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ON GROUND-BASED MEASUREMENTS OF THE OPTICAL PROPERTIES OF THE ATMOSPHERE ALOFT

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INTRODUCTION

The continued study of image transmission problems through the atmosphere has made it possible to narrow down the large set of important physical factors involved in image transmission calculations to an irreducible core of three factors. For any given subregion X of the atmosphere and any given object Y in X, these three factors are:

Knowledge of

(1). The radiance distribution $N(x, \cdot)$ at each point x of X.

(2). The values $\alpha(x)$ of the volume attenuation function at each point x of X.

(3). The inherent radiance distribution $N_0(y, \cdot)$ at each point y of Y.

The basis of the selection of these three factors is given in the section on Fundamentals below. For the present, however, it is sufficient to point out that while increasingly greater detail in image transmission computations can be obtained by including correspondingly greater numbers of factors in the calculations such as the polarization factors, optical turbulence factors, etc., these latter factors are clearly of a secondary importance in the overwhelming majority of visibility prediction problems encountered in practice. While such additional factors are indispensible in certain special contexts they are not as universally fundamental as <u>un</u>polarized radiance distributions and volume attenuation values. Knowing only optical turbulence factors, for example, without knowing N and α will never yield a determinate solution to a visibility problem. Further, the extra expenditure in effort and time required to find the polarized components of a radiance distribution does not generally buy a correspondingly greater amount of utilizable information about image transmission phenomena.

Thus, by deliberately restricting attention to the small but essential set of factors, a sound practical understanding of general image transmission phenomena has been attained which otherwise would have been impossible had attention been directed at the outset to an intractably large array of physical factors. For example, by means of existing air-borne measuring and recording equipment, important and useful information on factor (1) in the atmosphere has been determined.[1] Furthermore, by limiting the large number of inherent optical properties to the single factor in (2), it has been possible to develop a virtually complete theory of its measurement under all practical conditions[2] by means of special air-borne measuring and recording equipment.

Despite these great inroads into the study of image transmission phenomena made possible by restricting attention to the above three-factor characterization, there remains essentially unsolved the important problem of how to measure factors (1) and (2) at all points in X from a fixed vantage point in X. The determination of factor (2) under such conditions is by far the more difficult of the two problems. This note is devoted primarily to a study of the determination of factor (2) within this context. Specifically, what means will allow $\alpha(x)$ to be determined at each point x aloft and yet allow the bulk of the measuring and recording equipment to remain on the ground?

This problem is not a new one. There are several suggested solutions in the literature, but each contains some vitiating assumption which restricts its universality or usefulness. In particular, this problem preoccupied Middleton repeatedly throughout his book on vision through the atmosphere.[3] In his words (p. 108), "... no method has yet been found for the determination of the [volume attenuation function] for a layer of air very far off the ground without going up there with rather formidable instrumentation ..." And again, (p. 227), "Up to now there is no satisfactory solution to the problem of measuring from the ground the [volume attenuation function] of layers at any height..." At present, seven years later, a general satisfactory solution has not been found.

We hope to accomplish two things in this note. The first is to suggest a possible solution of the problem of the determination from the ground of the α-values of the atmosphere aloft. The principal value of the suggestion lies in the fact that it incorporates no restrictive assumptions about the structures of either the light field or of the inherent optical properties of the atmosphere. In this sense the method goes beyond certain previous attempts to determine α by means of searchlights, fixed base lines on the ground, etc., (see e.g., chapter nine of reference 3).

The second goal of this note is of a more general nature and is reached in the section on Fundamentals. In that section we briefly present the justification of the choice of the factors (1), (2), (3) above, and go on to inquire what further information can be derived from knowledge of (1) and (2). The answer to this inquiry uncovers the fundamental importance of the radiance distribution N in modern studies of image transmission phenomena: Not only does knowledge of N permit the execution of immediate visibility computations in the usual ways but it may be considered as the fundamental datum of all pertinent knowledge about the inherent and apparent optical properties of an optical medium: <u>From knowledge of radiance distributions **alone** it is possible in principle to obtain complete solutions to image transmission problems</u>. The actual experimental realization of this principle must await the development of novel measuring and recording techniques. This will not present insuperable difficulties. More importantly, it must await understanding by the experimenters themselves and their consequent willingness to utilize it.

