

UNIVERSITY OF CALIFORNIA, SAN DIEGO  
SCRIPPS INSTITUTION OF OCEANOGRAPHY  
VISIBILITY LABORATORY  
SAN DIEGO, CALIFORNIA 92152

**DETERMINATION OF PATH RADIANCE FOR DOWNWARD PATH OF SIGHT  
FROM GROUND-BASED OR LOW-ALTITUDE MEASUREMENT**

Almerian R. Boileau

CONTRACT NO. F 19628-67-C-0181

SIO Ref. 68-24

August 1968

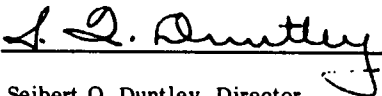
**Contract Monitor: Dr. Robert Fenn (CROA)  
Air Force Cambridge Research Laboratories  
Office of Aerospace Research**

Distribution of this document is unlimited. It may be released to the Clearinghouse, Department of Commerce, for sale to the general public.

Prepared for

AIR FORCE CAMBRIDGE RESEARCH LABORATORIES  
OFFICE OF AEROSPACE RESEARCH  
UNITED STATES AIR FORCE  
BEDFORD, MASSACHUSETTS 01730

Approved:



Seibert Q. Duntley, Director  
Visibility Laboratory

Approved for Distribution:



William A. Nierenberg, Director  
Scripps Institution of Oceanography

## **ABSTRACT**

This paper describes a method of obtaining the path radiance for a downward path of sight by the ground-based measurements of the path radiance of the reciprocal upward path of sight and the two corresponding sterisents.

## Determination of Path Radiance for Downward Path of Sight from Ground-Based or Low-Altitude Measurement\*

ALMERIAN R. BOILEAU  
*Visibility Laboratory, University of California,  
 San Diego, California 92152*  
 (Received 7 August 1967)

INDEX HEADINGS: Atmospheric optics, Radiometry.

WITH the advent of space travel an easy means of determining the path radiance for the downward path of sight is desirable. This communication describes such a method for the case of the optical standard atmosphere.<sup>1</sup> The required measurements are (1) the path radiance for the upward path of sight and (2) the related, reciprocal radiant sterisents.<sup>2</sup>

The path radiance is the integration of the sterisent over a path of sight, with each sterisent increment being weighted by the transmittance from the increment to the point of observation. For a horizontal path of infinite length the path radiance is given by the equation

$$N_{\infty}^*(z, 90^\circ, \phi) = \int_0^{\infty} N_*(z, 90^\circ, \phi) \cdot \exp[-\alpha(z)x] dx, \quad (1)$$

where  $N_*(z, 90^\circ, \phi)$  is the radiant sterisent measured at altitude  $z$  in the direction specified by the zenith angle  $\theta$  and the azimuth  $\phi$ . In this case  $N_*(z, 90^\circ, \phi)$  and  $\alpha(z)$  are constants. In an optical standard atmosphere, in the case of a path other than the horizontal, the attenuation coefficient varies exponentially with altitude, i.e.,

$$\alpha(z) = \alpha(0) \cdot e^{-z/k}, \quad (2)$$

and the sterisent, to a useful approximation, usually varies in the same manner, i.e.,

$$N_*(z, \theta, \phi) \approx N_*(0, \theta, \phi) \cdot e^{-z/k}, \quad (3)$$

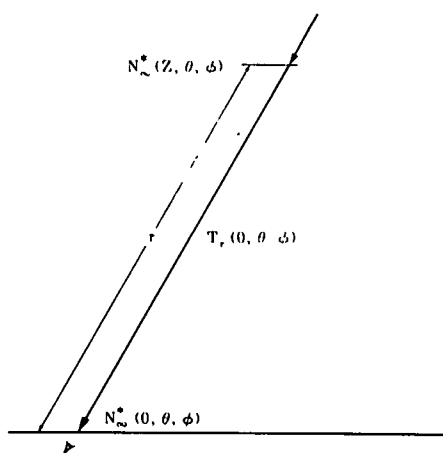


FIG. 1.  $N_*(0, \theta, \phi)$ , the radiance of path of length  $r$ , from zero altitude of altitude  $z$ , may be determined by Eq. (9). This requires the measurement of  $N_{\infty}^*(z, \theta, \phi)$  be made by airborne equipment.  $T_r(0, \theta, \phi)$  is usually calculated from the altitude profile of the attenuation coefficient.

