Limnol. Oceanogr., 34(8), 1989, 1484–1489

Dependence of the diffuse reflectance of natural waters on the sun angle

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Abstract

Through Monte Carlo simulation, the variation of the diffuse reflectance of natural waters with sun angle is found to be dependent on the shape of the volume scattering function (VSF) of the medium. It is shown that single scattering theory can be used to estimate the reflectance—sun angle variation given the VSF—and, conversely, the VSF over a limited range of scattering angles can be estimated from measurements of the variation of the reflectance with sun angle. The complex variation of reflectance with the incident illumination and surface roughness can be reduced to the variation of a single parameter: the downwelling distribution function in the absence of scattering. These observations are applicable to all but the most reflective of natural waters.

The irradiance ratio or irradiance reflectance at a depth z is defined according to $R(z) = E_u(z)/E_d(z)$, where E_u and E_d are the upwelling and downwelling irradiances at z (units given in list of symbols). When this is evaluated just beneath the sea surface it is referred to as the diffuse reflectance and indicated by R. R is interesting because it is relatively easy to measure (e.g. *absolute* calibration of the irradiance meter is not required), and it is used in the theory of ocean color remote sensing (Gordon and Morel 1983). In a series of Monte Carlo simulations of the transport of optical radiation in the ocean, Gordon et al. (1975) computed R for a homogeneous ocean as a function of the inherent optical properties of the water, the absorption coefficient a, the scattering coefficient b, and the volume scattering function $\beta(\alpha)$, using scattering phase functions $[P(\alpha) = \beta(\alpha)/b]$ measured by Kullenberg (1968) in the Sargasso Sea. These Monte Carlo simulations, as well as those discussed later, fully accounted for the effects of multiple scattering. In a limited number of cases $R(\vartheta_0)$, the diffuse reflectance as a function of the solar zenith angle ϑ_0, was also studied. It was concluded that $R(\vartheta_0)$ was a very weak function of ϑ_0—varying $<20\%$ for $0° \le \vartheta_0 \le 60°$. Later, Kirk (1984) presented a similar Monte Carlo study [with phase functions measured by Petzold (1972) in San Diego Harbor] which showed a variation in $R(\vartheta_0)$ over the same range of angles of as much as 50%. This difference is far greater than what might be expected due to differences in the computational procedure and thus requires explanation. If it can be verified that neither computation is in error, differences in the results can only lie in the fact that different scattering phase functions were used in the computations. In fact, Jerlov (1976, p. 149) stated that the variation of R and ϑ_0 "is a consequence of the shape of the scattering function," but provided no quantitative demonstration of the claim. Also, even for $\vartheta_0 = 0$, Plass et al. (unpubl.) have already shown that R depends on the shape of the scattering phase function, contrary to the conclusion of Gordon et al. (1975), Kirk (1984), and Morel and Prieur (1977) that R depends on the phase function principally through the backscattering coefficient b_b given by

$$b_b = 2\pi b \int_{\pi/2}^{\pi} P(\alpha) \sin \alpha \, d\alpha;$$

however, Plass et al. used phase functions that differed considerably from those observed in natural waters in order to demonstrate the dependence.

To investigate quantitatively the influence of the scattering phase function on the

Acknowledgments
This work received support from the Office of Naval Research under contract N00014-84-K-0451 and grant N00014-89-J-1985, and from the National Aeronautics and Space Administration under grant NAGW-273.

Significant symbols

a	Absorption coefficient, m^{-1}
α	Scattering angle
b	Scattering coefficient, m^{-1}
$\beta(\alpha)$	Volume scattering function, m^{-1} sr^{-1}
c	Attenuation coefficient $(a + b)$, m^{-1}
D_0	Downwelling distribution function ($\omega_0 = 0$)
e_f, e_b	Phase function parameters
$E_d(z)$	Downwelling irradiance at z, W $(m^2$ $nm)^{-1}$
$E_u(z)$	Upwelling irradiance at z, W $(m^2$ $nm)^{-1}$
$L(\vartheta, \varphi)$	Upwelling radiance just beneath the surface, W $(m^2$ $nm)^{-1}$
m	Refractive index of water
μ	$\cos \vartheta$
μ_w	$\cos \vartheta_{0w}$
φ	Azimuth angle of upwelled radiance
$P(\alpha)$	Scattering phase function (β/b), sr^{-1}
$R(D_0)$	Diffuse reflectance at D_0
$R(\vartheta_0)$	Diffuse reflectance at ϑ_0
$R(z)$	Irradiance ratio (E_u/E_d) at z
ϑ	Polar angle of upwelled radiance
ϑ_0	Solar zenith angle
ϑ_{0w}	Solar zenith angle below surface
ω_0	Scattering albedo (b/c)
z	Depth, m