THEORETICAL BASIS OF THE PROPOSED GROUND-BASED METHOD

Volume Attenuation Function and Beam Transmittance

We begin with a review of the relation between the volume attenuation function α and the beam transmittance function T_r .

If a path of sight of length r lies in a region X of constant index of refraction and constant volume attenuation function, then the relation between α and T_r is:

$$T_r = e^{-\alpha r} . \tag{1}$$

If a path of sight of length r lies in a region X of constant index of refraction (an assumption which is, for all practical purposes, valid in any subregion of the atmosphere) but in which α may possibly vary markedly from point to point along the path, then

$$T_r = \exp \left\{ - \int_0^r \alpha(r') \, dr' \right\} , \tag{2}$$

where r' can be measured from either of the endpoints of the path.

To point up the fact that T_r in (2) generally depends on the location of the endpoints of the path, we write (2) as:

$$T_r(x, \xi) = \exp \left\{ - \int_0^r \alpha(x + r'\xi) \, dr' \right\} . \tag{3}$$

In Equation (3) the symbol x denotes the location vector of the observation point (and is therefore an ordered triple of real numbers $x = (x_1, x_2, x_3)$), and ξ denotes the unit direction vector which determines the orientation of the path at x (Figure 1). The length r is now measured along the path from x . Thus the path in question is uniquely determined by the triple (x, ξ, r) . Suppose photons start out along (x, ξ, r) at its far endpoint $y = x + r\xi$, travel in the direction $\eta = -\xi$, and eventually end up at x . These incoming photons are usually detected by a radiance tube at x with its axis directed along ξ . The quantity $T_r(x, \xi)$ gives the fraction of the photons transmitted from y to x along (x, ξ, r) .

A basic property of $T_r(x, \xi)$, which follows immediately from (3) is its <u>reciprocity property</u> which states that:

$$T_r(x, \xi) = T_r(y, \eta) , \tag{4}$$

where

$$y = x + r\xi , \qquad \eta = -\xi . \tag{5}$$

Thus the beam transmittance of the path (x, ξ, r) through a region of constant index of refraction—but over which α may be arbitrary—equals the beam transmittance of the path (y, η, r) , and the common value for each path depends jointly only on x and y (or equivalently, on either (x, ξ, r) or (y, η, r)). Hence the beam transmittance function may be said to be symmetrical with respect to x and y, and may be written as $T(x,y) = T(y,x)$ to point up this fact. However, we will retain the customary notation defined in (3), since it is more natural for our present purposes.

We now discuss a property of $T_r(x,\xi)$ which depends in an unsymmetric way on the endpoints x and y, and which, in fact, holds the key to the present method of the determination of α. Suppose the expression (3) is differentiated with respect to r, holding the observation point x and direction vector ξ fixed. We then obtain the formula:

$$\frac{dT_r(x,\xi)}{dr} = -T_r(x,\xi)\,\alpha(y). \tag{6}$$

Thus by letting (x,ξ,r) grow in the direction ξ, holding x fixed, formula (6) shows that the negative logarithmic derivative of T_r is precisely the value $\alpha(y)$ at the moving endpoint of the path:

$$\boxed{-\frac{d}{dr}\ln T_r(x,\xi) = \alpha(y).} \tag{7}$$

Herein lies the heart of the proposed method: Suppose that, by some means, $T_r(x,\xi)$ can be measured for a path (x,ξ,r) whose observation point x is fixed on the ground and whose length r changes in a known manner in the arbitrary but fixed direction ξ. Then the rate of change of $-\ln T_r(x,\xi)$ is precisely the desired value $\alpha(y)$ of α at the instantaneous location $y = x + r\xi$ of the moving endpoint of the path (x, ξ, r).

We now turn to a discussion of a possible means of determining $T_r(x, \xi)$.

