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AN ADAPTATION OF THE METHOD OF PROBIT ANALYSIS TO PSYCHOPHYSICAL THRESHOLD DATA

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1.0 THE PROBLEM

The objective of this development is to adapt the methods of probit analysis to automatic desk calculator computation by personnel relatively untrained and unoriented in probability techniques and in methods of psychophysical determination of visual thresholds.

2.0 ASSUMPTIONS

The assumption is made that the deviations of threshold measurements indicate a normal distribution, either directly or by transformation.

3.0 FACTS BEARING ON THE PROBLEM

3.1 The type of problem to which this method is currently applied is discussed in Reference 3. Briefly it consists of determining the threshold contrast of visual targets. The threshold in this case is defined as the contrast at which there is probability of 0.5 of detecting the target. The contrast of the target is varied to give from nearly 1.0 to 0.0 of detection probabilities. Forty presentations of each of six target contrasts were made in restricted random order. One of the six contrasts used was made equal to zero, in order to determine false-positive frequency. A yes-no indicator response was made by the observer and recorded.

3.2 The data to be fitted consist of the number of trials and number of positive responses at each target contrast used in an experiment. The result desired is the 50 percent threshold of detection.

3.3 The calculators available were the Friden Model STW Automatic Calculators.

3.4 The calculator operators available were college undergraduates with no requirement for knowledge of probability of theory and techniques or of the experiment itself.

3.5 The tables used were the ones in Reference 1. In this case they were photocopied and ring-bound for convenient access.

3.6 The basis of this system is the probit analysis method set out in Reference 1. Briefly, probit analysis is a least squares, maximum likelihood, iterative method of fitting distribution functions utilizing a 5-biased, normal distribution; that is, normal (5,1). The adaptation is almost entirely from the material in Reference 1. The adaptation is set up in self-explanatory form. A very helpful study was made of Reference 2, on this same subject.

4.0 DISCUSSION

4.1 The calculator operator was given the calculation forms (Figures I, a, b, c), the step description sheets (Figures II, a, b, c), and the tables. The calculation forms have each step sequentially numbered. In the rare cases where values are not in the tables they can be from the formulas given in the step description sheets or can be taken from tables of the normal probability function, such as those on page 237, Reference 4. The step description sheets are numbered to correspond to the calculation forms. Figures I and II should be sufficient to demonstrate the method. The Appendix, a glossary of notation is included for the benefit of the reader. The notation of Reference 1 is generally retained. Note that the auxiliary χ^2 computation is that generally accepted (page 416, Reference 5).

4.2 This system has been used in two different branches of the Visibility Laboratory by operators of the type specified in Section 3.3 above; that is, college undergraduates without training in probability and in the experiments involved. The results (and the operators bear this out) show that the system is easily mastered and can be carried out with a minimum, if any, explanation or coaching. These forms are now in regular use in the Laboratory.

4.3 This system has been used on other types of problems than threshold problems. These were types where a fitted distribution was desired from data which appeared to have an approximately normal distribution, or where an approximation to a normal distribution could be obtained by transformation such as a log-normal distribution. For instance, distribution functions of sighting ranges of ships at sea were developed by this method.

5.0 CONCLUSIONS

5.1 The system described here is suitable for use by operators of college undergraduate level and without training in probability theory and techniques, and without training in the experiments being analyzed.

5.2 The system has general application to fitting distribution functions where an approximately normal distribution exists or can be obtained by transformation.

6.0 ACKNOWLEDGEMENTS

6.1 The Vision Research Branch and Visibility Branch of the Laboratory were a great help in this project in testing out the method and assisting in development of the forms.

6.2 Gratitude is due Doctor John H. Taylor, Head of the Vision Research Branch of the Laboratory, for presenting the opportunity to work on this problem.

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(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	
x	n	r	$p' = \frac{r}{n}$	$p(C) = \frac{p' - C}{1 - C}$	EP(I)	Y	w(II)	nw	y(IV)	nwx	nwy	
3	All	All	2	2	3	2	3	3	3	6	6	Significant Figures

Sums:

(13)	(14)	(15)
Σnw	Σnwx	Σnwy

Roman numerals indicate Table number.