diffuse reflectance, I have recalculated R using standard Monte Carlo techniques and also with a new backward Monte Carlo code developed for another purpose (Gordon 1985). The results of the new computations are in complete agreement with the old, i.e. when the Petzold (1972) phase function is used the results from either code agree well with Kirk (1984) and when the Kullenberg (1968) phase function is used the results agree with Gordon et al. (1975). This agreement suggests that the computations of both Kirk and Gordon et al. were correct and that the difference is in fact due to the specific phase functions used in the two studies. It also suggests that the shape of the variation of R with ϑ_0 can provide some information about $\beta(\alpha)$. In what follows, it is shown that, given $\beta(\alpha)$, the single scattering approximation can be used to specify the variation of R with ϑ_0, and, conversely, given $R(\vartheta_0)$, it is possible to invert the process and estimate $\beta(\alpha)$ over a range of scattering angles from ~60° to 180°.

Let an infinitely deep homogeneous ocean, with inherent optical properties a, b, and $\beta(\alpha)$ be illuminated by the direct solar beam. This beam upon transmittance through the air–sea interface (assumed flat) has a solar zenith angle in the water of ϑ_{0w}, where $m \sin \vartheta_{0w} = \sin \vartheta_0$ and m is the refractive index of water. In the single scattering approximation (e.g. *see* Jerlov 1976), this refracted solar beam is then scattered *once* by the medium, producing an upwelling radiance $L(\vartheta, \varphi)$ just beneath the surface, where ϑ and φ are the polar (measured from the zenith) and the azimuth (measured from the solar azimuth) angles of the direction in which the radiance is propagating. The upwelling irradiance just beneath the surface is given by:

$$E_u(0) = \int_0^{2\pi} d\varphi \int_0^{\pi/2} L(\vartheta, \varphi) \cos \vartheta \sin \vartheta \, d\vartheta.$$

The downward irradiance just beneath the surface consists of two parts: the irradiance from the transmitted solar beam and the irradiance produced by the reflection of $L(\vartheta, \varphi)$ from the interface. In the single scattering approximation this latter part is at most 0.5% of the former for the application reported here and will be ignored. In this case, the single scattering approximation to the reflectance just beneath the sea surface is given by

$$R(\vartheta_0) = \frac{1}{c}\int_0^{2\pi} d\varphi \int_0^1 d\mu \frac{\mu}{\mu + \mu_w} \beta(\alpha) \quad (1)$$

where $\cos \alpha = -\mu\mu_w + [(1 - \mu^2)(1 - \mu_w^2)]\cos \varphi$, and $\mu_w = \cos \vartheta_{0w}$, $\mu = \cos \vartheta$, and $c = a + b$. This expression, which is an excellent approximation for $b \ll a$ (or equivalently for $\omega_0 \equiv b/c \ll 1$), provides a direct link between $R(\vartheta_0)$ and $\beta(\alpha)$ in this limit. Gordon (1973) has shown that for a medium like the ocean, which scatters strongly in the forward direction, the usefulness of Eq. 1 can be extended to larger values of ω_0 by replacing c by $a + b_b$. For example, with the sun at zenith, this replacement in Eq. 1 reproduced Monte Carlo computations of R to within 0.5% for $\omega_0 < 0.6$ and 12% for $\omega_0 < 0.85$. Thus, since the ϑ_0 dependence of R in Eq. 1 is independent of whether c is retained or replaced by $a + b_b$, one might reasonably expect it to approximate the variation of R with sun angle, even for rather large values of ω_0.