Beam Transmittance and Radiance Ratios

Suppose there are two parallel paths (x_1, ξ, r), (x_2, ξ, r) of common length r in X such as those depicted in Figure 2. Then the radiances at the endpoints of each path are generally representable as (see (A1) in the section on Fundamentals, below):

$$N(x_1, \xi) = T_r(x_1, \xi)\, N(y_1, \xi) + N_r^*(x_1, \xi), \tag{8}$$

$$N(x_2, \xi) = T_r(x_2, \xi)\, N(y_2, \xi) + N_r^*(x_2, \xi), \tag{9}$$

where

$$y_i = x_i + r\xi, \quad i = 1, 2. \tag{10}$$

From the definitions of T_r and N_r^* it is easy to show that in all real media with continuous α on (x, ξ, r), $0 \le r < \infty$ (a valid general assumption for the real atmosphere, for example) the functions $T_r(\cdot, \xi)$ and $N_r^*(\cdot, \xi)$ are <u>continuous</u>. Therefore, in any real medium it is possible to choose x_1 and x_2 sufficiently close together so that for a given common pair (ξ, r) the pairs of values $T_r(x_1, \xi)$, $T_r(x_2, \xi)$

and $N_r^*(x_1,\xi)$, $N_r^*(x_2,\xi)$ are within any preassigned neighborhood, however small. Thus it is always possible to choose x_1 and x_2 sufficiently close so that, as far as any instrumental accuracy is concerned,

(a) $T_r(x_1,\xi) = T_r(x_2,\xi) = T_r(x,\xi)$ where $x = (x_1 + x_2)/2$.

(b) $N_r^*(x_1,\xi) = N_r^*(x_2,\xi) = N_r^*(x,\xi)$ where $x = (x_1 + x_2)/2$.

It then follows from (8) and (9) that, under conditions (a) and (b),

$$\Delta N_r(x,\xi) = T_r(x,\xi)\,\Delta N_o(y,\xi), \tag{11}$$

where

$$\Delta N_r(x,\xi) \equiv N(x_1,\xi) - N(x_2,\xi), \tag{12}$$

$$\Delta N_o(y,\xi) \equiv N(y_1,\xi) - N(y_2,\xi). \tag{13}$$

Therefore, we can use (11) to estimate $T_r(x,\xi)$ if we know $\Delta N_r(x,\xi)$ and $\Delta N_o(y,\xi)$:

$$\boxed{T_r(x,\xi) = \frac{\Delta N_r(x,\xi)}{\Delta N_o(y,\xi)}} \tag{14}$$

It is important to point out that while $N_r^*(\cdot,\xi)$ and $T_r(\cdot,\xi)$ are necessarily <u>continuous</u> mathematical functions on X , (so that conditions (a) and (b) universally hold) we cannot assert the same about the function $N(\cdot, \xi)$. However, it is this very fact--namely the possibility of the discontinuity of $N(\cdot, \xi)$ --which allows relatively large differences ΔN_r and ΔN_0 to be obtained--and used to advantage in (14)--whenever $\| x_1 - x_2 \|$ is made small.

Volume Attenuation Function and Radiance Ratios

Combining (7) and (14), we have, under the assumptions (a) and (b) above,

$$\boxed{-\frac{d}{dr} \ln \left[\frac{\Delta N_r(x, \xi)}{\Delta N_0(y, \xi)} \right] = \alpha(y) \quad \text{where} \quad y = x + r\xi} \tag{15}$$

We now consider some ways in which (15) can be realized in actual experimental procedures.

EXPERIMENTAL PROCEDURES

Two-Source Method

For the first procedure to be considered, suppose $N(y_1, \xi)$ and $N(y_2, \xi)$ are independent of y_1 and y_2, and are of known and different magnitudes. In actual practice they may be obtained by using two bright steady light sources of known spectral characteristics, and which are attached to some moving probe in X, such as a rocket. They may be held apart at some small fixed distance $\| y_1 - y_2 \| = \| x_1 - x_2 \|$ on the body of the probe. Suppose that the probe moves along the path of variable length defined by (x, ξ), $x = (x_1 + x_2)/2$, and has a speed $v(t)$ at time t. Hence at any time t the fixed point x and the probe define a path $(x, \xi, r(t))$ in X, where $r(t) = \int_0^t v(t')\,dt'$. A radiance recorder at x plots at every instant t the determinable quantity $A(t) = \ln[\Delta N_r(x, \xi)]$. Then, in accordance with (15), we have

$$\boxed{-\frac{1}{v(t)} \frac{dA(t)}{dt} = \alpha(x + r(t)\xi) \quad .} \tag{16}$$

