(z, +) + h(z, -)$, the scalar irradiance.

Formula (8) is developed as follows:

The operational definition of the absorption function, $a(z)$, is:

$$a(z) = \frac{1}{h(z)} \cdot \frac{d\bar{H}(z, +)}{dz} \quad (\text{Preisendorfer 1957}) \quad (9)$$

Now, by definition of $\bar{K}(z)$, we have:

$$\frac{d\bar{H}(z, +)}{dz} = -\bar{H}(z, +) \bar{K}(z) \quad (10)$$

and

$$\bar{H}(z, +) = \bar{H}(0, +) e^{-\int_0^z \bar{K}(z') dz'}$$

Furthermore, between the two depths, $z - \frac{\Delta z}{2}$, $z + \frac{\Delta z}{2}$, we have by definition of $\bar{K}(z)$:

$$\bar{H}\left(z + \frac{\Delta z}{2}, +\right) = \bar{H}\left(z - \frac{\Delta z}{2}, +\right) e^{-\int_{z - \frac{\Delta z}{2}}^{z + \frac{\Delta z}{2}} \bar{K}(z') dz'} \quad (11)$$

$\bar{K}(z)$ may be considered constant over the interval $z - \frac{\Delta z}{2}$, $z + \frac{\Delta z}{2}$ when Δz is sufficiently small. Therefore, from (11):

$$\int_{z - \frac{\Delta z}{2}}^{z + \frac{\Delta z}{2}} \bar{K}(z') dz' = \bar{K}(z) \int_{z - \frac{\Delta z}{2}}^{z + \frac{\Delta z}{2}} dz' = \bar{K}(z) \Delta z =$$

$$= \ln \left[\frac{\bar{H}(z - \frac{\Delta z}{2})}{\bar{H}(z + \frac{\Delta z}{2})} \right] \quad (12)$$

Substituting in (9) the values of $\frac{d\bar{H}}{dz}$ as given in (10) and $\bar{K}(z)$ as given in (12) we have the convenient operational formula for $a(z)$ (8).

OTHER PHYSICAL MEASUREMENTS

In addition to the radiance distribution measurements described above, it was possible to measure some of the water properties by direct methods. The function $K(z, -)$ was obtained by direct measurement using a flat-plate collector to obtain relative values of $H(z, -)$ at various depths. The collecting surface of this instrument, shown in Figure 4, is carefully designed to collect radiant flux in accordance with the law:

$$P(\theta) = P(0) \cos \theta, \text{ i.e., the surface is a Lambert collector. } \quad (13)$$

The volume attenuation function α was measured with an hydrophotometer shown in Figure 5. The length of the path of the transmitted beam is $r = 1$ meter. This instrument is designed to

be insensitive to changes in coupling (between the radiant flux and the detector) when the index of refraction of the medium changes over a wide range. It is also designed to minimize the effect of forward scattered light within the beam. Recent work by Preisendorfer (1958b) makes it possible to predict that the error created by forward scattered light within the beam will be of the order of 0.3% in the value of α .

RESULTS

The experimental values obtained at a depth of 29 meters for eight optical properties of Lake Pend Oreille are given in Table III. The total scattering coefficient for collimated light, ξ , can be found from a and α by Equation 14:

$$\alpha(z) = a(z) + s(z) \quad (14)$$

The table includes this calculated value of ξ and also the directly measured value of $K(z,-)$ for $z = 29$ meters.

DISCUSSION

These measurements were carried out in the spring of 1957 before the start of the spring plankton bloom and well before the development of any thermocline. The lake was tested extensively to determine its homogeneity and was found to be very nearly homogeneous by every test. Therefore the values of $\alpha(29)$, $a(29)$ and $s(29)$

TABLE III

THE OPTICAL PROPERTIES OF LAKE PEND OREILLE AT DEPTH 29 METERS
AND FOR A WAVELENGTH BAND 64 MILLIMICRONS WIDE PEAKING AT 480 μ

Property	April 28, 1957	March 16, 1957	
	Computed from N(29, θ , ϕ)	Measured Directly	Calculated Indirectly
K(29, -)	0.169/m	0.184/m	
K(29, +)	0.164/m		
R(29, -)	0.0227		
D(29, -)	1.31		
D(29, +)	2.78		
a(29)	0.117/m		
α (29)		0.442/m	
s(29)			0.325/m

are representative values for all depths z in the lake. The values of the properties given here should approach the minimum values for the year. The values $K(z,-)$, $K(z,+)$, $R(z,-)$, $a(z)$, $s(z)$ would be expected to rise in the upper layers of water during the plankton bloom and would probably be higher for all depths during the heavy spring run-off.