where the constant  $k$  is 9150 m or 30 000 ft. Thus, in an optical standard atmosphere, for an infinitely long vertical path, the path radiance is

$$N_{\infty}^*(0, 0^\circ, \phi) = N_*(0, 0^\circ, \phi) \times \int_0^{\infty} e^{-z/k} \cdot \exp[-\alpha(0) \int_0^z \exp(-u/k) du] dz. \quad (4)$$

$N_*(0, 0^\circ, \phi)$  and  $\alpha(0)$  are constants and have been removed from their respective integrals and, to avoid ambiguity, the variable  $u$  has been substituted for  $z$  in the second integral

With the substitution of  $\alpha(0) \sec\theta$  in place of  $\alpha(0)$  in the transmittance factor of the equation, the equation for determining the path radiance for any path of infinite length in the upward direction becomes

$$N_{\infty}^*(0, \theta, \phi) = N_*(0, \theta, \phi) \int_0^{\infty} e^{-z/k} \cdot \exp[-\alpha(0) \sec\theta \int_0^z \exp(-u/k) du] dz. \quad (5)$$

Similarly the equation for determining the path radiance for any path of infinite length in the downward direction is

$$N_{\infty}^*(\infty, \pi - \theta, \phi + \pi) = N_*(0, \pi - \theta, \phi + \pi) \int_0^{\infty} e^{-z/k} \cdot \exp[-\alpha(0) \sec\theta \int_z^{\infty} \exp(-u/k) du] dz \quad (6)$$

These two equations differ in only two particulars, viz., (1) the two sterisents are for the same altitude but for reciprocal directions, and (2) the limits of the  $du$  integrals are from 0 to  $z$  in one equation and  $z$  to  $\infty$  in the other.

When Eq. (6) is divided by Eq. (5) and the quantity  $N_{\infty}^*(0, \theta, \phi)$  is transposed to the right side of the equation, the following expression is obtained:

$$N_{\infty}^*(\infty, \pi - \theta, \phi + \pi) = N_{\infty}^*(0, \theta, \phi) \left[ \frac{N_*(0, \pi - \theta, \phi + \pi)}{N_*(0, \theta, \phi)} \right] \times \frac{\int_0^{\infty} e^{-z/k} \cdot \exp[-\alpha(0) \sec\theta \int_z^{\infty} \exp(-u/k) du] dz}{\int_0^{\infty} e^{-z/k} \cdot \exp[-\alpha(0) \sec\theta \int_0^z \exp(-u/k) du] dz} \quad (7)$$

The right-hand factor, the fraction formed by the two integrals, is integrable and is equal to unity for the altitudes (limits of integra-

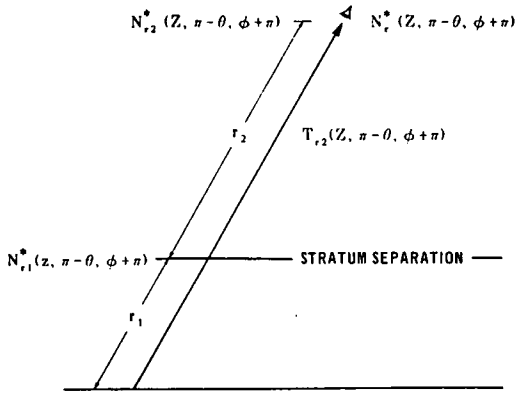


FIG. 2. Total path radiance for the downward path of sight  $r=r_1+r_2$  is the sum of the path radiance of path  $r_1$  weighted by the transmittance of path  $r_2$  plus the path radiance of path  $r_2$ . This is shown by Eq. (10).

tion) as shown or any other range of altitude. Thus the equation becomes

$$N_r^*(Z, \pi - \theta, \phi + \pi) = N_r^*(z, \theta, \phi) \left[ \frac{N_{r1}^*(z, \pi - \theta, \phi + \pi)}{N_r^*(z, \theta, \phi)} \right]. \quad (8)$$

The applicability of Eq. (8) must be considered for two conditions; first when the atmosphere is virtually an optical standard atmosphere and, second, when the atmosphere is stratified or made up of layers, each having the characteristics of an optical standard atmosphere but with a discontinuity of the exponential function at the boundary between the layers. In the first case the path radiance for the downward path of sight for the total atmosphere is calculated from the values of the three optical quantities required by Eq. (8), measured at the ground level. If the desired path

radiance is for a shorter path of sight, e.g., from sea level to a definite altitude  $Z$ , it is necessary to determine the path radiance for the upward path of sight to altitude  $Z$ , usually by<sup>3</sup>

$$N_r^*(0, \theta, \phi) = N_\infty^*(0, \theta, \phi) - N_\infty^*(Z, \theta, \phi) T_r(0, \theta, \phi), \quad (9)$$

as illustrated in Fig. 1. Then the desired path radiance for the downward path of sight is calculated by use of Eq. (8).