The left-hand side of (16) is the directly computable quantity associated with the directly observable radiances $N(x_1, \xi)$ and $N(x_2, \xi)$ and the known speed $v(t)$. The result of the operation at time t yields $\alpha(x + r(t)\xi) = \alpha(t)$ the value of α at time t —or equivalently, the value of α at the instantaneous position of the probe.

Periodic-Source Method

For the second procedure suppose there is a single continually blinking source of radiant flux on the probe. More precisely, let a source on the probe have a periodic radiance $N(t)$ for all $t \geq 0$, such that the fundamental cycle of period T is defined by $N(t) = N_1$ for $0 \leq t \leq t_1$ and $N(t) = N_2$ for $t_1 < t \leq T$. Then during any one cycle the range of inherent radiance values is of magnitude $\Delta N_0 = |N_2 - N_1|$, and the corresponding range of apparent radiances observed at x is of magnitude ΔN_r.

We now assume that the period of the radiance cycle is sufficiently small so that over any period $(t, t+T)$ we have essentially $v(t)$ for the speed of the probe, and that the derivative of $A(t)$ may be approximated by a finite incremental quotient:

$$\boxed{-\frac{1}{v(t)} \frac{A(t+T) - A(t)}{T} \cong \alpha(x + r(t)\xi)} \tag{17}$$

If the fundamental cycle takes place over a sufficiently small period T then it might be more convenient to evaluate $\alpha(x + r(t)\xi)$ by means of quantities $A(t)$ which do not belong to adjacent cycles, but rather which are n cycles apart:

$$\boxed{-\frac{1}{v(t)} \frac{A(t+nT) - A(t)}{nT} \cong \alpha(x + r(t)\xi).} \tag{18}$$

Discussion of the Preceding Methods

The arguments for and against the above procedures take extremely simple forms and are mutually independent. The argument for them observes that they are based on exact theoretical results and thereby have the highest degree of potential accuracy; no vitiating assumptions are made about the nature of the existing light field in X . On the other hand, the methods will require a high degree of instrumental accuracy and dependability, much higher than present-day instrumental procedures. Whether the procedures will actually be realized depends in the last analysis on possibly unforseen and urgent needs for α -determinations by ground-based methods. An accurate and theoretically sound method then stands ready for such contingencies.

Still Further Methods

Suppose that the probe can carry a radiance tube and also some telemetering equipment for sending the radiance distribution data to the ground where it is recorded and tabulated. Suppose further that the probe contains a source of known inherent radiance ${}_1N_0$. (If the probe were a rocket this source could possibly be provided by the exhaust flame of the rocket itself.)

Now returning to Equation (15) we interpret ΔN_r and ΔN_0 as follows. First

$$\Delta N_r = {}_1N_r - {}_2N_r ,$$

is, as before, measurable directly from the ground, but now ${}_1N_r$ is the apparent radiance of the known source on the probe, and ${}_2N_r$ is the apparent radiance of the immediate background of the probe. Second,

$$\Delta N_0 = {}_1N_0 - {}_2N_0 \ ,$$

where ${}_2N_0$ is the radiance of the background of the probe as seen from the probe. This information is contained in the telemetered radiance distribution data. Combining these differences according to (15), we can estimate α once again at the variable point $x + r\xi$. Therefore, radiance distribution data can be continuously collected by such a probe and be sent back to the recording equipment, while simultaneous observations of the probe from the ground will yield values of α.

Active and Passive Probes

The two-source and periodic-source methods above are examples of the use of <u>passive probes</u>. These, by definition, simply move through X and act as radiance markers whose apparent radiances are observed from the ground and are there recorded and used in running calculations such as those summarized by Equations (16) - (18). The probe discussed in the paragraph above is an example of an <u>active probe</u>: it carries with it equipment to sense and telemeter radiance distribution data in its immediate environment. It need not carry recording and tabulating equipment. One of the main results in the next section shows that a single active probe of the N-distribution type can, in principle, supply all data necessary for the complete determination of the inherent and apparent optical properties of the region X which it probes.