All of the constants reported here are associated with a bandwidth of 64 m μ wide centered at 480 m μ .

In Table III the value of $K(29,-)$, measured directly, was obtained under overcast lighting conditions in March whereas the value computed from $N(29,0,\phi)$ is for clear sunny condition in April. These conditions may account for the difference in the two tabulated values of $K(29,-)$.

Table IV gives relative values of irradiance and scalar irradiance as obtained by the integration procedure. The array of properties for this range of depths may be readily computed from the tabular values.

One of the prime objectives of making these measurements was to provide numbers which could be used to confirm certain theoretical relationships which relate the various optical properties. For example, it has been shown (Preisendorfer 1958a) that in general:

$$K(z,-) - R(z,-)K(z,+) = a(z) \left[D(z,+)R(z,-) + D(z,-) \right]$$

Substituting values from column 1 of Table III gives

$$.165 \sim .161$$

TABLE IV

INTEGRATED RELATIVE VALUES OF IRRADIANCE AND SCALAR IRRADIANCE				
z (meters)	$H(z,-)$	$H(z,+)$	$h(z,-)$	$h(z,+)$
4.24	721000	15500	899000	41900
10.42	329000	6040	413000	16500
16.58	109000	2230	141000	6190
28.96	13100	298	17200	830
41.30	1660	39.0	2190	108
53.71	221	5.19	289	14.3

CAPTION TO FIGURE I

The computational route on the flow chart and intermediate steps are found by following the connecting lines in an upward direction from the property of interest.

Notation is as follows:

$N(z, \theta, \phi)$	Radiance at depth z in direction (θ, ϕ) .
$\frac{N_r}{N_o}$	Ratio of the radiance of a beam after passage through a water path of length r to its initial radiance.
$h(z, -)$	Downwelling scalar irradiance at depth z .
$H(z, -)$	Downwelling irradiance at depth z .
$h(z, +)$	Upwelling scalar irradiance at depth z .
$H(z, +)$	Upwelling irradiance at depth z .
T_r	Beam transmittance for path of length r .
$s(z)$	Total scattering function. at depth z .*
$D(z, -)$	Downwelling distribution function.
$D(z, +)$	Upwelling distribution function.
$R(z, -)$	Reflectance of downwelling light by a hypothetical horizontal plane in the water at depth z .
$K(z, -)$	Attenuation function for downwelling irradiance.
$K(z, +)$	Attenuation function for upwelling irradiance.
$a(z)$	Volume absorption function at depth z .*
$\alpha(z)$	Volume attenuation function at depth z .*

*In homogeneous media the term "function" as applied to these properties is replaced by the term "coefficient," since in homogeneous media these properties do not change with depth. The other functions generally vary with depth even in homogeneous media.