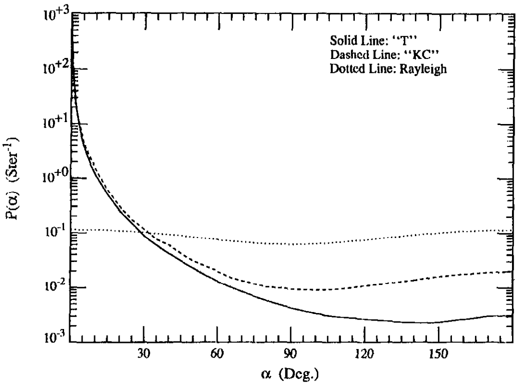

Fig. 1. Phase functions used in the present study.

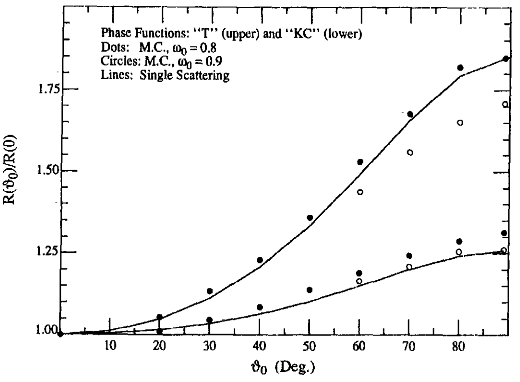

Fig. 2. R as a function of ϑ_0. Curves are the result of the single scattering approximation (Eq. 1).

The efficacy of Eq. 1 in this respect is examined by comparison with exact (Monte Carlo) multiple scattering computations carried out with two scattering phase functions: "KC," the Kullenberg (1968) phase function measured at 460 nm in the Sargasso Sea and used by Gordon et al. (1975); and "T," the mean of the three particle phase functions measured in turbid water at 530 nm by Petzold (1972) and used in the computations of Kirk (1984). These are shown in Fig. 1 along with phase function for the molecular (Rayleigh) scattering of pure water. The individual Monte Carlo simulations of $R(\vartheta_0)$ have a statistical error of no more than ± 0.5–1% for phase function KC and ± 1–2% for T. The resulting comparison between Eq. 1 and the Monte Carlo (M.C.) computations is presented in Fig. 2, where it is seen that the analytical computation of $R(\vartheta_0)/R(0)$ agrees with the Monte Carlo computations with a maximum error of $<5\%$ for $\omega_0 = 0.8$ and $\lesssim 10\%$ for $\omega_0 = 0.9$. The general increase in R with ϑ_0 is because the minimum scattering angle for photons to be redirected to the surface increases with ϑ_0 according to $\alpha = (\pi/2) - \vartheta_{0w}$. Since $P(\alpha)$ increases rapidly with decreasing α for $\alpha < \pi/2$ (Fig. 1), portions of the phase function that are larger than those for $\pi/2 < \alpha < \pi$ increasingly contribute to R as ϑ_0 increases. This is particularly evident for phase function T for which there is a strong contrast in the values of $P(\alpha)$ above and below $\alpha = \pi/2$. In the case of scattering by pure seawater (not shown), i.e. Rayleigh scattering, the analytical computation has a maximum error of $\lesssim 0.5\%$ for the largest value of ω_0 encountered for pure seawater (~ 0.3 near 400 nm).

It is seen that for $\omega_0 = 0.8$ there is a clear trend for the multiple scattering computations of $R(\vartheta_0)/R(0)$ to be above the single scattering results because, at each scattering event after the first, some photons can scatter through progressively smaller angles and still be redirected toward the surface. Since $P(\alpha)$ increases with decreasing α for $\alpha \lesssim \pi/2$, multiple scattering should increase $R(\vartheta_0)/R(0)$ above the single scattering result when photons can scatter a few times before being absorbed. Since phase function T is larger at very small scattering angles than KC (25% of the scattering events for T have $\alpha \leq 1°$, compared to 5% for KC), the effect is larger for KC. For larger values of ω_0, this effect disappears because photons scatter many times before reaching the surface and the information concerning the direction at which the photon entered the water becomes lost. Thus, for small ω_0 the exact value of $R(\vartheta_0)/R(0)$ should be close to that given by Eq. 1; however, as ω_0 increases, $R(\vartheta_0)/R(0)$ will initially increase above the single scattering value and then eventually decrease below it as $\omega_0 \rightarrow 1$.