Figure I,a

(16) $\frac{1}{\Sigma nw} =$

(17) $\bar{x} = \frac{\Sigma nwx}{\Sigma nw} =$

(18) $\bar{y} = \frac{\Sigma nwy}{\Sigma nw} =$

(19) $\Sigma nwx^2 =$

(20) $\Sigma nwxy =$

(21) $\Sigma nwy^2 =$

(22) $\frac{\Sigma^2 nwx}{\Sigma nw} =$

(23) $\frac{\Sigma nwx \Sigma nwy}{\Sigma nw} =$

(24) $\frac{\Sigma^2 nwy}{\Sigma nw} =$

(25) $\Delta = Sxx$

(26) $\Delta = Sxy$

(27) $\Delta = Syy$

(28) $b = \frac{Sxy}{Sxx} =$

(29) $a = \bar{y} - b\bar{x} =$

(31) $\frac{S^2xy}{Sxx} =$

(32) $\Delta = \chi^2 =$

(33) ; ______ $< P_{\chi^2}$ (VI) $<$ ______

(30) $Y = a + bx =$ x

If $P < 0.05$ repeat from step 7 with new Y line (Step 30)

(34) $m = \frac{5-a}{b} =$

(35) $\sigma = \frac{1}{b} =$

(36) $\sigma_m^2 = \frac{1}{b^2}\left[\frac{1}{\Sigma nw} + \frac{(m-\bar{x})^2}{Sxx}\right] =$

(37) $\sigma_m =$

(38) $\sigma_a^2 = \frac{\Sigma nwx^2}{Sxx} =$

(39) $\sigma_a =$

(40) $\sigma_b^2 = \frac{1}{Sxx} =$

(41) $\sigma_b =$

(42) $\sigma_\sigma^2 = \frac{\sigma^4}{Sxx} =$

(43) $\sigma_\sigma =$

Fiducial limits

(44) t (VII) =

(45) $g = \frac{t^2\sigma_b^2}{b^2} =$

(46)

$x \pm t\sigma_m = x +$ $> x > x -$

or

$$x + \frac{g}{1-g}(x - \bar{x}) \pm \frac{t}{b(1-g)}\left[\frac{1-g}{\Sigma nw} + \frac{(x-\bar{x})^2}{Sxx}\right]^{\frac{1}{2}} =$$

$= x +$ $> x > x -$

Figure 1b

(47)	(48)	(49)	(50)	(51)	(52)				
n	$p(c)$	Y	$P(I)$	nP	$\frac{n(p-P)^2}{P}$				

(53)

χ^2 [ ] Return to step (33)

Figure I,c

Step	Description
1	x is the independent variable, whatever measure is used to identify groups of data.
2	n is the total number of events, trials, presentations at each x.
3	r is number of positive responses.
4	$p' = \frac{r}{n}$ is the raw probability.
5	$p(C) = \frac{p' - C}{1 - C}$; adjusted probability, where C is the "population value" (Finney) or expected false positive proportion.
6	EP(p) is the empirical probit from Table I, and defined as $EP = \quad EP: p(C) = \int_{-\infty}^{EP-5} \frac{1}{2\pi} e^{-\frac{1}{2} u^2} du$.
7	Y is the trial probit and is found by plotting EP against x, estimating a straight line fit giving more importance to EP near 5. Y are points on the trial line corresponding to data x.
8	W(Y,C) is the weight given to a set of data for a given s. From Table II. $W = \frac{Z^2}{PQ}$, where Z is the frequency ordinate corresponding to Y, Q = 1-P
9	nW is the weighting coefficient