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Basic Measurements

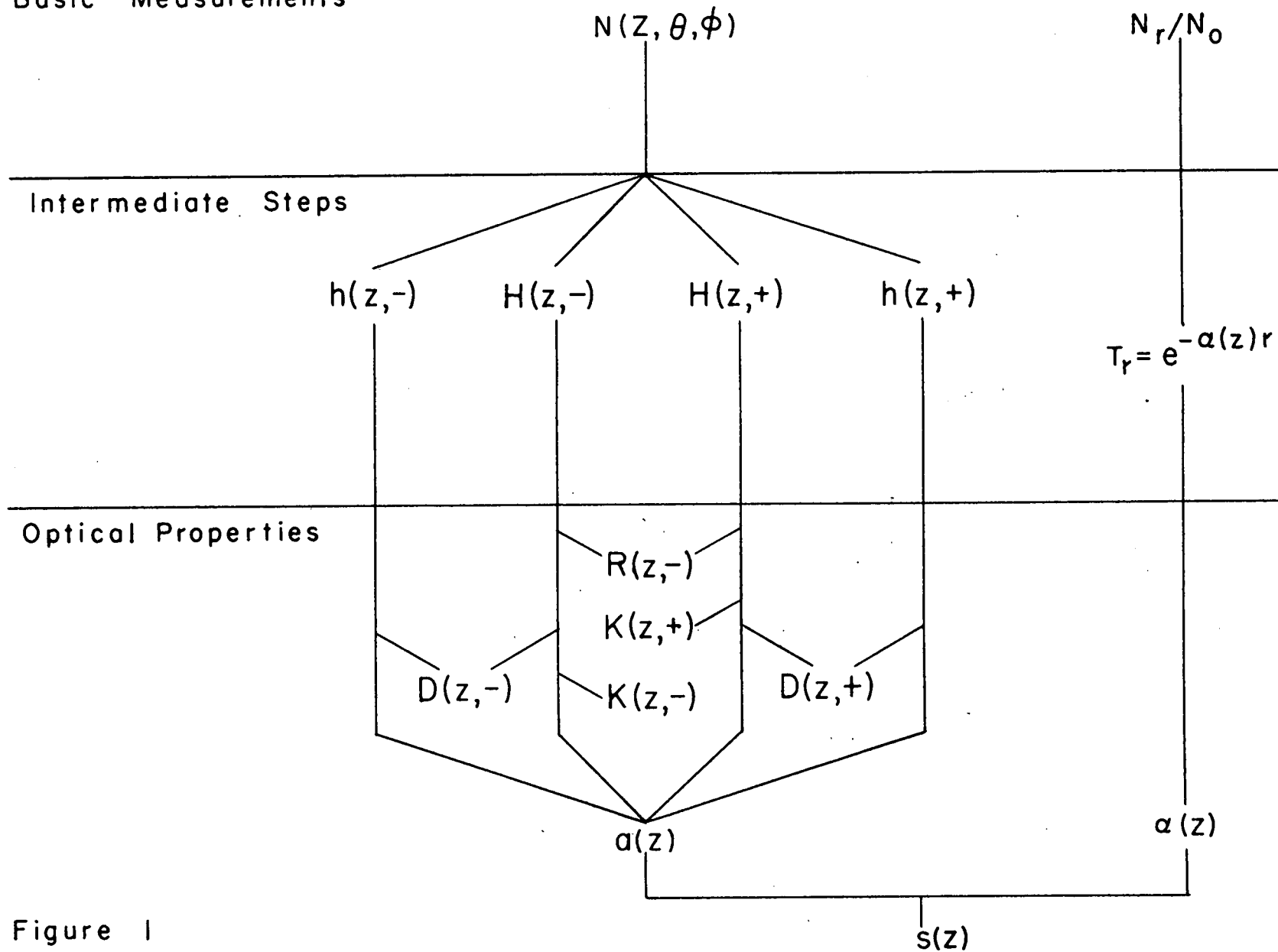