FUNDAMENTALS

Why N, α and N_o are Sufficient for Practical Visibility Calculations

We will show that knowledge of N, α and N_o as defined in factors (1), (2) and (3) of the Introduction is sufficient to solve, in principle, all practical problems requiring the apparent radiance and apparent contrast of an object in X.

In Figure 1, as before, let x be an observation point and ξ the direction of observation at x, and let (x, ξ, r) denote the path, determined by x and ξ, of length r (measured from x). If $N(y, \xi)$ is the field radiance at y in the direction ξ, then the general solution of the integrodifferential equation for N along (x, ξ, r) states that

$$N(x, \xi) = T_r(x, \xi)\, N(y, \xi) + N_r^*(x, \xi), \qquad (A1)$$

where

$$N_r^*(x, \xi) = \int_0^r T_{r'}(x, \xi)\, N_*(x', \xi)\, dr' \qquad (A2)$$

and where

$$N_*(x', \xi) = \int_{\Xi} \sigma(x'; \xi : \xi')\, N(x', \xi')\, d\Omega(\xi'), \qquad (A3)$$

and, finally, where

$$x' = x + r'\xi \,, \quad 0 \leq r' \leq r \,.$$

In other words, (A1) states that the observed radiance $N(x, \xi)$ may be written quite generally as the sum of the transmitted radiance $T_r(x, \xi)\, N(y, \xi)$ and the path radiance $N_r^*(x, \xi)$ (the space light) associated with (x, ξ, r). Equation (A1) holds regardless of the structures of α, σ, and N along (x, ξ, r).

It follows from (A1) that if $N(x, \cdot)$ is known at all points x of X along with α (and therefore T_r is known by means of Equation (3)) we can find the path radiance $N_r^*(x, \xi)$ for any path (x, ξ, r) in X:

$$N_r^*(x, \xi) = N(x, \xi) - T_r(x, \xi)\, N(y, \xi) \,. \tag{A2}$$

Now suppose there is an object at y which has an inherent radiance $N_0(y, \xi)$ in the direction ξ, and suppose that it does not appreciably perturb the natural radiance distributions along (x, ξ, r). Then from (A1) with the general value $N(y, \xi)$ replaced by $N_0(y, \xi)$, we have, by means of (A2):

$$N_r(x, \xi) = \left[N_0(y, \xi) - N(y, \xi) \right] T_r(x, \xi) + N(x, \xi) \,, \tag{A3}$$

where $N_r(x, \xi)$ is the apparent radiance of the object as seen from x in the direction ξ. In this way $N_r(x, \xi)$ is completely determined knowing factors (1), (2) and (3) of the Introduction.

The associated apparent contrast is, of course,

$$C_r(x,\xi) = \left[\frac{N_o(y,\xi) - N(y,\xi)}{N(x,\xi)} \right] T_r(x,\xi) \,, \tag{A4}$$

or

$$C_r(x,\xi) = C_o(y,\xi) \frac{N(y,\xi)}{N(x,\xi)} T_r(x,\xi) \,, \tag{A5}$$

in its usual general form.

How N and α Can Help Determine Other Inherent Optical Properties

Besides being of use in obtaining estimates of the apparent radiance of distant objects, the radiance distributions $N(x,\cdot)$ and α can be used to find the values of the volume absorption function a and the values of the volume scattering function σ at each point of X .

Specifically, we can, in principle, determine the value $a(x)$ from knowledge of $N(x,\cdot)$ alone using the divergence method.[4] For, by examining the following divergence relation for radiant flux in a scattering-absorbing medium:

$$\nabla \cdot \underset{\sim}{H}(x) = -a(x)\, h(x)$$

and recalling that

$$\underset{\sim}{H}(x) = \int_{\Xi} \xi\, N(x,\xi)\, d\Omega(\xi)$$

and that

$$h(x) = \int_{\Xi} N(x, \xi)\, d\Omega(\xi)$$

are directly obtainable from $N(x, \cdot)$, we merely perform the following operation:

$$a(x) = - \frac{\nabla \cdot \underset{\sim}{H}(x)}{h(x)} \tag{A6}$$

to obtain $a(x)$.