The near-agreement between $R(\vartheta_0)/R(0)$ and that predicted by single scattering indicates that measurements of $\beta(\alpha)$ over the appropriate range of angles ($41° \lesssim \alpha \leq 180°$) can be used to predict the dependence of R on ϑ_0. However, measurement of $\beta(\alpha)$ over this entire range is not really necessary. Gordon (1976) showed that b_b could be accurately determined without knowing the full scattering function by fitting measurements of β at only three angles, $\alpha = 45°$, 90°, and 135°, to an analytic equation first used by Beardsley and Zaneveld (1969):

$$\beta(\alpha) = \frac{\beta(90°)}{(1 - e_f \cos \alpha)^4 (1 + e_b \cos \alpha)^4} \quad (2)$$

where e_f and e_b are adjustable parameters. The fits of phase functions T and KC to Eq. 2 are shown as solid lines in Fig. 3. The fit is excellent for scattering between about 40° and 160°, correctly reproducing the significant variation around 90°. Note, however, that Eq. 2 is a very poor approximation at scattering angles <25°–30° and is in error by a factor of 10^3 or more near 0°. For Rayleigh scattering (not shown), Eq. 2 provides an excellent fit for all scattering angles. Computation of $R(\vartheta_0)/R(0)$ using Eq. 1 (single scattering) and the fits of the phase function to Eq. 2 (Fig. 3) agree with those using Eq. 1 and the actual phase functions with an error of less than ±2%. On this basis I conclude that measurement of $\beta(45°)$, $\beta(90°$, and $\beta(135°)$ are sufficient to describe the variation of R with ϑ_0.

It is of interest to know whether the process above can be inverted, i.e. given $R(\vartheta_0)/R(0)$ is it possible to recover any information concerning $\beta(\alpha)$? To examine this question, I have assumed that $R(\vartheta_0)$ is measured (in this case *simulated* by Monte Carlo) at 10° increments from 0° to 89°. This "data" is then fit to Eq. 1, with $\beta(\alpha)$ given by Eq. 2, by nonlinear least-squares to determine the unknown parameters e_f and e_b. This procedure can only yield an estimate of $P(\alpha)/P(90°)$ over the angular range $41° \lesssim \alpha \leq 180°$. The value of $P(90°)$ or $\beta(90°)$ must be estimated independently. Gordon et al. (1975) have shown that $R(0) \propto b_b/(a + b_b)$. Thus, I have determined $P(90°)$ by requiring that the inverted and true phase functions have the same value of b_b. Then both phase functions will yield the same $R(0)$. The resulting "inverted" phase functions for $\omega_0 = 0.8$ (solid curves) and 0.9 (dashed curves) are presented in Fig. 4 and demonstrate that in this restricted case, i.e. measurement of $R(\vartheta_0)$ for the full domain of ϑ_0 and the absence of an atmosphere and surface waves, an estimate of the general shape of $\beta(\alpha)$ for $60°–70° \lesssim \alpha \leq 180°$ can be retrieved from measurements of $R(\vartheta_0)$.

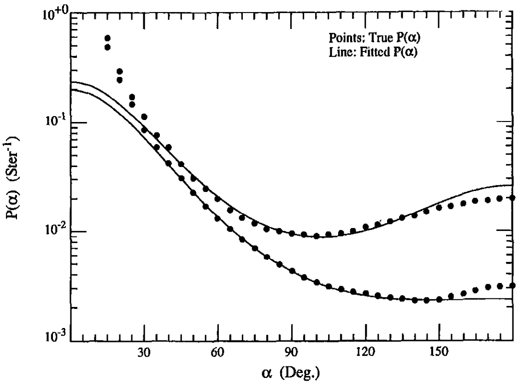

Fig. 3. Phase functions KC (upper) and T (lower). Lines are fits with Eq. 2 using $P(\alpha)$ evaluated at $\alpha =$ 45°, 90°, and 135°.

The conclusions here are based on Monte Carlo simulations of an idealized ocean, i.e. a flat, homogeneous ocean with $\omega_0 \leq 0.9$ in the absence of the atmosphere. How applicable are they to a real ocean? From the analysis of Gordon (1987) one expects $\omega_0 \leq 0.9$ in natural waters except in intense plankton blooms (and then only in the green part of the visible spectrum) or regions with a high concentration of nonabsorbing suspended particles (e.g. white sand) in the water. The principal effect of the atmosphere is to add a quasi-diffuse component (skylight) to the irradiance incident on the sea surface. Similarly, replacing the flat surface with a rough surface renders the incident light field *beneath* the surface more diffuse. Because of the diffuse nature of the downwelling irradiance beneath the surface, these observations suggest that Eq. 1 cannot be applied *directly* to a rough ocean illuminated by the sun and the sky; however, it is possible to characterize the surface and the incident radiance distributions in such a manner that Eq. 1 is still useful.