Figure 1

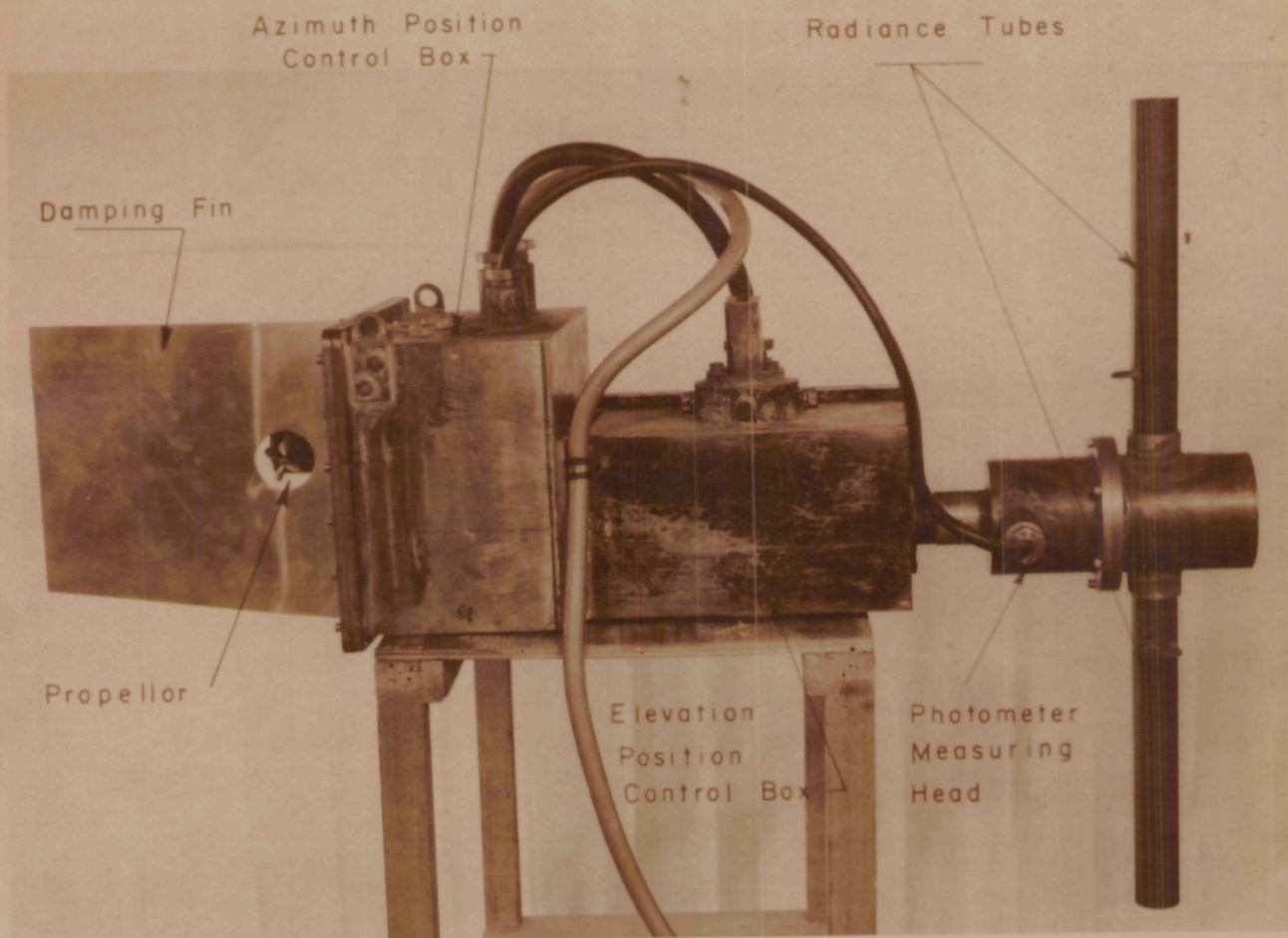


Figure 2

Underwater Photometer