In the second case, when the atmosphere is stratified, the path radiance for the downward path of sight is the sum of the individual path radiances of the various strata weighted by the transmittance from the point of determination of each path radiance to the point of observation. This is illustrated for an atmosphere composed of two strata in Fig. 2. The applicable equation is

$$N_r^*(Z, \pi - \theta, \phi + \pi) = N_{r1}^*(z, \pi - \theta, \phi + \pi) T_{r2}(Z, \pi - \theta, \phi + \pi) + N_{r2}^*(Z, \pi - \theta, \phi + \pi), \quad (10)$$

where  $N_{r1}^*(z, \pi - \theta, \phi + \pi)$  and  $N_{r2}^*(Z, \pi - \theta, \phi + \pi)$  are calculated by Eq. (8). The transmittance is found by

$$T_{r2}(Z, \pi - \theta, \phi + \pi) = \exp \left[ -\alpha(0) \sec \theta \int_{z_1}^z e^{-z/k} dz \right], \quad (11)$$

where  $\alpha(0) = \alpha(z) e^{z/k}$ .

All of the quantities in the right-hand terms of the four preceding equations are either measurable with ground-based or airborne equipment, or are calculable from such measured quantities.

\* The research reported herein was sponsored by, but does not necessarily constitute the opinion of, the Air Force Cambridge Research Laboratories, Office of Aerospace Research, under Contract No. F 19628-67-C-0181.

<sup>1</sup> An optical standard atmosphere is defined as an atmosphere in which the scattering particles are the same at all altitudes but in which the relative number of particles per unit volume decreases exponentially with altitude. S. Q. Duntley, J. Opt. Soc. Am. 38, 179 (1948).

<sup>2</sup> The term recommended by the Optical Society of America Nomenclature Committee, J. Opt. Soc. Am. 57, 854 (1967), for path function defined in Ref. 3.

<sup>3</sup> S. Q. Duntley, A. R. Boileau, and R. W. Preisendorfer, J. Opt. Soc. Am. 47, 499 (1957).

## DOCUMENT CONTROL DATA - R&amp;D

*(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)*

1 ORIGINATING ACTIVITY (Corporate author)

Visibility Laboratory  
University of California  
San Diego, California 92152

2a REPORT SECURITY CLASSIFICATION

UNCLASSIFIED

2b GROUP

3 REPORT TITLE

DETERMINATION OF PATH RADIANCE FOR DOWNWARD PATH OF SIGHT FROM GROUND-BASED  
OR LOW-ALTITUDE MEASUREMENT

4 DESCRIPTIVE NOTES (Type of report and inclusion dates)

Scientific Report

5 AUTHOR(S) (Last name, first name, initial)

Boileau, Almerian R.

6 REPORT DATE

August 1968

7a TOTAL NO OF PAGES

5

7b NO OF REFS

3

8a. CONTRACT OR GRANT NO

F 19628-67-C-0181

9a ORIGINATOR'S REPORT NUMBER(S)

SIO Ref. 68-24

b OAR

9b OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10 AVAILABILITY LIMITATION NOTICES

Distribution of this document is unlimited. It may be released to the Clearinghouse,  
Department of Commerce, for sale to the general public.

11 SUPPLEMENTARY NOTES

Journal of the Optical Society of America  
Vol. 58, No. 4, 586-587, April, 1968

12 SPONSORING MILITARY ACTIVITY

Air Force Cambridge Research Laboratories  
Office of Aerospace Research  
L. G. Hanscom Field, Bedford, Massachusetts 01730

13 ABSTRACT

This paper describes a method of obtaining the path radiance for a downward path of sight by the ground-based measurements of the path radiance of the reciprocal upward path of sight and the two corresponding sterisents.

14.	KEY WORDS	LINK A		LINK B		LINK C	
		ROLE	WT	ROLE		ROLE	WT
	Atmospheric Optics Radiometry						