The key to such a characterization lies in a recent set of Monte Carlo simulations (Jerome et al. 1988) showing that $R(\vartheta_0)/R(0) = 1/\cos \vartheta_{0w}$. Plotting $R(\vartheta_0)/R(0)$ for the present computations against $1/\cos \vartheta_{0w}$ re-

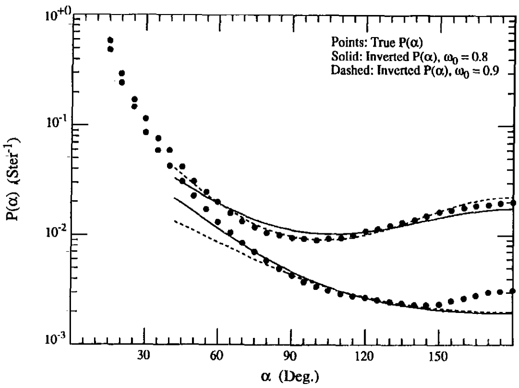

Fig. 4. Phase functions KC (upper) and T (lower). Solid and dashed curves are the result of inverting Eq. 1 for $\omega_0 = 0.8$ and 0.9.

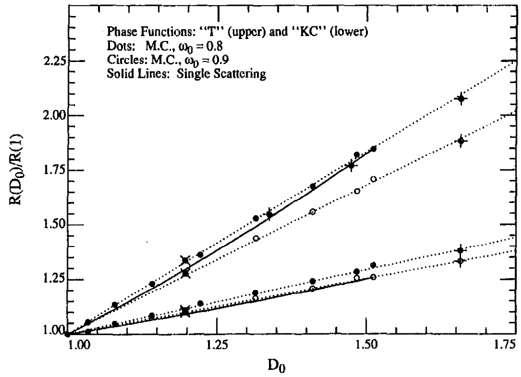

Fig. 5. R as a function of D_0. Symbols with a + are for a rough surface; those with an × are for a diffuse incident irradiance. Solid curves are the result of the single scattering approximation for a flat ocean with no atmosphere (Eq. 1).

sults in a straight line with a slope k which depends mostly on the phase function but somewhat on ω_0. The value of k is approximately unity when the same phase function used by Jerome et al. (1988) is used. The values of k for phase functions KC and T are ~0.85 and 1.15. To try to extend this linear dependence on the illumination geometry, it is noted that when the illumination incident on a flat ocean is in the form of a parallel beam from the sun, the downwelling distribution function—the downwelling scalar irradiance divided by the downwelling irradiance (Preisendorfer 1961)—just beneath the surface in the *absence* of scattering, D_0, is exactly $1/\cos\vartheta_{0w}$. In cases with parallel beam illumination of a flat ocean, as well as situations with more complex illumination, D_0 has been used previously to remove the geometrical properties of the incident light field from the downwelling irradiance attenuation coefficient (Gordon 1989; Gordon et al. 1975), in effect normalizing the coefficient to that which would be measured in the absence of the atmosphere with the sun at zenith. Thus, it is natural to ask if D_0 might be used to simplify the analysis of $R(\vartheta_0)/R(0)$ in more complex situations.