* Measuring Head and Positioning Equipment

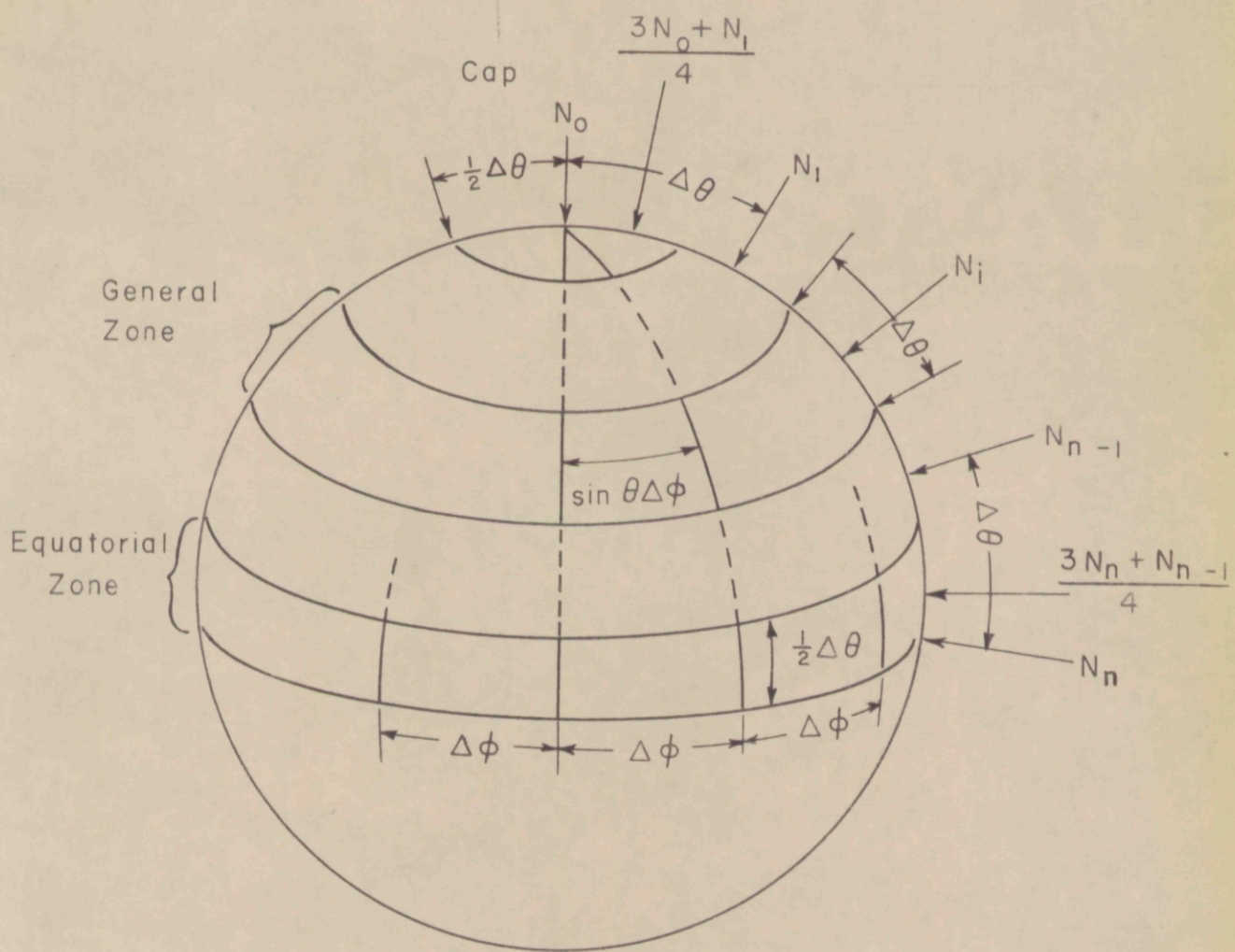


Figure 3