To find the volume scattering function value $\sigma(x; \xi; \xi')$ at x for the two directions ξ and ξ', we observe first that from (A2) we may operationally define $N_*(x, \xi)$ as the following limit of observable quantities:

$$N_*(x, \xi) = \lim_{r \to 0} \frac{N_r^*(x, \xi)}{r} , \tag{A7}$$

which, in view of (A2), may be written:

$$N_*(x, \xi) = \lim_{r \to 0} \frac{N(x, \xi) - T_r(x, \xi) N(y, \xi)}{r} , \tag{A8}$$

Usually, there is a length $r_0 > 0$ with the property that, for all $r \leq r_0$ the quotient in (A8) is essentially constant and equal to the limiting value on the left of (A8):

$$N_*(x, \xi) = \frac{N(x, \xi) - T_{r_0}(x, \xi)\ N(y, \xi)}{r_0} . \tag{A9}$$

Therefore, with the aid of (A3), σ may be defined as the solution of the integral equation:

$$\left[N(x,\xi) - T_{r_0}(x,\xi)\,N(y,\xi)\right] = r_0 \int_{\Xi} \sigma(x;\xi;\xi')\,N(x,\xi')\,d\Omega\,, \quad \text{(A10)}$$

which depends explicitly only on the known radiance distributions $N(x,\cdot)$ and α. Practical solution procedures for (A10) have been considered in detail.[5]

Why $N(x,\cdot)$ Can be the Fundamental Datum

By reviewing all that has been said so far, we arrive at a most interesting conclusion, which perhaps has more philosophical than practical utility: <u>By a complete and accurate determination of only $N(x,\cdot)$ in X it is possible, in principle, to determine all currently known inherent optical properties of X (e.g., α, a, σ) and to solve all the important current image transmission problems (N_r, C_r) in X</u>. Thus, from the present point of view of image transmission phenomena, the pertinent visibility computations for object Y in an optical medium X are determinate once we know:

(I) $N(x,\cdot)$ at each x of X.

(II) $N_0(y,\cdot)$ at each y of Y.

The conditions (I) and (II) can of course be generalized to the polarized case.

The proof of this principle is obtained by accumulating the appropriate statements of the present paper, namely the general interpretation of (15), along with (A3), (A4), (A6) and (A10).

How a Fundamental Empirical Methodology can be Eventually Attained

From what has been developed above, we may conclude that--in the final analysis--the most fundamental empirical operation that can be performed in an optical medium X is the empirical determination of the radiance distribution $N(x, \cdot)$ at each point x of X. The correspondingly fundamental mode of solution of image transmission problems in real optical media then devolves on a methodology which embodies the following ordered sequence of procedures:

(A) Methodical and continual measurement, recording and tabulation of $N(x, \cdot)$ by means of telemetering probes or mobile laboratories in X.

(B) Determination and tabulation of all inherant and apparent optical properties of X from the $N(x, \cdot)$ data by means of ground-based specially programmed automatic computors.

(C) Computation and tabulation of apparent radiances N_r and apparent contrasts C_r of given objects in by means of the specially programmed automatic computors, using the stored tabulations of step (B).