Clearly, the simplest procedure for trying to extend the analysis to more complex situations is to plot $R(\vartheta_0)/R(0)$ [or equivalently, $R(D_0)/R(1)$] as a function of D_0, i.e. to replace the $1/\cos\vartheta_{0w}$ used by Jerome et al. (1988) by the more general D_0. To test the applicability of such a procedure, I have carried out Monte Carlo simulations for two other incident distributions (beneath the surface): a distribution resulting from an incident collimated beam combined with wind-induced surface roughness and one resulting from uniform radiance incident on a flat surface. For the wind-ruffled surface case, the surface was described by a Cox and Munk (1954) slope distribution characteristic of a 7.2 m s^{-1} wind speed and independent of wind direction. The computations were carried out for $\vartheta_0 = 40°$, 60°, 70°, and 80°. In contrast to the larger values of ϑ_0 the 40° and 60° calculations showed only a small (<2%) increase in $R(\vartheta_0)/R(0)$ over the flat ocean, so the effect of surface roughness on the validity of $R(\vartheta_0)/R(0)$ computed from Eq. 1 is small for $\vartheta_0 \lesssim 60°$. A separate computation was carried out at each value of ϑ_0 to determine D_0 for the rough surface. The resulting values of $R(D_0)/R(1)$ for $\vartheta_0 =$ 70° and 80° are plotted against D_0 in Fig. 5 (symbols with a +) along with the results taken from Fig. 2. Note that in each case, the simulated values of $R(D_0)/R(1)$ are linearly related to D_0 and that the rough ocean cases fall with excellent accuracy along the same lines as their flat ocean counterparts.

The simulations carried out with totally diffuse incident irradiance falling on a flat ocean (symbols with an × in Fig. 5) show that the same linear relationship is satisfied in this case as well. The results of these computations show that for a given ω_0 and scattering phase function

$$R(D_0) = k(D_0 - 1)R(1) + R(1),$$

i.e. the variation of R with the incident illumination and the surface roughness can be completely explained through their effect on D_0. (An accurate scheme for estimating D_0 from simple irradiance and wind speed measurements is given by Gordon 1989.) The parameter k depends mostly on the scattering phase function, and, for $\omega_0 \lesssim 0.9$, it can be approximated with single scattering theory.

My main conclusion here—that single scattering can be used to characterize the variation of R with the incident radiance distribution—should be applicable to a real ocean if highly reflective waters ($\omega_0 > 0.9$) are avoided.

References

BEARDSLEY, G. F., AND J. R. V. ZANEVELD. 1969. Theoretical dependence of the near-asymptotic apparent optical properties on the inherent optical properties of sea water. J. Opt. Soc. Am. **59:** 373–377.

COX, C., AND W. MUNK. 1954. Measurements of the roughness of the sea surface from photographs of the sun's glitter. J. Opt. Soc. Am. **44:** 838–850.

GORDON, H. R. 1973. Simple calculation of the diffuse reflectance of the ocean. Appl. Opt. **12:** 2803–2804.

———. 1976. Radiative transfer in the ocean: A method for determination of absorption and scattering properties. Appl. Opt. **15:** 2611–2613.

———. 1985. Ship perturbation of irradiance measurements at sea 1: Monte Carlo simulations. Appl. Opt. **23:** 4172–4182.

———. 1987. A bio-optical model describing the distribution of irradiance at the sea surface resulting from a point source imbedded in the ocean. Appl. Opt. **26:** 4133–4148.

———. 1989. Can the Lambert-Beer law be applied to the diffuse attenuation coefficient of ocean water? Limnol. Oceanogr. **34:** 1389–1409.

———, O. B. BROWN, AND M. M. JACOBS. 1975. Computed relationships between the inherent and apparent optical properties of a flat homogeneous ocean. Appl. Opt. **14:** 417–427.

———, AND A. Y. MOREL. 1983. Remote assessment of ocean color for interpretation of satellite visible imagery: A review. Springer.

JERLOV, N. G. 1976. Marine optics, 2nd ed. Elsevier.

JEROME, J. H., R. P. BUKATA, AND J. E. BURTON. 1988. Utilizing the components of vector irradiance to estimate the scalar irradiance in natural waters. Appl. Opt. **27:** 4012–4018.

KIRK, J. T. O. 1984. Dependence of relationship between inherent and apparent optical properties of water on solar altitude. Limnol. Oceanogr. **29:** 350–356.

KULLENBERG, G. 1968. Scattering of light by Sargasso Sea water. Deep Sea Res. **15:** 423–432.

MOREL, A., AND L. PRIEUR. 1977. Analysis of variations in ocean color. Limnol. Oceanogr. **22:** 709–722.

PETZOLD, T. J. 1972. Volume scattering functions for selected natural waters. Scripps Inst. Oceanogr. Visibility Lab. SIO Ref. 72-78.

PREISENDORFER, R. W. 1961. Application of radiative transfer theory to light measurements in the sea. Int. Union Geod. Geophys. Monogr. 10, p. 11–30.