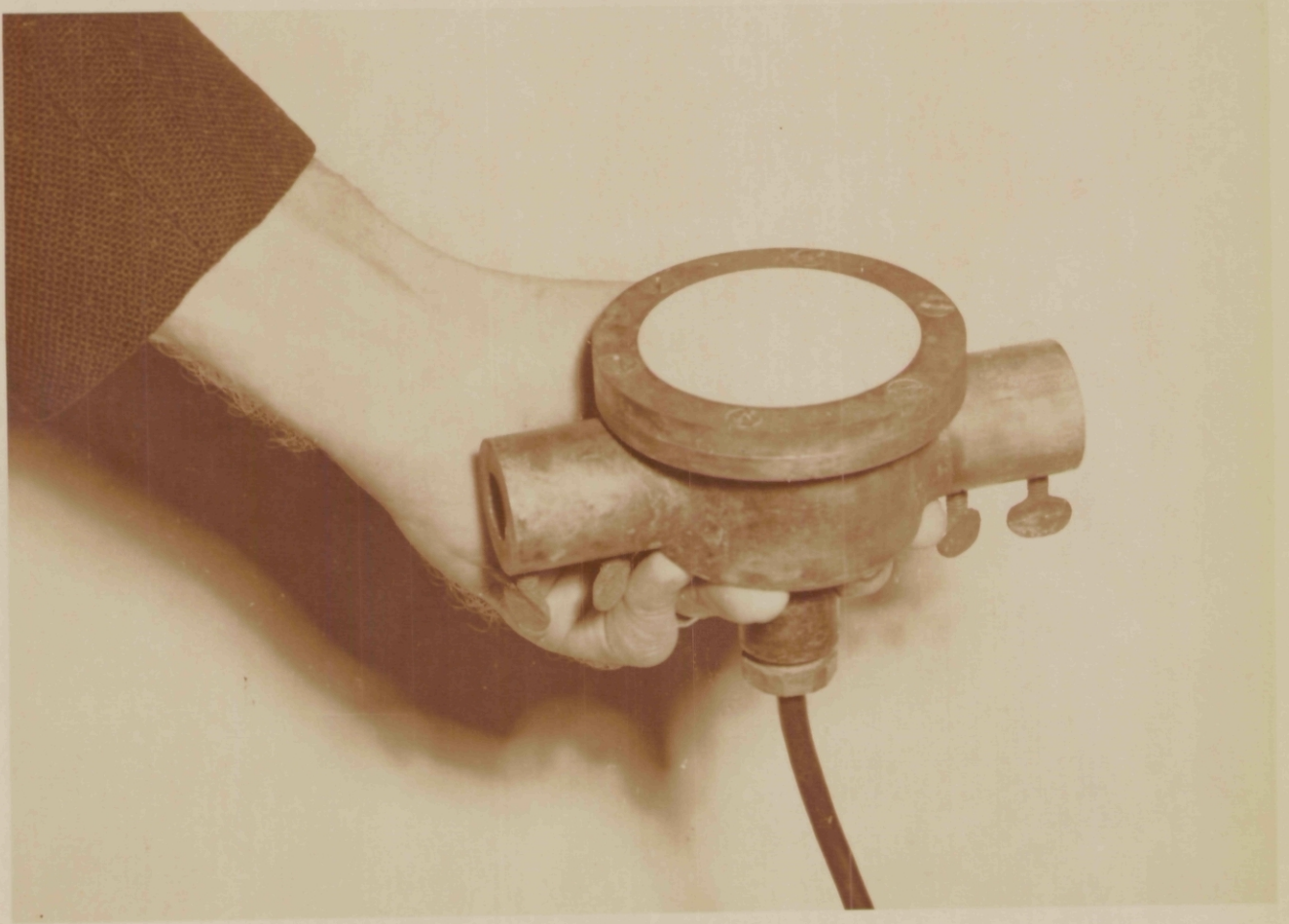


Figure 4

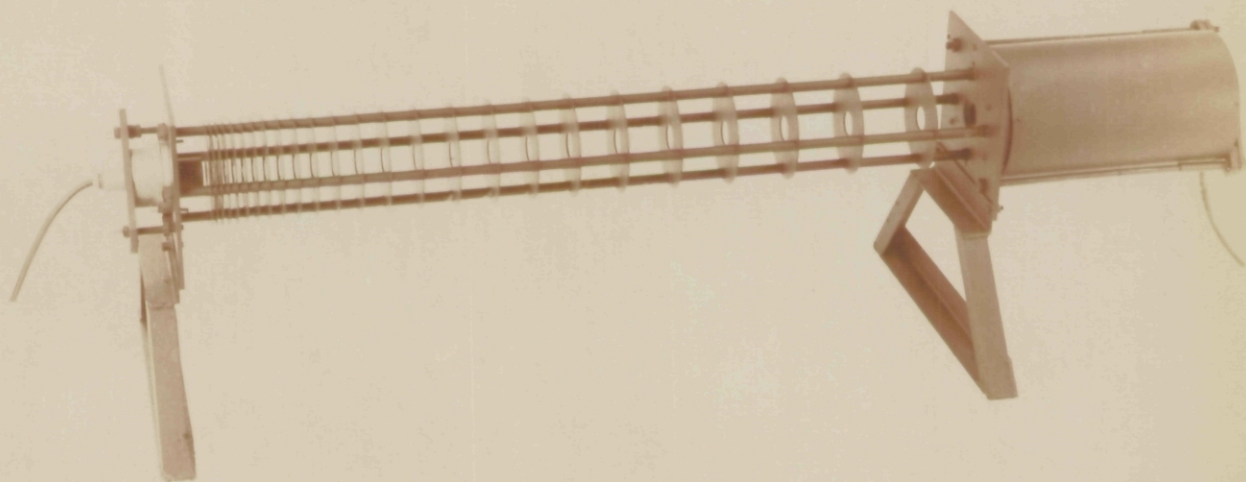


Figure 5