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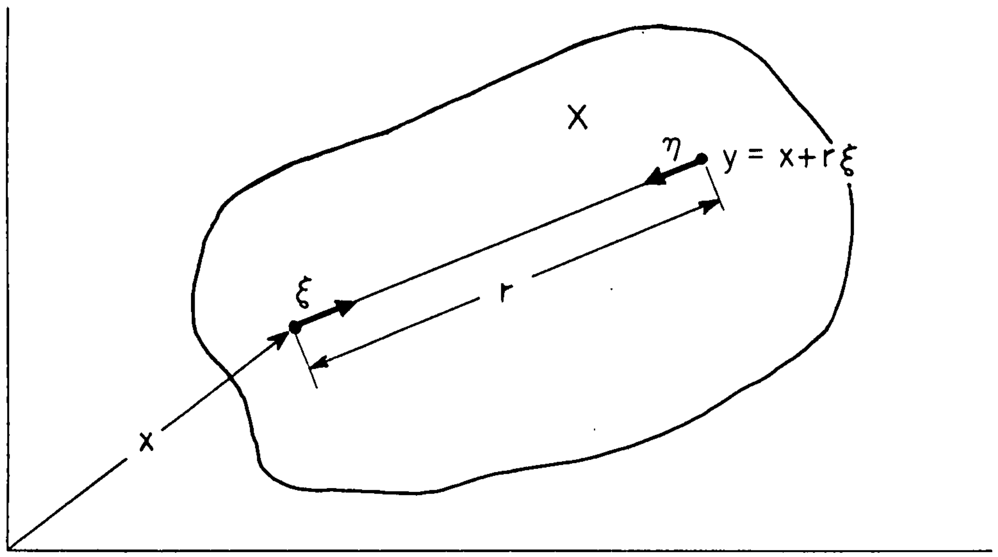

Figure 1

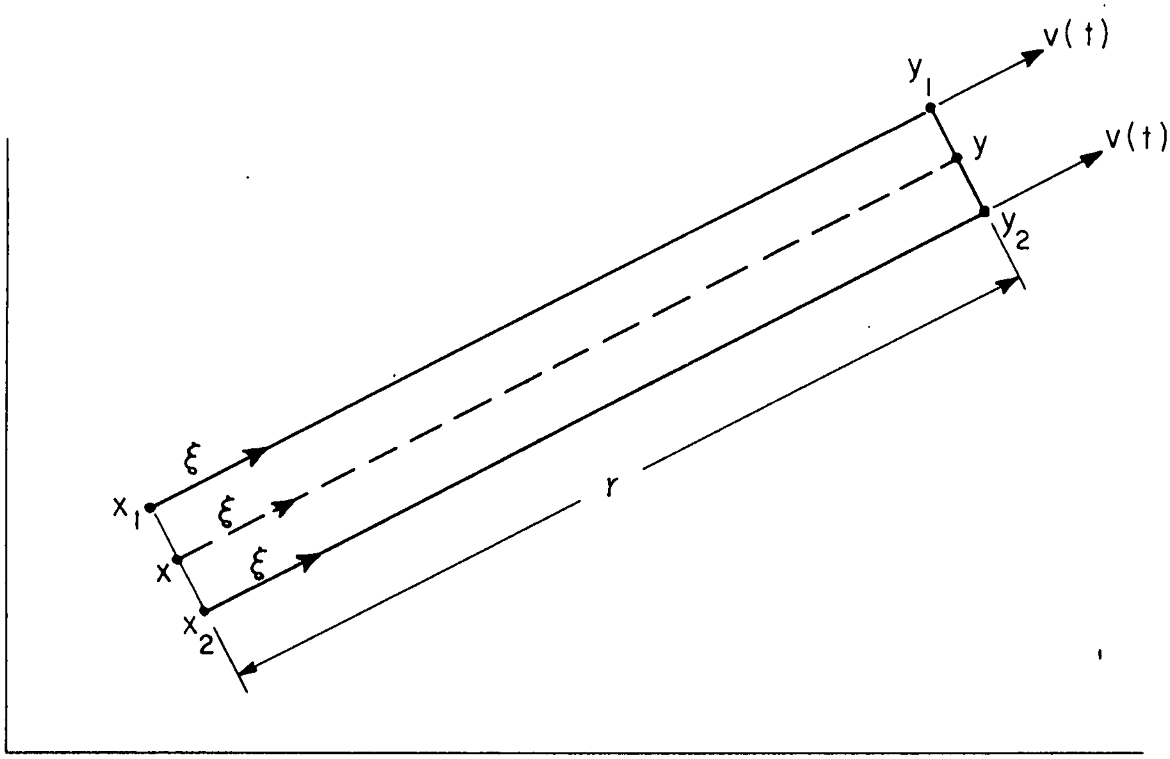

Figure 2

Rudolph W. Preisendorfer