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EXTRACTION AND EXTENSION OF MATRICES IN COMPUTER CALCULATIONS

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EXTRACTION AND EXTENSION OF MATRICES IN COMPUTER CALCULATIONS

By

William Hadley Richardson

1. The problems involved here are: The extraction of a submatrix from a matrix; and the extension of a matrix to a matrix of larger dimensions with the original matrix as a given submatrix of the larger and the rest of the larger matrix composed of zero submatrices.

This is generally a trivial problem but of interest in computer calculations where submatrices must be separated from massive matrices which are stored in memory and matrix multiplication subroutines are available.

2. To extract a submatrix Y_{22} from a matrix $Y = (Yij)$, $i, j = (1,2,3)$ (see Figure 1), define a matrix $X = (X_i)$ such that:

X has the same number of rows as Y_{22}.

$X_1 = 0$ with same number of columns as Y_{12} has rows.

$X_2 = I$ with same number of rows and columns as Y_{22} has rows.

$X_3 = 0$ with same number of columns as Y_{32} has rows.

Define a matrix $Z = (Z_j)$ such that:

Z has same number of columns as Y_{22}.

$Z_1 = 0$ with same number of rows as Y_{21} has columns.

$Z_2 = I$ with same number of rows and columns as Y_{22} has columns.

$Z_3 = 0$ with same number of rows as Y_{23} has columns.

Then $X\ Y\ Z = (Y_{2j})\ Z = Y_{22}$.

The method is completely general if it is considered that, when Y_{22} is a corner submatrix, certain of the Y_{ij} become empty matrices. For instance, if the submatrix, Y_{22}, to be extracted is in the upper left-hand corner the Y_{i1} and Y_{1j} are empty matrices and in consequence X_1 and Z_1 are empty matrices.

3. Extending a matrix is a similar operation in reverse. To extend a matrix, Y_{22}, to a larger matrix, $Y = (Y_{ij})$, i,j = (1,2,3), which has the original matrix as a submatrix and the rest of the larger matrix zeros, (see Figure 2), define a matrix $X = (X_i)$ such that:

X has the same number of columns as Y_{22} has rows.

$X_1 = 0$ with the same number of rows as Y_{12}.

$X_2 = I$ with the same number of rows and columns as Y_{22} has rows.

$X_3 = 0$ with the same number of rows as Y_{32}.

Define a matrix $Z = Z_j$ such that:

Z has same number of rows as Y_{22} has columns.

$Z_1 = 0$ with same number of columns as Y_{21}.

$Z_2 = I$ with same number of rows and columns as Y_{22} has columns

$Z_3 = 0$ with same number of columns as Y_{23}.

Then $XY_{22}Z = (Y_{i2})Z = (Y_{ij}) = Y$.

The generality of method is as stated in paragraph 2 above.

4. <u>Proofs:</u>

4.1. Extractor: $XYZ = Y_{22}$

$$XY = (XY_j)$$

$$XY_1 = X_1Y_{11} + X_2Y_{21} + X_3Y_{31}$$

$$= OY_{11} + IY_{21} + OY_{31}$$

$$= Y_{21}$$

$$XY_2 = X_1Y_{12} + X_2Y_{22} + X_3Y_{32}$$

$$= OY_{12} + IY_{22} + OY_{32}$$

$$= Y_{22}$$

$$XY_3 = X_1Y_{13} + X_2Y_{23} + X_3Y_{33}$$

$$= OY_{13} + IY_{23} + OY_{33}$$

$$= Y_{23}$$

So $XY = (Y_{21}, Y_{22}, Y_{23}) = (Y_{2j})$

Then $XYZ = (Y_{2j})Z$

$$XYZ = Y_{21}Z_1 + Y_{22}Z_2 + Y_{23}Z_3$$

$$= Y_{21}O + Y_{22}I + Y_{23}O$$

$$= Y_{22}$$

Q.E.D.