Figure II, a

Step	Description
10	$y = \frac{p - P}{Z} + Y$. y is the working probit, found from Table IV.
11,12	Compute and record each value.
13,14,15	Sum appropriate columns.
16	$\frac{1}{nW}$ is used later as a factor and in checking numerical work.
17,18	$\bar{x}$ and $\bar{y}$ are means of the variables in the working probit distribution.
19,20,21	Done by accumulative multiplication from steps 11 and 12.
22,23,24	Self-explanatory.
25,26,27	Δ stands for difference in a normal algebraic subtraction. Retain sign.
28,29	Self-explanatory. a and b are constants of new probit function.
30	New probit function.
31	Self-explanatory.
32	Δ is algebraic difference and is χ^2 for this fit.
33	P_{χ^2} is found from Table VI using as number of degrees of freedom 2 less than number of values of x in data. If $P_{\chi^2} < 0.05$ (or the significant level chosen) repeat from step 7, using the new probit function to obtain new Y values for each x. <u>Note</u>: If recycling is indicated and there are very small or very large values of Y or p(C) combine the r for these values so that $r > 3$ and use third sheet to find χ^2, since the difference approximation is not valid.

Figure II, b

Step	Description
34	m is mean of computed distribution.
35	σ is standard deviation of computed distribution.
36,37	σ_m is standard deviation of mean of computed distribution.
38,39	σ_a is standard deviation of intercept of computed probit line on Y axis.
40,41	σ_b is standard deviation of slope of computed probit line.
42.43	σ_σ is standard deviation of computed σ .
44	t is from Table 7 using chosen level of significance and degrees of freedom used in 33.
45	g is a constant used in obtaining fiducial limits.
46	If g is much less than 1 use the first formula, otherwise the second.
47	n is from (2).
48	r is from (3).
49	Y is from (30).
50	P corresponds to Y and is obtained from Table I
51,52	Self-explanatory.
53	Sum values in (53) and re-enter system at step (33).

Figure II, c

NOTATION

a	=	$\bar{y} - b\bar{x}$; intercept of linearized distribution on y axis
b	=	$\frac{Sxy}{Sxx}$; slope of linearized distribution.
C	:	proportion of false responses to control events.
EP	=	$[EP: b(C) = \int_{-\infty}^{EP-5} \frac{1}{2\pi} e^{-1/2\, u^2} du]$; empirical probit.
g	=	$\frac{t^2 \sigma_b^2}{b^2}$; a constant used to develop fiducial limits.
m	=	$\frac{5 - a}{b}$; mean abscissa of the computed distribution.
n	:	total number of events (trials, presentations).
P	=	$\int_{-\infty}^{Y-5} \frac{1}{2\pi} e^{-1/2\, u^2} du$; cumulative probability corresponding to Y.
p'	=	$\frac{r}{n}$; raw cumulative probability.
p(C)	=	$\frac{p' - C}{1 - C}$; adjusted probability.
p_{χ^2}	=	χ^2 equivalent probability.
Q	=	1 - P.
r	:	number of positive reactions.
Sxx	=	$\Sigma nwx^2 - \frac{\Sigma^2 nwx}{\Sigma nw}$
Sxy	=	$\Sigma nwxy - \frac{\Sigma nwx \Sigma nwy}{\Sigma nw}$
Syy	=	$\Sigma nwy^2 - \frac{\Sigma^2 nwy}{\Sigma nw}$

t : dependent variable of Fisher's t distribution found in table under desired fiduciary level.

$W = \frac{Z^2}{PQ}$; weight applied to data at

x : independent variable under study .

$\bar{x} = \frac{\Sigma nwx}{\Sigma nw}$; weighted mean abscissa from data.

Y : ordinate of estimated curve or fitted curve.

$y = \frac{p - P}{Z} + Y$

$\bar{y} = \frac{\Sigma nwy}{\Sigma nw}$; weighted mean of working probits.

Z : ordinate of the frequency function.

χ^2 : chi-squared measure of goodness of fit of computed distribution to data.

σ : standard deviation of computed distribution.

$\sigma_b = \left[\frac{1}{Sxx}\right]^{\frac{1}{2}}$; standard deviation of slope, b.

$\sigma_m = \frac{1}{b}\left[\frac{1}{\Sigma nw} + \frac{(m-\bar{x})^2}{Sxx}\right]^{\frac{1}{2}}$; standard deviation of mean of distribution.

$\sigma_\sigma = \frac{\sigma^2}{(Sxx)^{\frac{1}{2}}}$; standard deviation of the computed distribution standard deviation.