4.2. Extender: $XY_{22}Z = Y$

$XY = (XY_i)$

$XY_1 = X_1Y_{22} = 0Y_{22} = 0$

$XY_2 = X_2Y_{22} = IY_{22} = Y_{22}$

$XY_3 = X_3Y_{22} = 0Y_{22} = 0$

So $XY = (0, Y_{22}, 0) = (Y_{i2})$

Then $XYZ = (Y_{i2})\ Z = (XYZ_{ij})$

$XYZ_{11} = Y_{12}Z_1 = 00 = 0$

$XYZ_{12} = Y_{12}Z_2 = 0I = 0$

$XYZ_{13} = Y_{12}Z_3 = 00 = 0$

$XYZ_{21} = Y_{22}Z_1 = Y_{22}0 = 0$

$XYZ_{22} = Y_{22}Z_2 = Y_{22}I = Y_{22}$

$XYZ_{23} = Y_{22}Z_3 = Y_{22}0 = 0$

$XYZ_{31} = Y_{32}Z_1 = 00 = 0$

$XYZ_{32} = Y_{31}Z_2 = 0I = 0$

$XYZ_{33} = Y_{31}Z_3 = 00 = 0$

And $XYZ = \begin{pmatrix} 0 & 0 & 0 \\ 0 & Y_{22} & 0 \\ 0 & 0 & 0 \end{pmatrix} = Y$

Q.E.D.

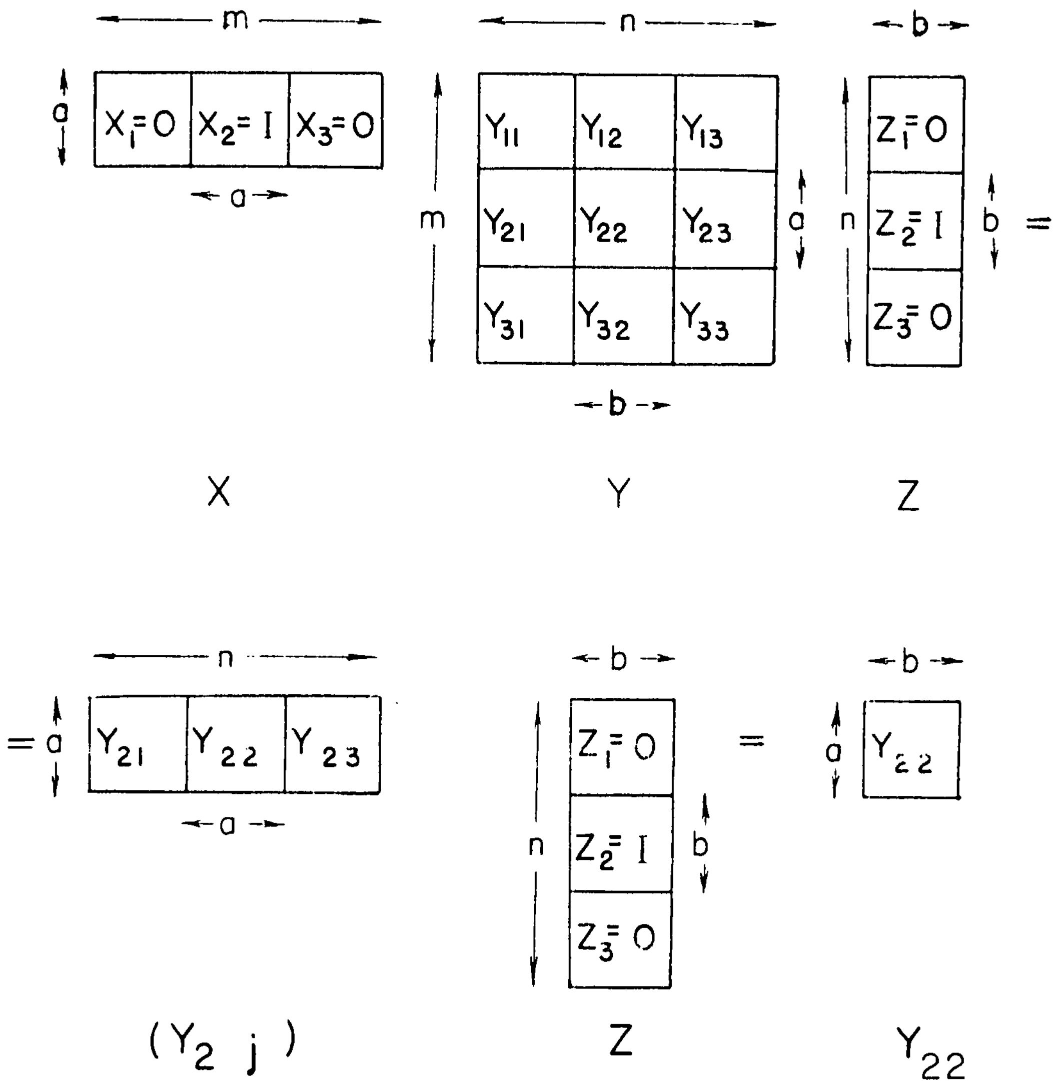

FIGURE 1

EXTRACTION

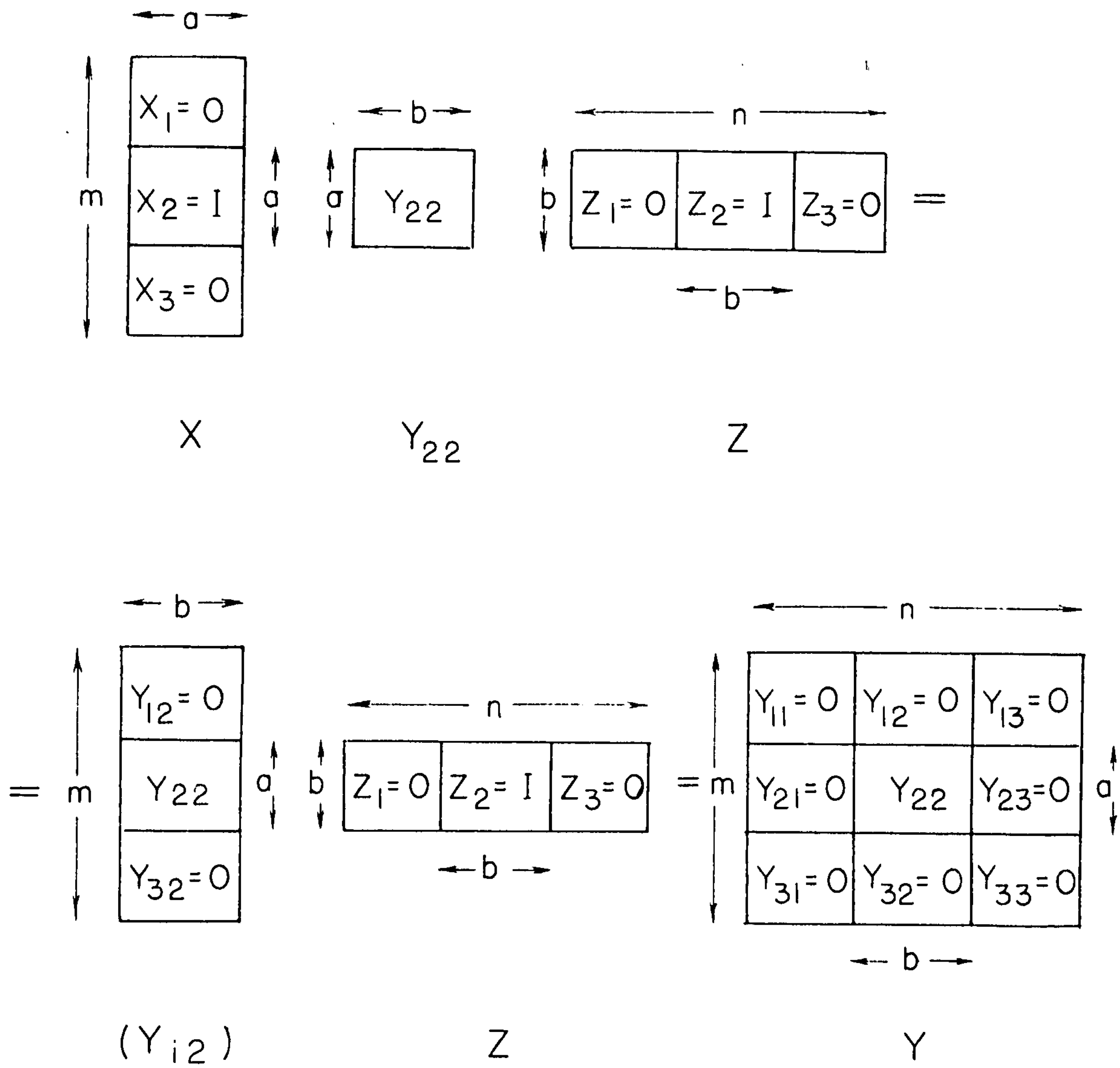

FIGURE 2

EXTENSION